Hyperviscosity in the BLAST High Order Finite Element Hydrodynamics Code

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Abstract: Hyperviscosity has proven to be an effective artificial viscosity in several hydrodynamic codes [1]. We adapt hyperviscosity for the high-order finite elemenent curvilinear hydrodynamics code BLAST [2]. The FEM formulation of the operators used in hyperviscosity are shown to converge under mesh refinement. Hyperviscosity can reduce wall heating when used as a limiter and vanishes on smooth problems.

Hyperviscosity

- In order to effectively represent shocks (discontinuities) with the Euler equations it is necessary to use **artificial viscosity**
- Hyperviscosity is a form of artificial viscosity that uses higher-order gradients to "focus" viscosity on the shock region
- Hypervisocsity modifies the viscosity tensor, Q, in the momentum and energy equations of Euler's equations:

$$\textbf{Momentum Conservation:} \quad \rho\frac{\mathrm{D}\mathbf{v}}{\mathrm{d}t} = -\nabla p + \nabla\cdot\mathbf{Q}$$

$$\textbf{Energy Conservation:} \quad \rho\frac{\mathrm{D}e}{\mathrm{d}t} = -p\left(\nabla\cdot\mathbf{v}\right) + \mathbf{Q}:\nabla\mathbf{v}$$

- Hyperviscosity uses the s field, which is the Frobenius Norm of the symmetrized velocity gradient:

$$\epsilon = \tfrac{1}{2}\left(\nabla v + v\nabla\right)$$

$$s = \sqrt{(\epsilon:\epsilon)}$$

- The bulk and shear viscosity terms both have a coefficient that depends on the Laplacian of the s field,:

$$\mu^* = C_\mu\rho\overline{|\nabla^2(sL^4)|}$$

$$\beta^* = C_\beta\rho\overline{|\nabla^2(\nabla\cdot u)|L^4 H(\nabla\cdot u)}$$

The overbar represents a smoothing operation and L is a grid-dependent length scale.

BLAST

The main features of the BLAST hydro code are [2]:

- Supports 2D (triangles, quads) and 3D (tets, hexes) unstructured curvilinear meshes.
- High order field representations.
- Exact discrete energy conservation by construction.
- Reduces to classical staggered-grid hydro under simplifying assumptions.

Laplacian Operator in Discrete Finite Element Space

In the finite element method operators are written in a weak-variational form. The Laplacian of the s-field is:

$$x = \Delta s \quad\rightarrow\quad \int x\phi_i = \int \Delta s\phi_i$$

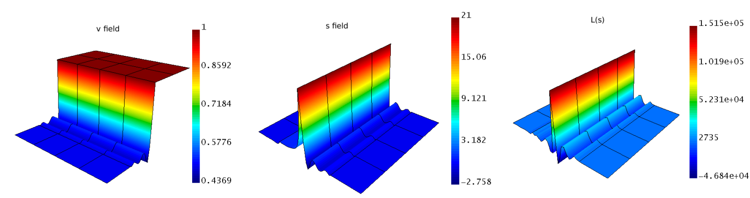
Using integration by parts the right hand side becomes:

$$\int \Delta s\phi_i = -\int_\Omega \nabla s\cdot\nabla\phi_i + \int_{\partial\Omega}\nabla s\cdot n\phi_i$$

After replacing s and x with a trial function ϕ_j and inserting some FEM terminology the system system looks like:

$$\mathbf{M}\alpha = \mathbf{S}\beta$$

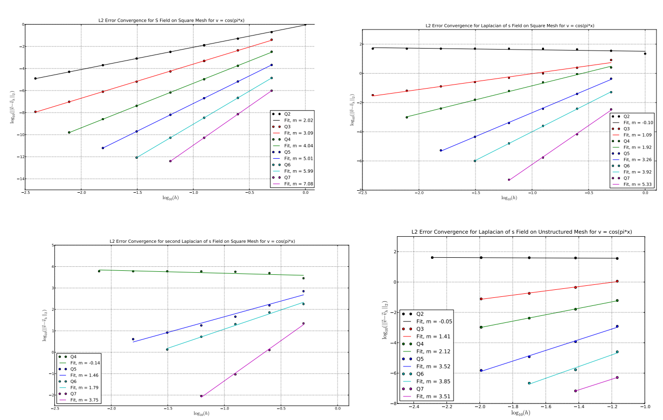
Where $\mathbf{M}$ and $\mathbf{S}$ are the mass and stiffness matrices repsectively. Multiple Laplacian operations can be obtained by taking higher powers of $\mathbf{M}^{-1}\mathbf{S}$.



Discontinuous Velocity Field for Q6 with two mesh refines

Convergence of Discrete Laplacian

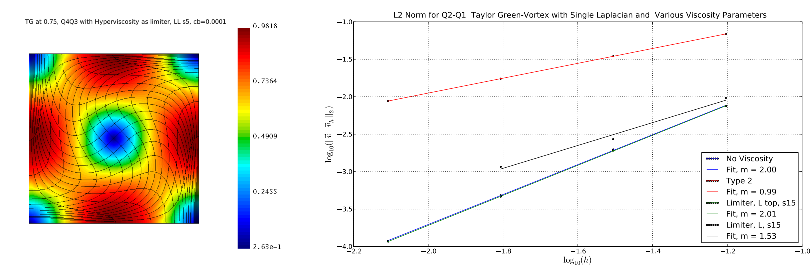
- The s field, Δs and $\Delta^2 s$ converge to the analytic solution under mesh refinement on structured and unstructured grids
- The same convergence rates are obtained when the Lumped Mass Matrix is used
- For convergence, the order of the basis function must be greater than or equal to the number of gradient operations: Q_k for v with $\nabla^n v$ where $k >= n$.



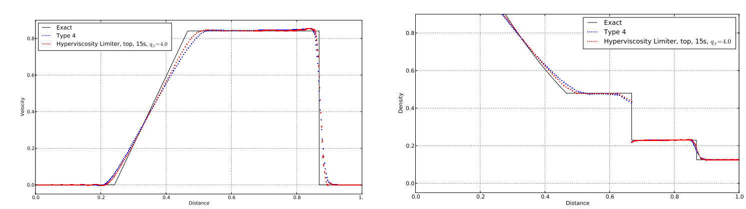
Hyperviscosity as a Limiter in Blast

Artificial viscosity methods based on a single spatial gradient can give too much viscosity in non-schock regions. Hyperviscosity will yield much smaller viscosity in these regions, making it ideal for use as a limiter:

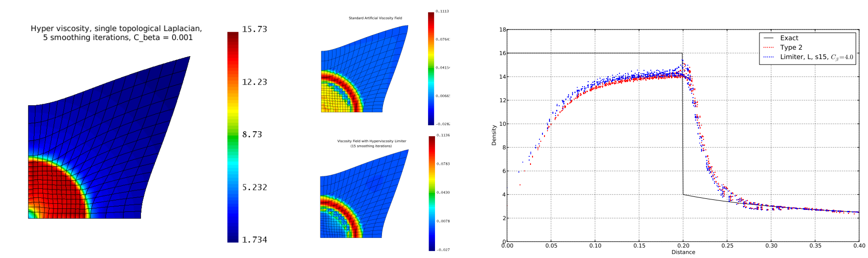
$$\mu = min(\mu, \mu_{hyper})$$



Velocity field for the Taylor-Green problem with hyperviscosity limiter at $t = 0.75s$ and convergence plot for different viscosity parameters



Comparison of velocity vs. distance and density vs. distance for Sod problem at $t = 0.2$ with the Q2-Q1 method



Density and viscosity fields for the Noh problem at $t = 0.6$ with the Q2-Q1 method hyperviscosity limiter and a comparison of density fields for the Noh problem

Conclusion

The operations required to form the hyperviscosity coefficients can be performed in the higher-order finite element space. When tuned correctly, hyperviscosity effectively reduces viscous heating in non-shock regions of the problem and does not overdamp shocks.

[1] A. W. Cook, W. H. Cabot, Hyperviscosity for shock-turbulence interactions, Journal of Computational Physics, Volume 203, Issue 2.

[2] V. A. Dobrev, Tz. V. Kolev, and R. N. Rieben. High order curvilinear finite element methods for Lagrangian hydrodynamics. SIAM J. Sci. Comp., 2012. to appear, also available as LLNL technical report LLNL-JRNL-516394.

This work performed under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory under Contract DE-AC52-07NA27344.

LLNL-POST-570172