Boundary and Interface Fitting of High-Order Meshes

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CASC
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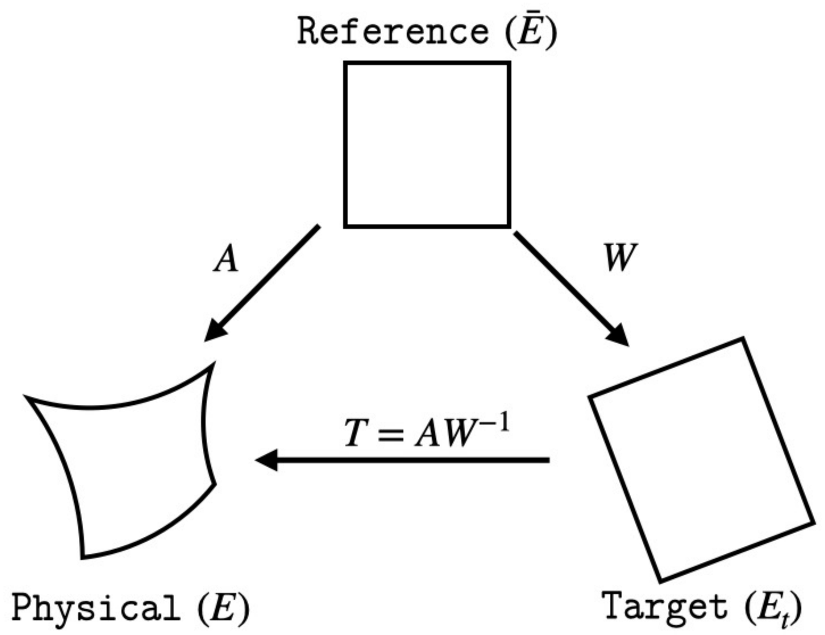
Overview

- Target Matrix Optimization Paradigm (TMOP) for r-adaptivity
  - Tangential relaxation and interface fitting
- Recent developments
  - Change in formulation
  - Marking to better define material interface for fitting
  - Combination of geometric primitives to define complicated level-sets
  - Use of background mesh to define the fitting function
  - Boundary fitting

Target Matrix Optimization Paradigm (TMOP)

- Define a transformation W for every element in the mesh.
- Use W along with the Jacobian of the transformation, A, of the physical element in the mesh to define T.
- Quality metric $\mu(T)$ measures the deviation between the active and target Jacobian transformation.



- Using the quality metric and the Jacobian transformation T, the TMOP objective function is defined as:

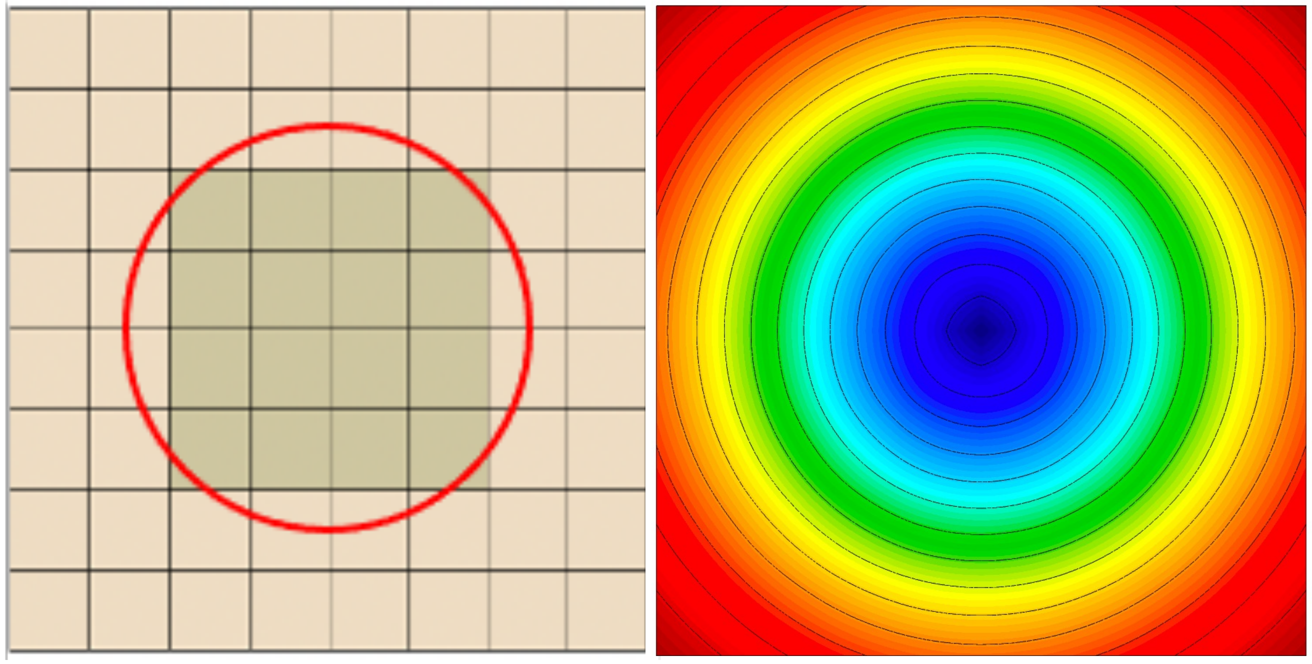
$$F_\mu(\mathbf{x}) = \sum_{E(\mathbf{x}_E)} \int_{E_t} \mu(T(\mathbf{x}))\, d\mathbf{x}_t$$

- r-adaptivity - $F(\mathbf{x})$ is minimized using a technique such as the Newton's method to optimize the mesh.

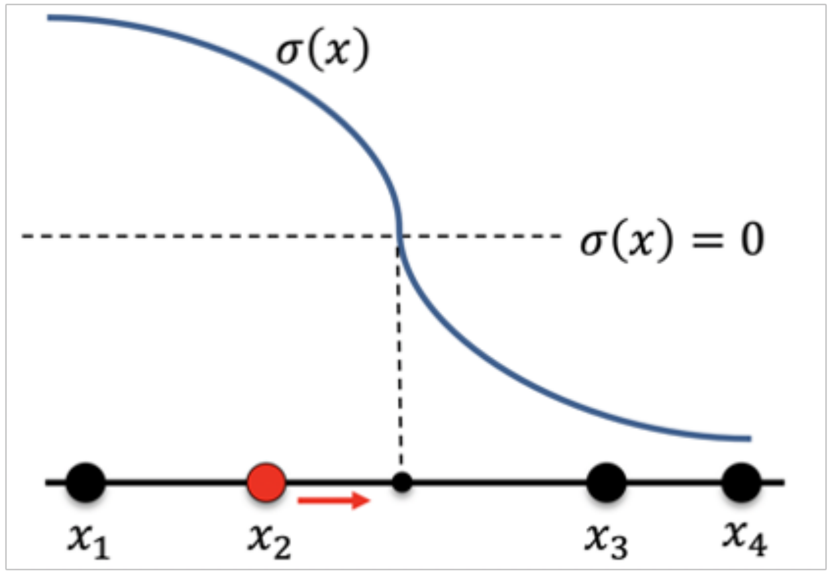
Old formulation - Tangential Relaxation and Interface Fitting

- The surface of interest is given as a discrete function ($\sigma(\mathbf{x})$).
- Penalty formulation (quality / fitting tradeoff).

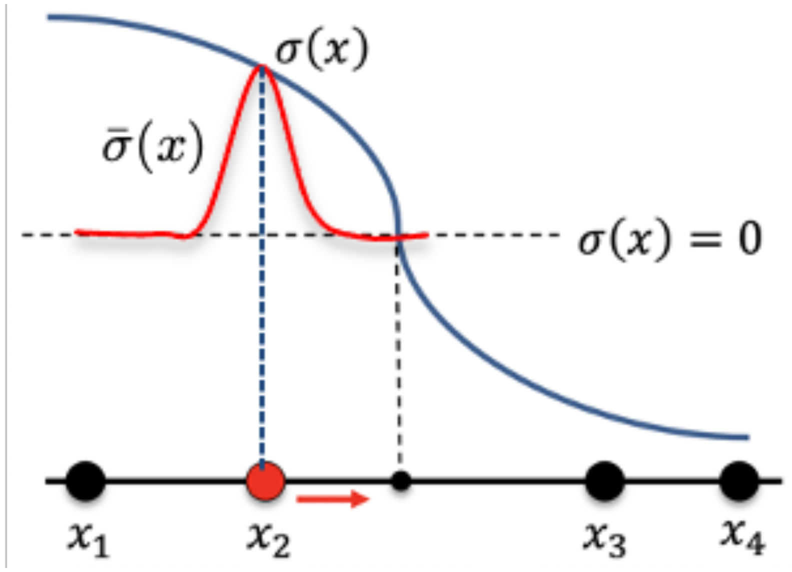
$$F(x) = F_\mu + w_\sigma \int_\Omega \bar{\sigma}(x)^2$$



Multimaterial mesh with target level set

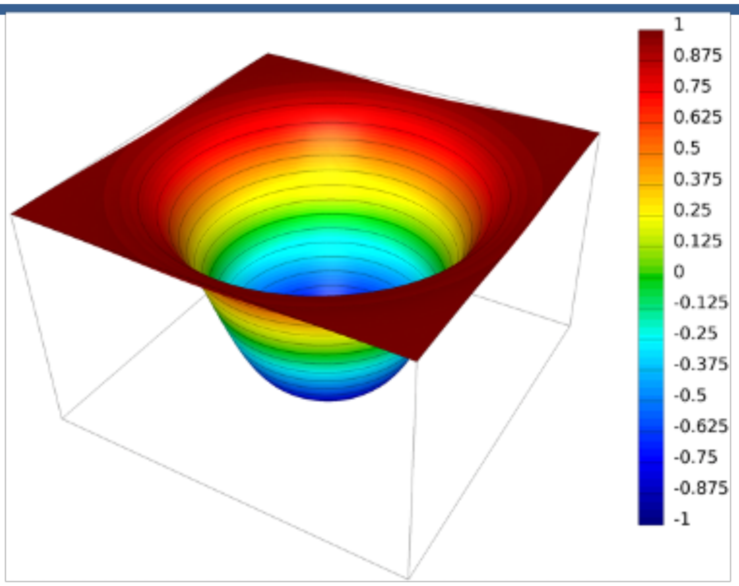


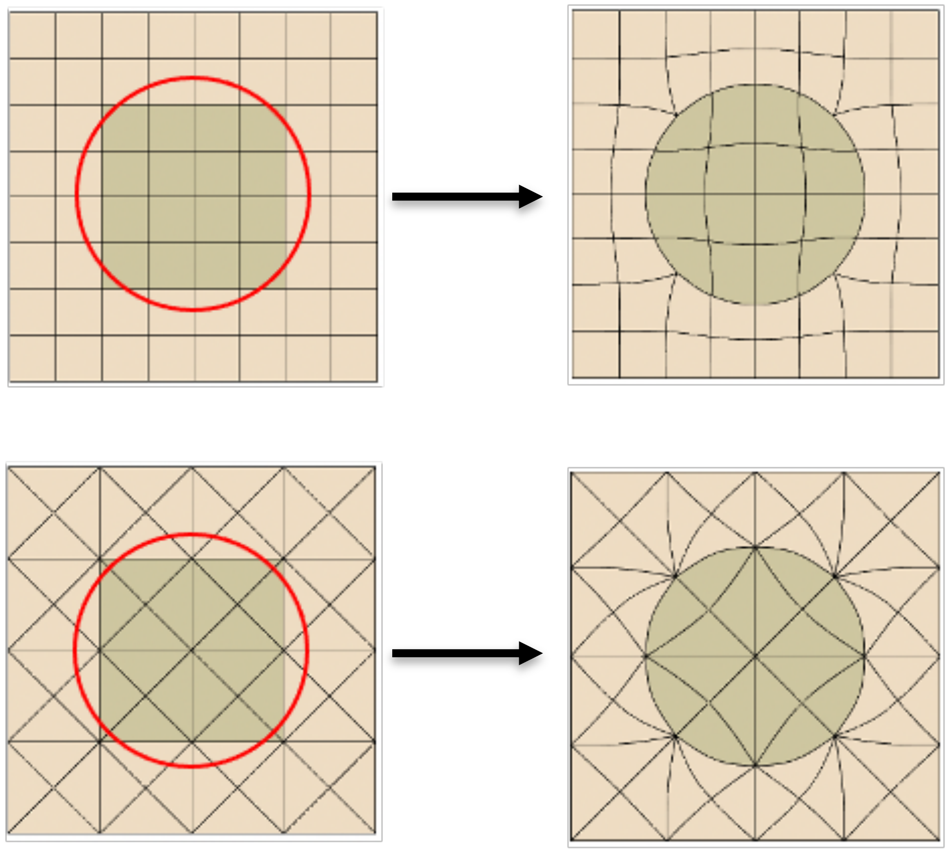
$$\bar{\sigma}_i = \begin{cases} \sigma_i & \text{if } i \in \mathcal{S}, \\ 0 & \text{otherwise.} \end{cases}$$

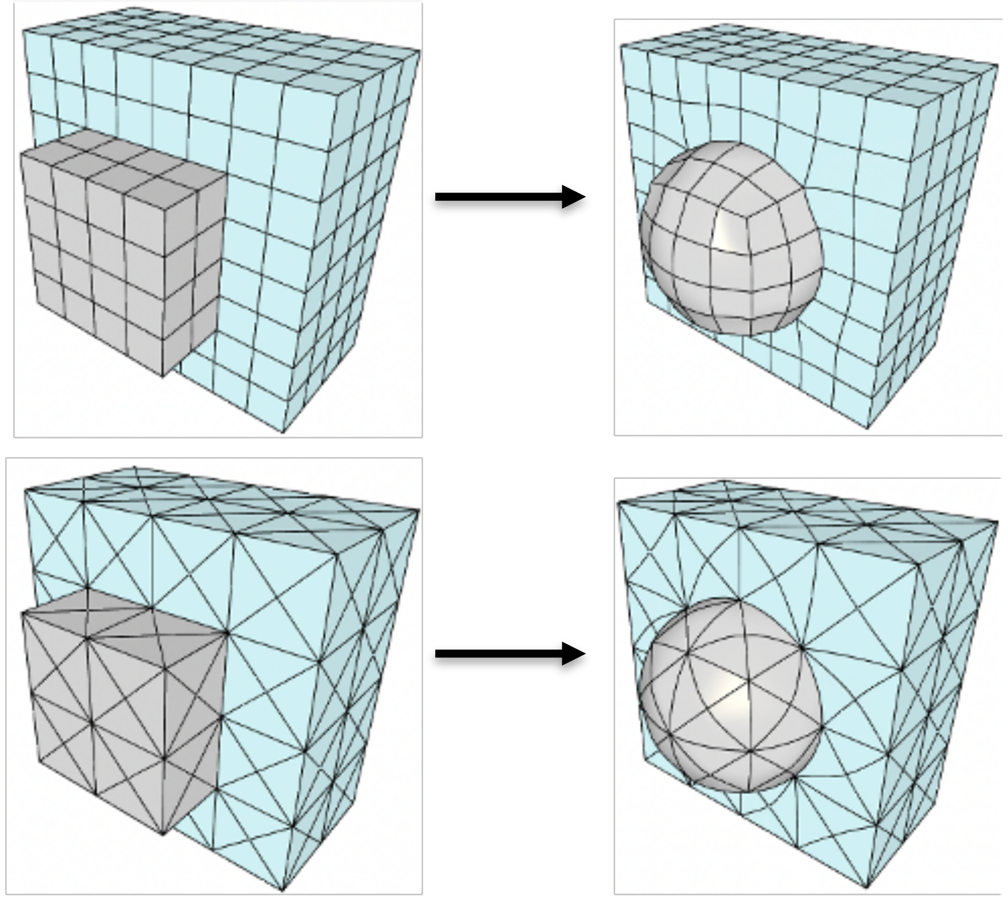


Old formulation - Tangential Relaxation and Interface Fitting

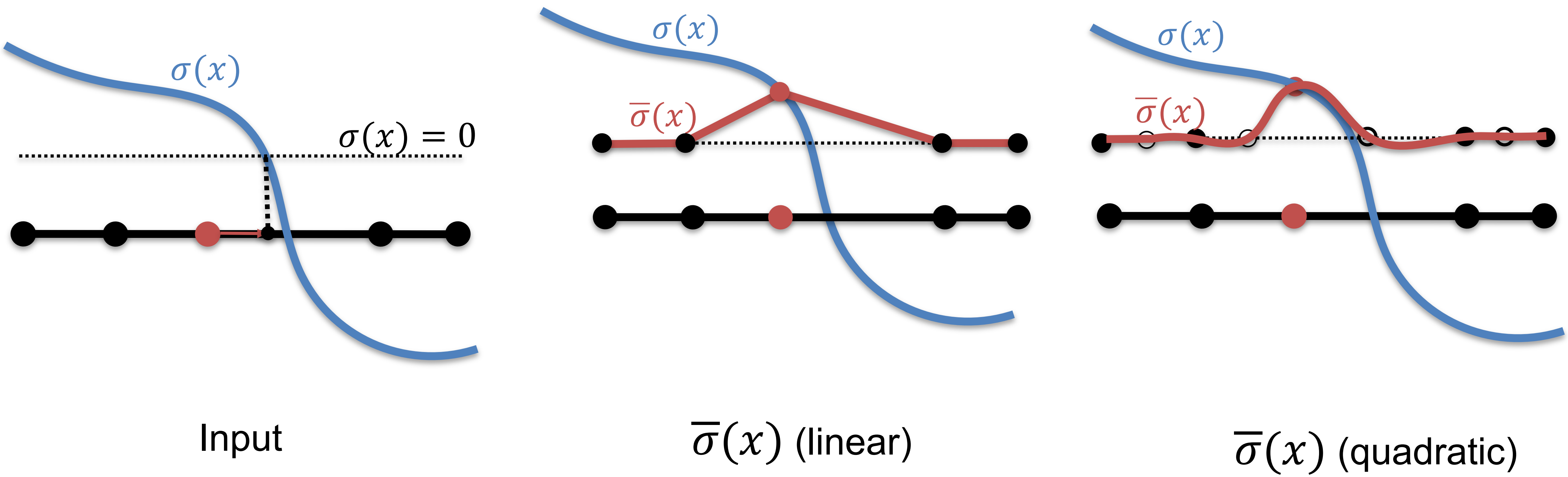
- In all cases σ is a discrete FE function.
  - Ball at the center of the domain.







Changes to Problem Formulation

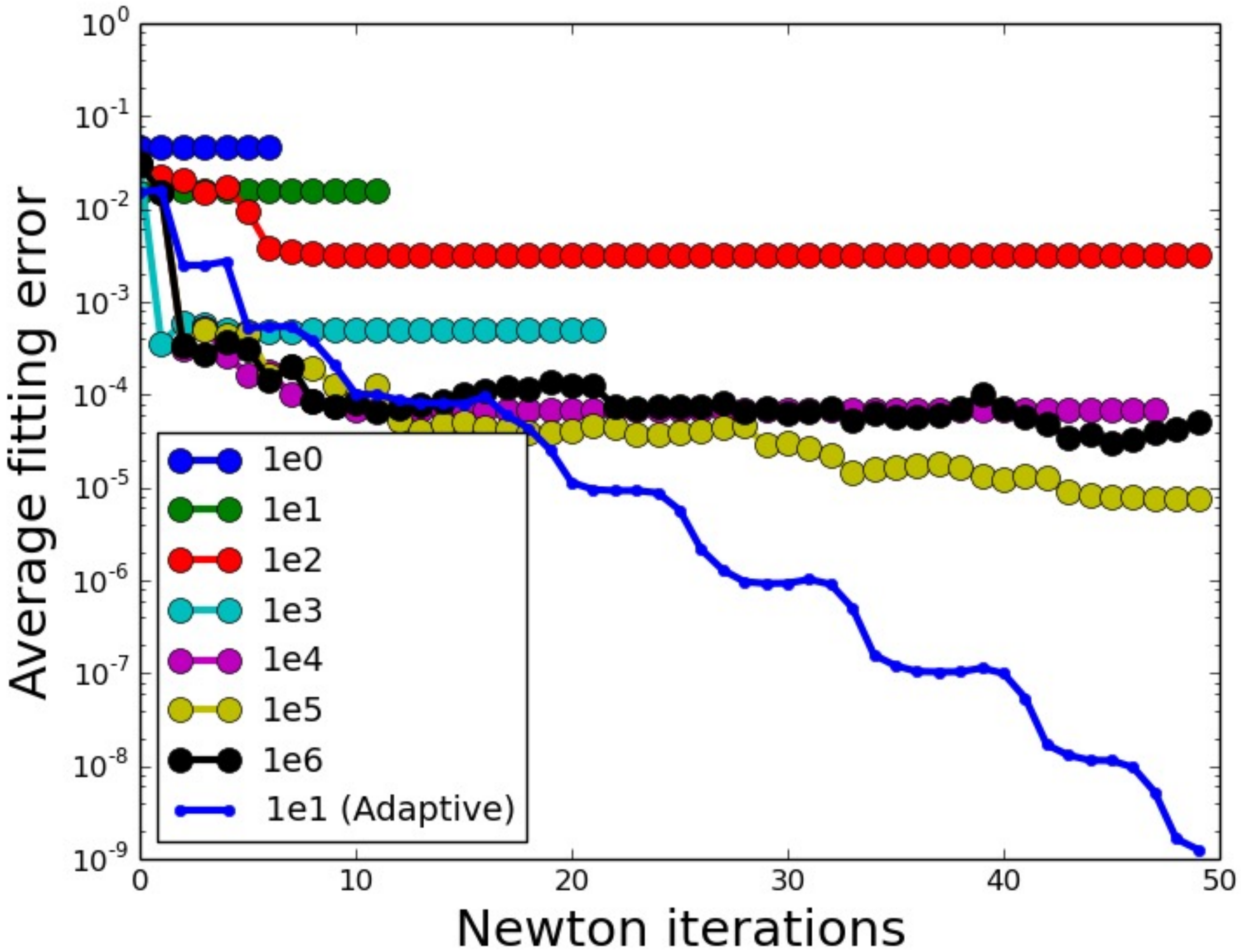


- We now use σ instead of $\overline{\sigma}$, and introduce a disrete mask ($\mathcal{M}$) that zeros out the contribution from/to all degrees of freedom that are not marked.

Changes to Problem Formulation

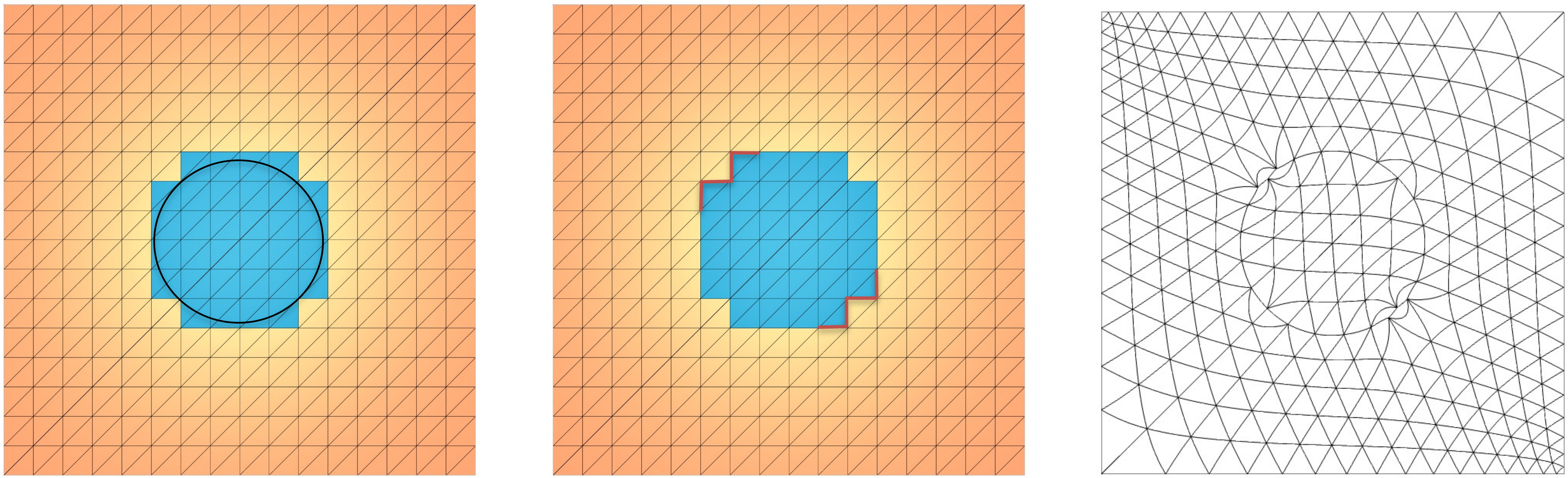
- The weight for interface fitting is currently somewhat heuristic.

$$F(\mathbf{x}) = F_\mu(\mathbf{x}) + w_\sigma \underbrace{\int_\Omega \sigma(\mathbf{x})^2}_{F_\sigma(\mathbf{x})}$$



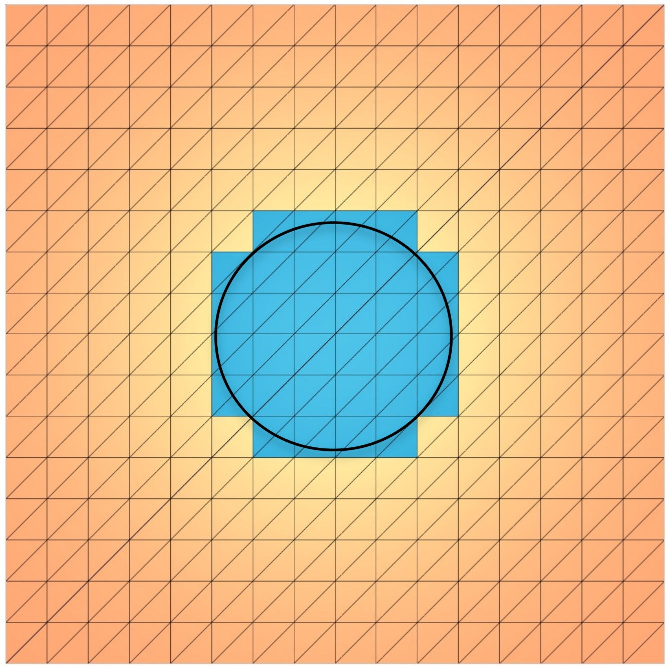
Marking strategy

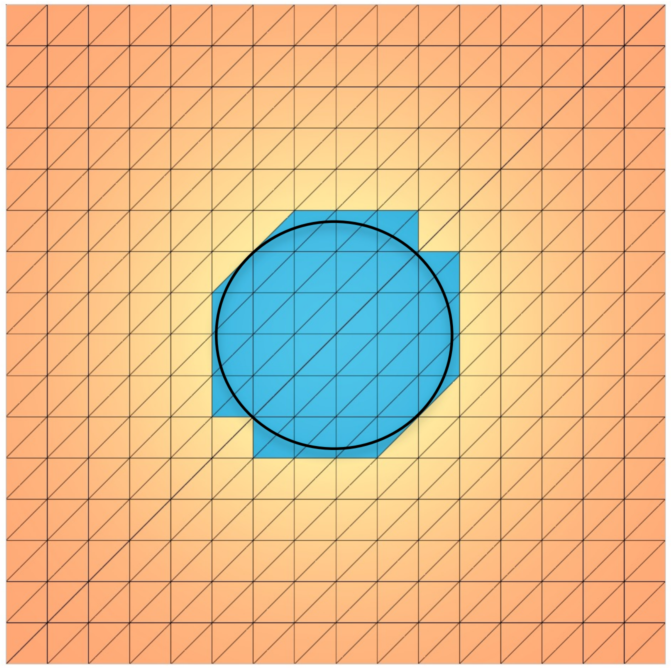
- Marking for interface fitting is not trivial and impacts the quality of the final mesh.

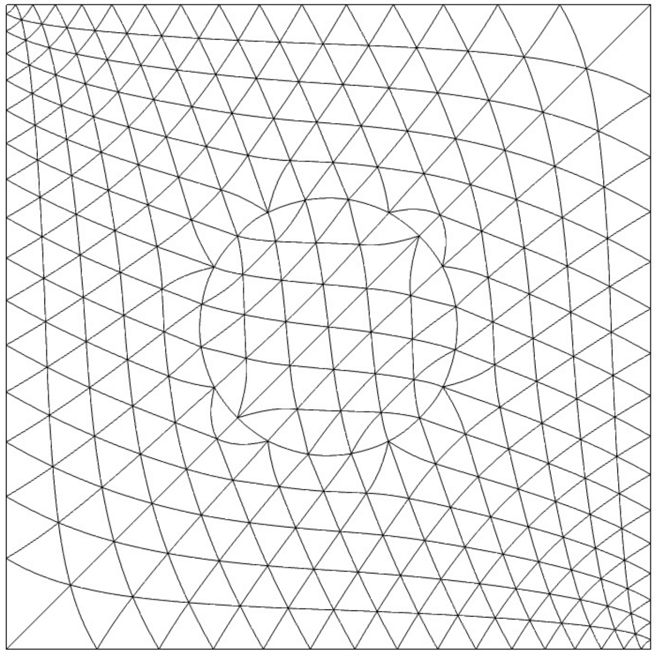


Marking strategy

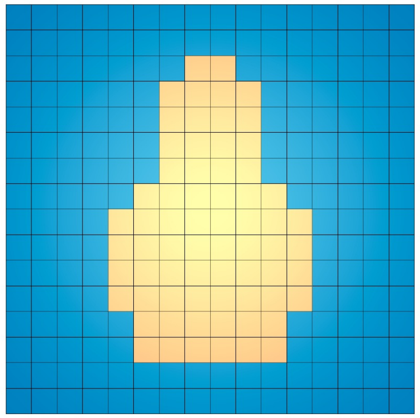
- Two pass approach
  - Mark nodes as usual
  - Change marked nodes for an element if all but 1 of its faces is marked.

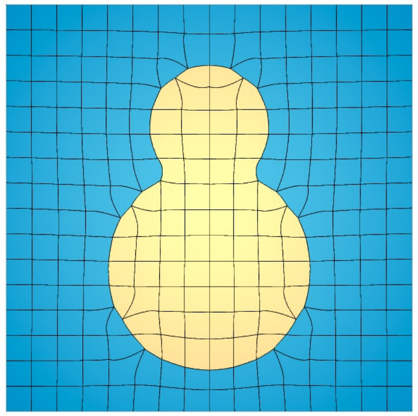






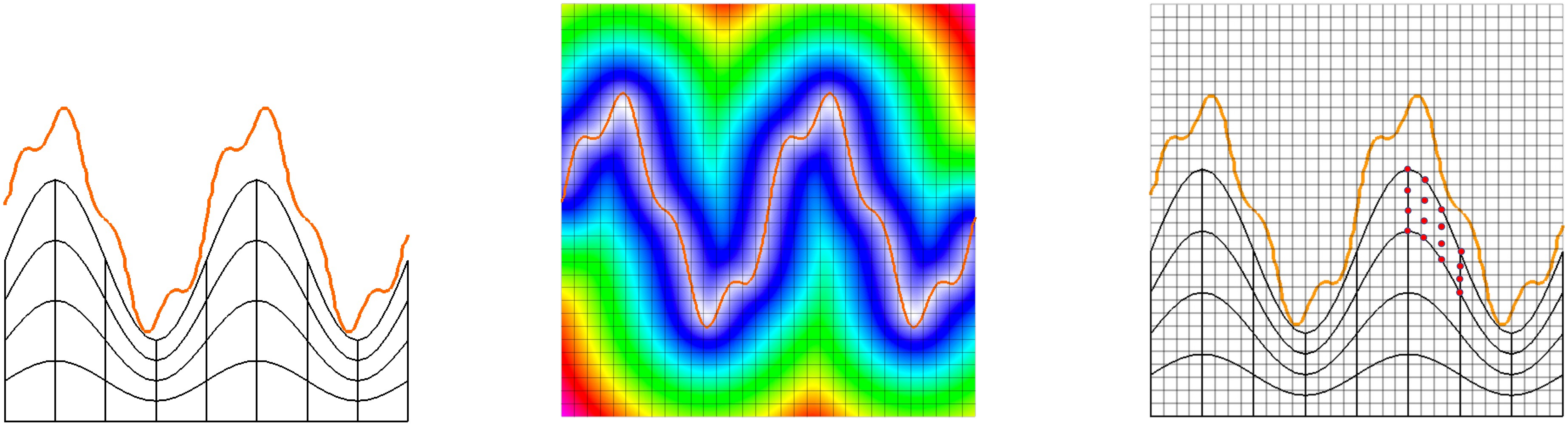
- What if two faces of a Quadrilateral or a Tetrahedron are marked?
  - Split such elements?
  - Increase interface fitting weight around such elements?





Use of Background Mesh

- In the existing framework, we cannot fit the boundary nodes to the target level set if it is outside the domain covered by the mesh.



Current mesh and target level set

Level set on a background mesh

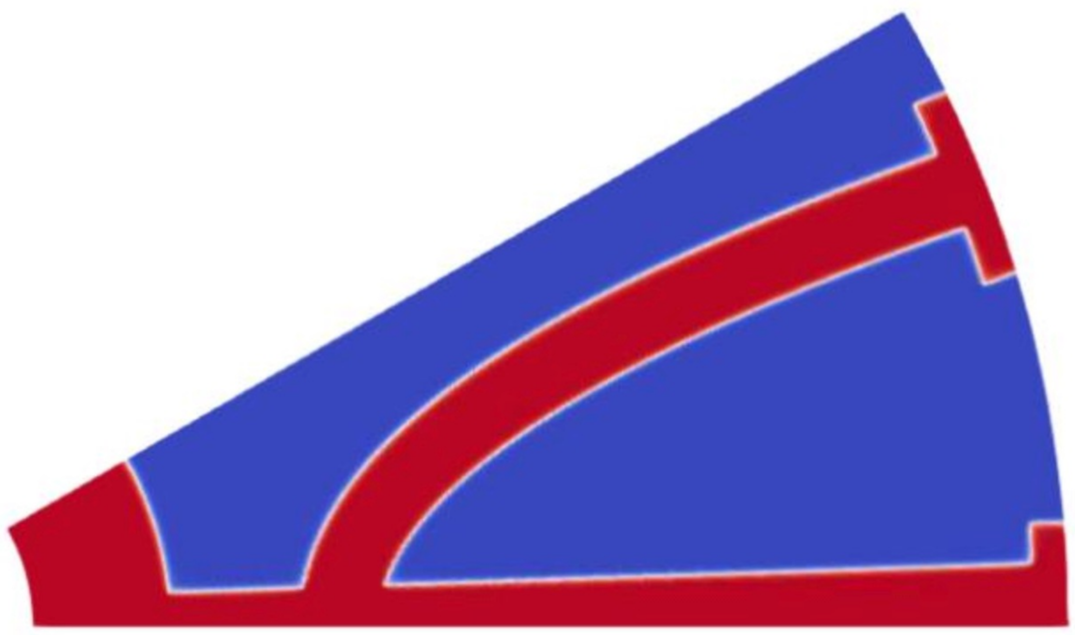
Relevant information is transferred from background mesh current mesh

- We have introduced the capability to use a background mesh to define the level set and communicate relevant information from background mesh to current mesh during mesh optimization.

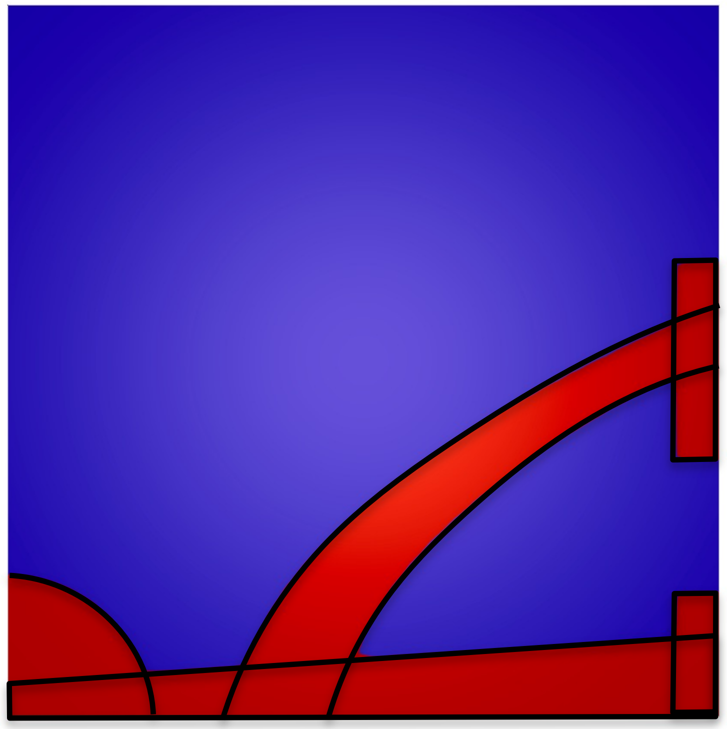
- Background meshes are also crucial if the current mesh is not fine enough to accurately capture the sharp target level set.

Constructing Target Curvilinear Shapes using Geometric Primitives

- Using ideas from constructive solid geometry (CGS), we have enabled use of multiple level set functions to define complicated curvilinear domains.
- Function for each geometric primitive tells wether the given point is inside (+1) or outside (-1).



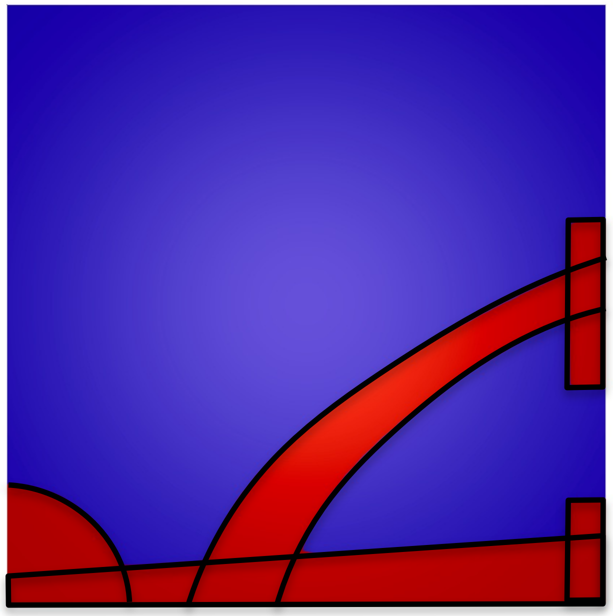
Reactor design problem



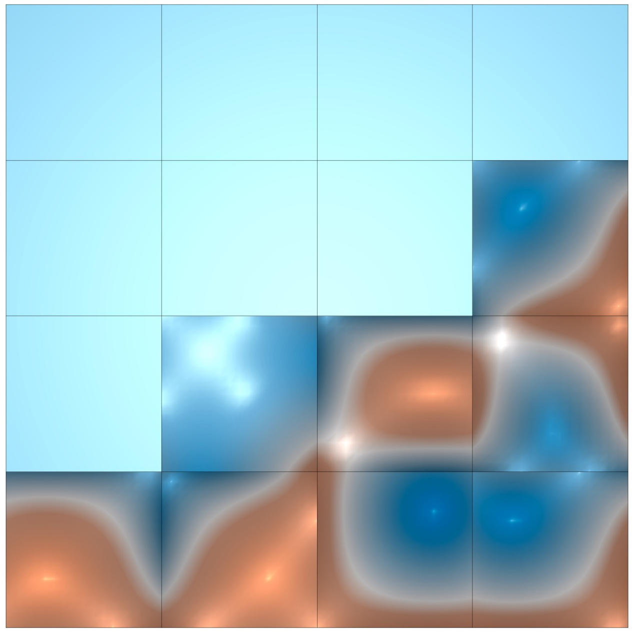
Multimaterial interface defined using geometric primitives: circle, parabola, rectangle, trapezium

Constructing Appropriate Background Mesh for Non-Trivial Level Sets

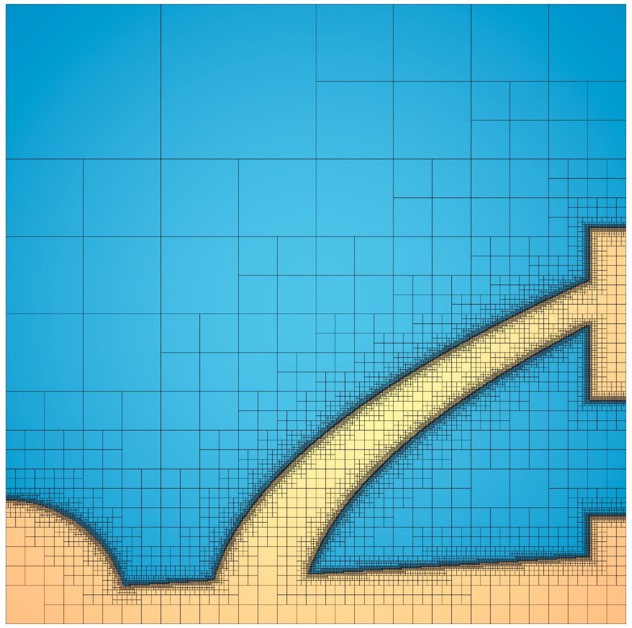
- To define an appropriate background mesh, we start with a coarse mesh and use AMR to increase resolution around the zero level set.
  - Determine distance from the 0 level set to use for interface fitting.



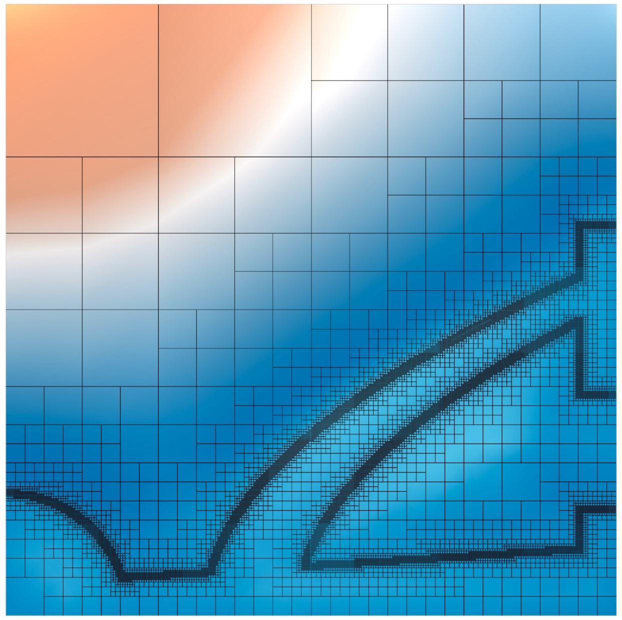
Target domain



Level set on coarse background mesh

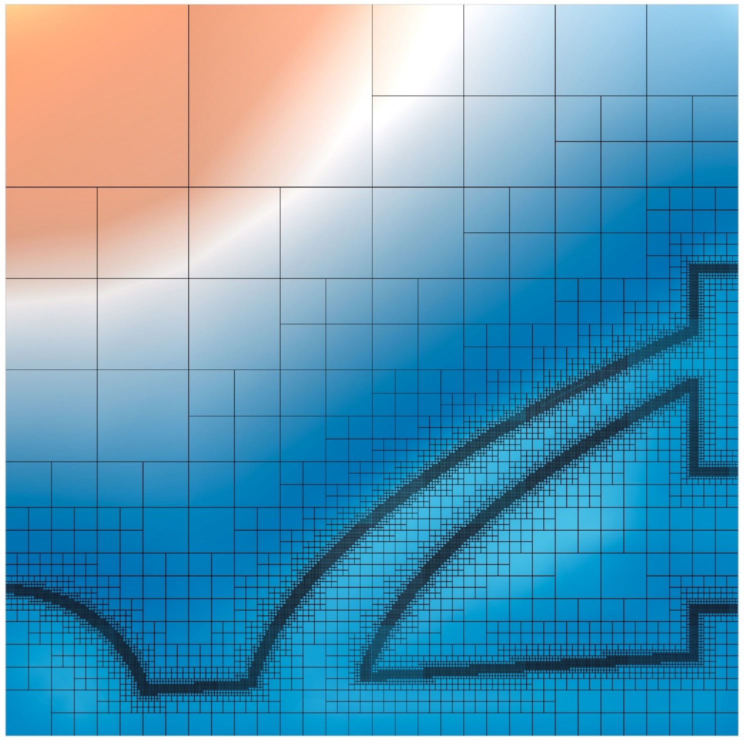


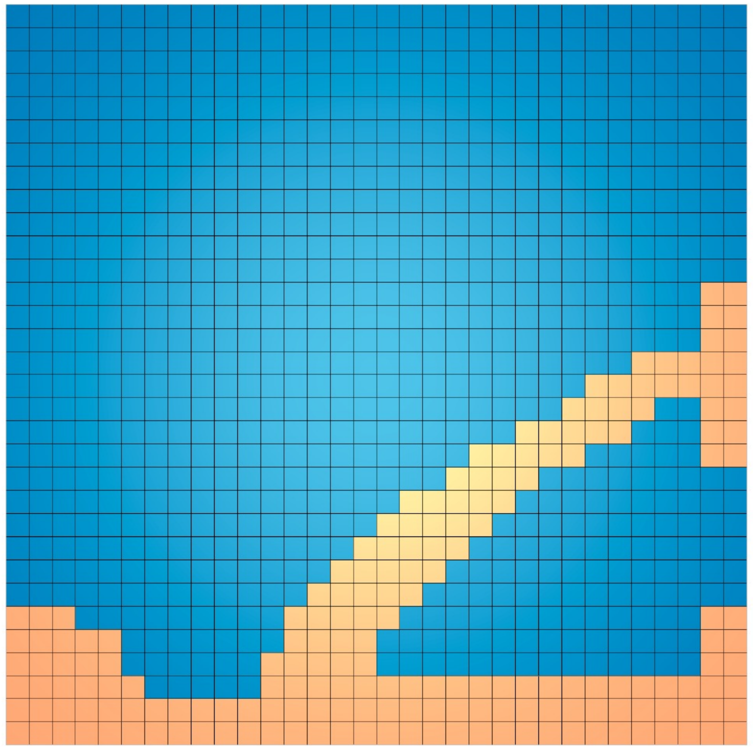
Level set on AMR mesh

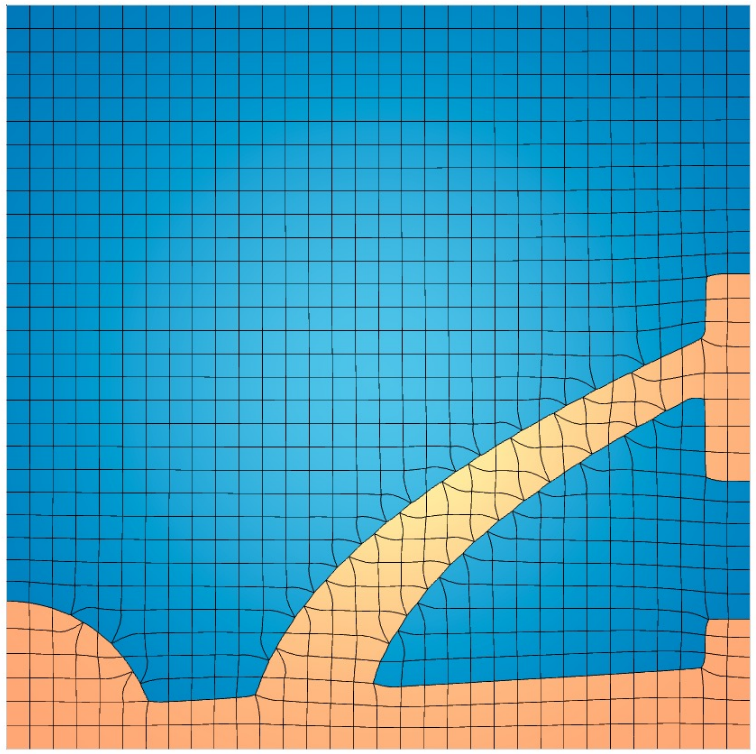


Distance function, used for $\sigma(\mathbf{x})$

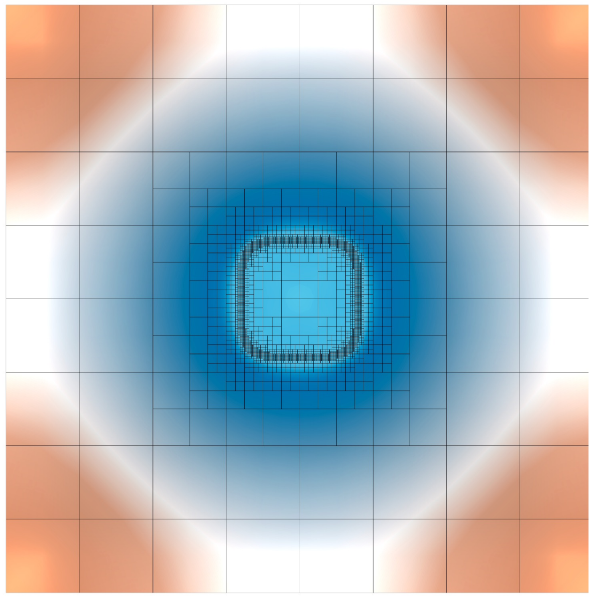
Interface and Boundary Fitting for Reactor Design


Distance function, used for $\sigma(\mathbf{x})$

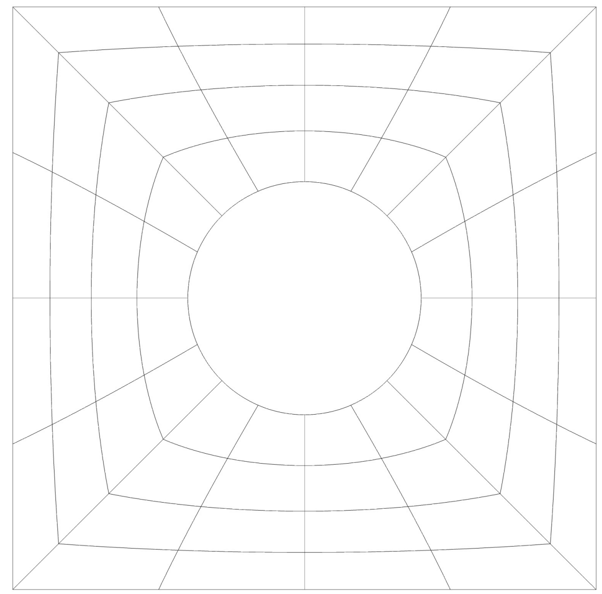

Input mesh with marked interface


Optimized mesh

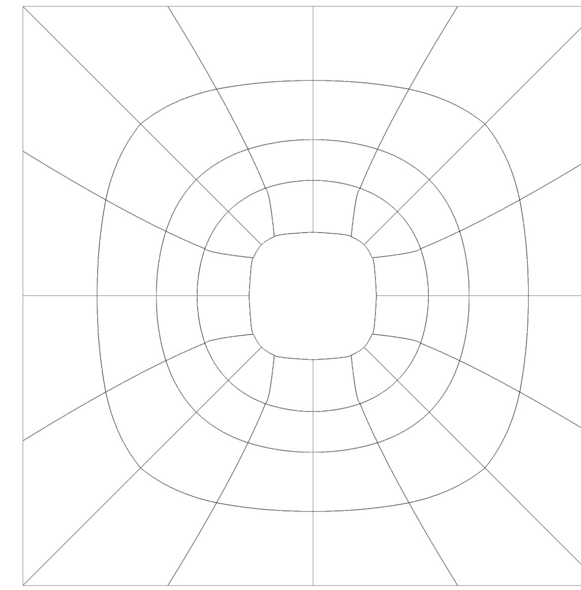
Boundary and Interface Fitting Applications



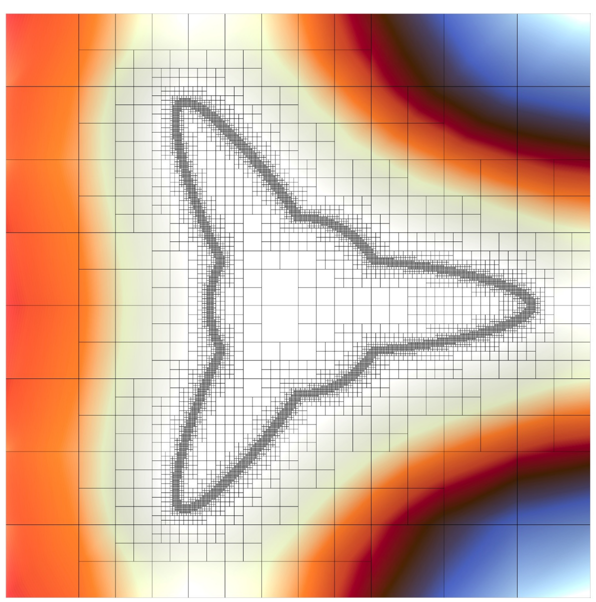
$\sigma(\mathbf{x})$ on background mesh



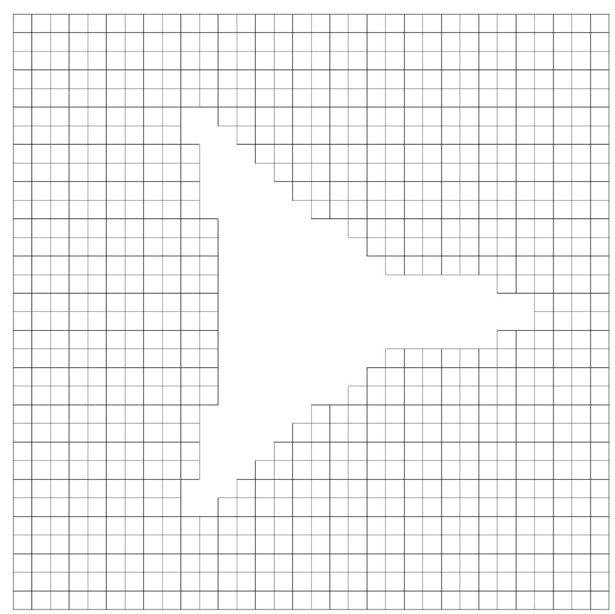
Initial mesh



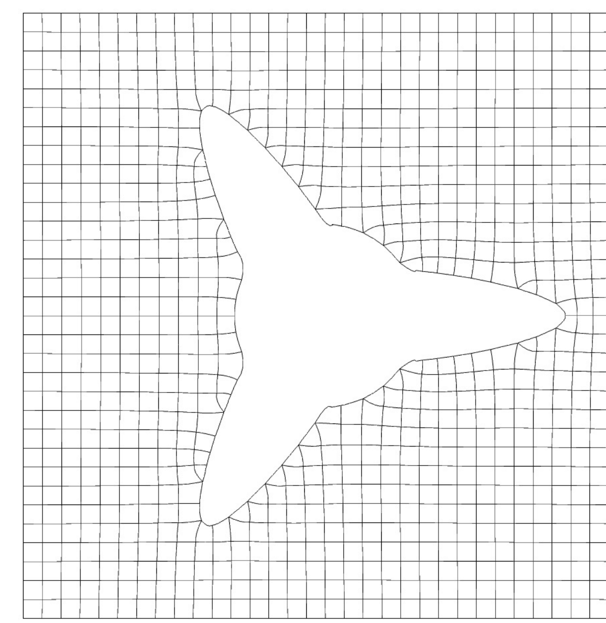
Optimized mesh



$\sigma(\mathbf{x})$ on background mesh



Initial mesh



Optimized mesh

Summary & Future Work

- The boundary and interface fitting method has been improved
  - New formulation.
  - Updated marking strategy.
  - Method to combine geometric primitives for defining complicated level-sets.
  - Use of a background mesh adapted to the level-set.
- Future work
  - Improve the formulation to handle sharp features of the level-sets.
  - Improve marking strategy.

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