High-Order Mesh Optimization

ethos
Enabling Technologies for High-Order Simulations

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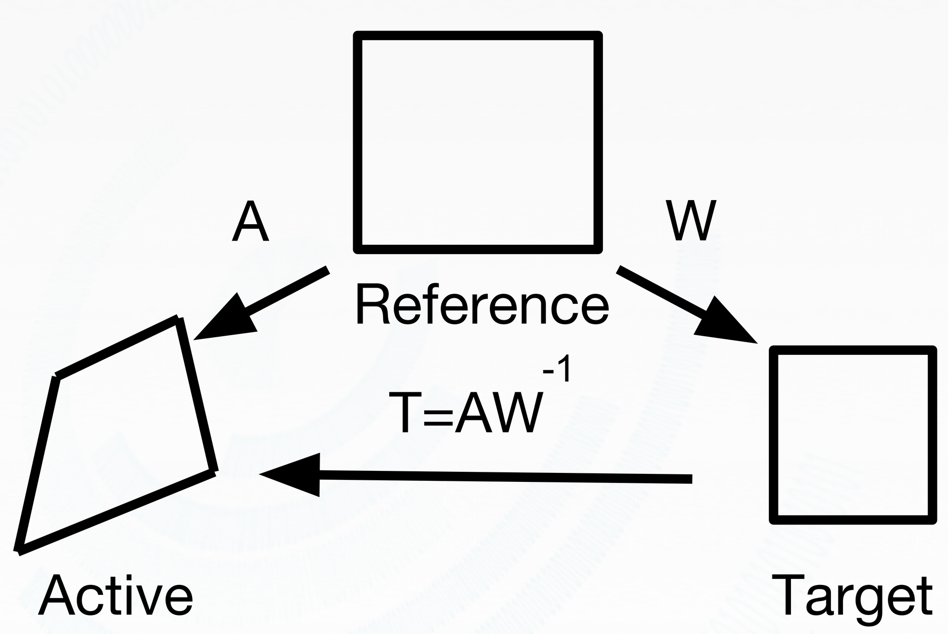
Motivation

High-Order methods are increasingly important for HPC simulations. High-order meshes can be very beneficial but are difficult to control due to their rich sub-zonal properties [4]. ETHOS develops rigorous theory for high-order mesh quality based on TMOP and VM [1,2,3] and uses it to produce mesh optimization algorithms for a wide spectrum of users. Synergistic with ECP: work is partially motivated by and benefits the CEED co-design center and its apps. The algorithms are easy-to-use and freely available through open-source software [5].

Approach

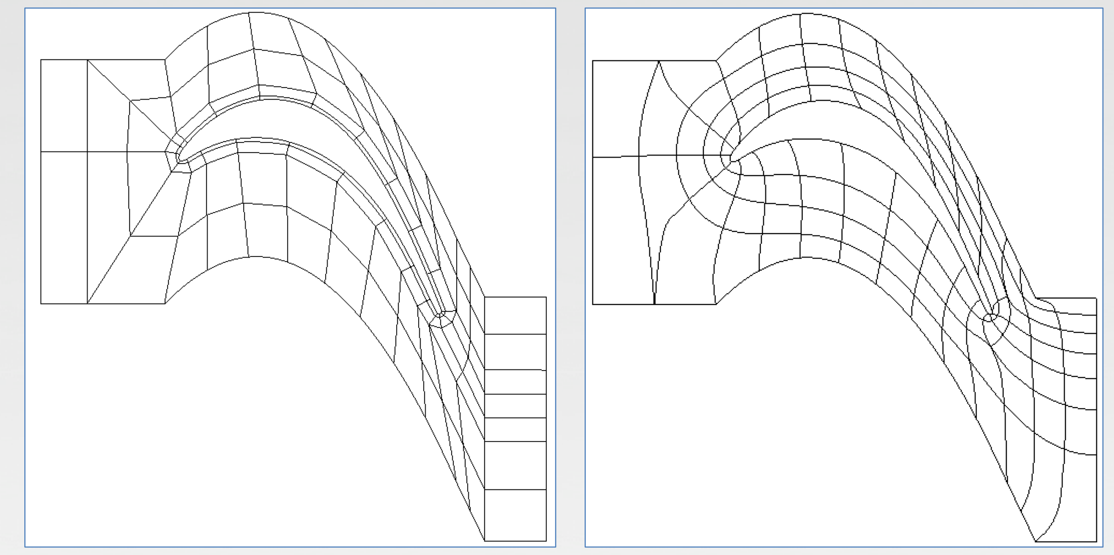
Extend Variation Minimization and the Target-Matrix Optimization Paradigm [1,2,3] to HO meshes.

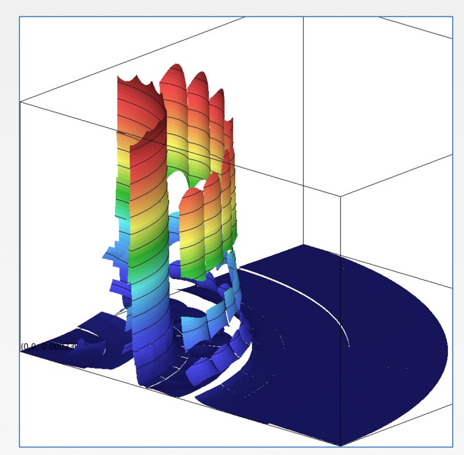


1. **Application-specific target elements, W.**
   Examples: Ideal / Ideal with specified size.
2. **Point-based mesh quality metric $\mu(T)$**
   Can measure shape / size / alignment:
   $$\mu_2^{shape} = \frac{|T|^2}{2\det(T)} - 1 \qquad \mu_{55}^{size} = 0.5\,(\det(T) - 1)^2$$
3. **Minimize a variational objective function**
   $$\frac{\partial F(\boldsymbol{x})}{\partial \boldsymbol{x}} = 0\,, \quad \text{where} \quad F(x) = \sum_K \int_{K_t} \mu(T(x))$$
   Constraints: $\det(T) > 0$ *at all integration pts.*
   Hessian-based methods require $\partial^2\mu/\partial T^2$.
4. **Additional capabilities:** *tangential relaxation, space-dependent compositions of metrics, limited node movement, mesh untangling.*

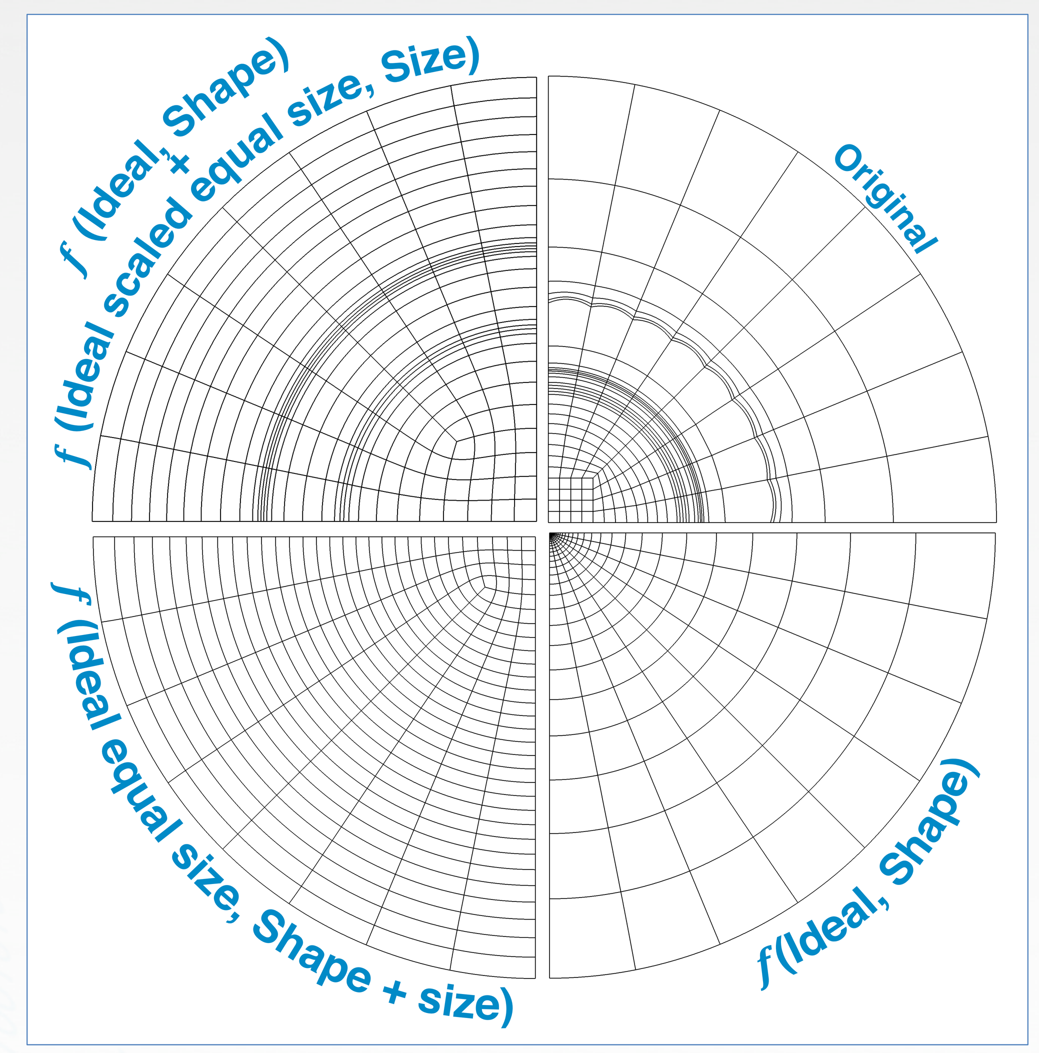
Results

Shape optimization of a 4th order turbine blade

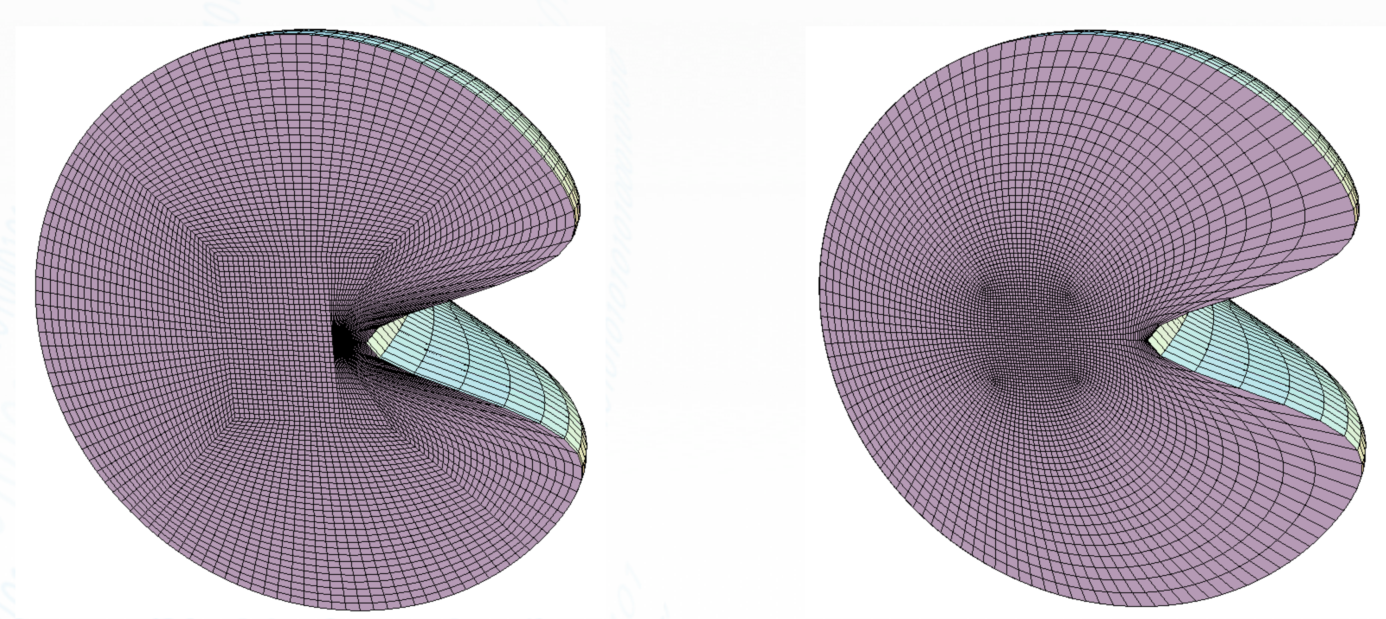




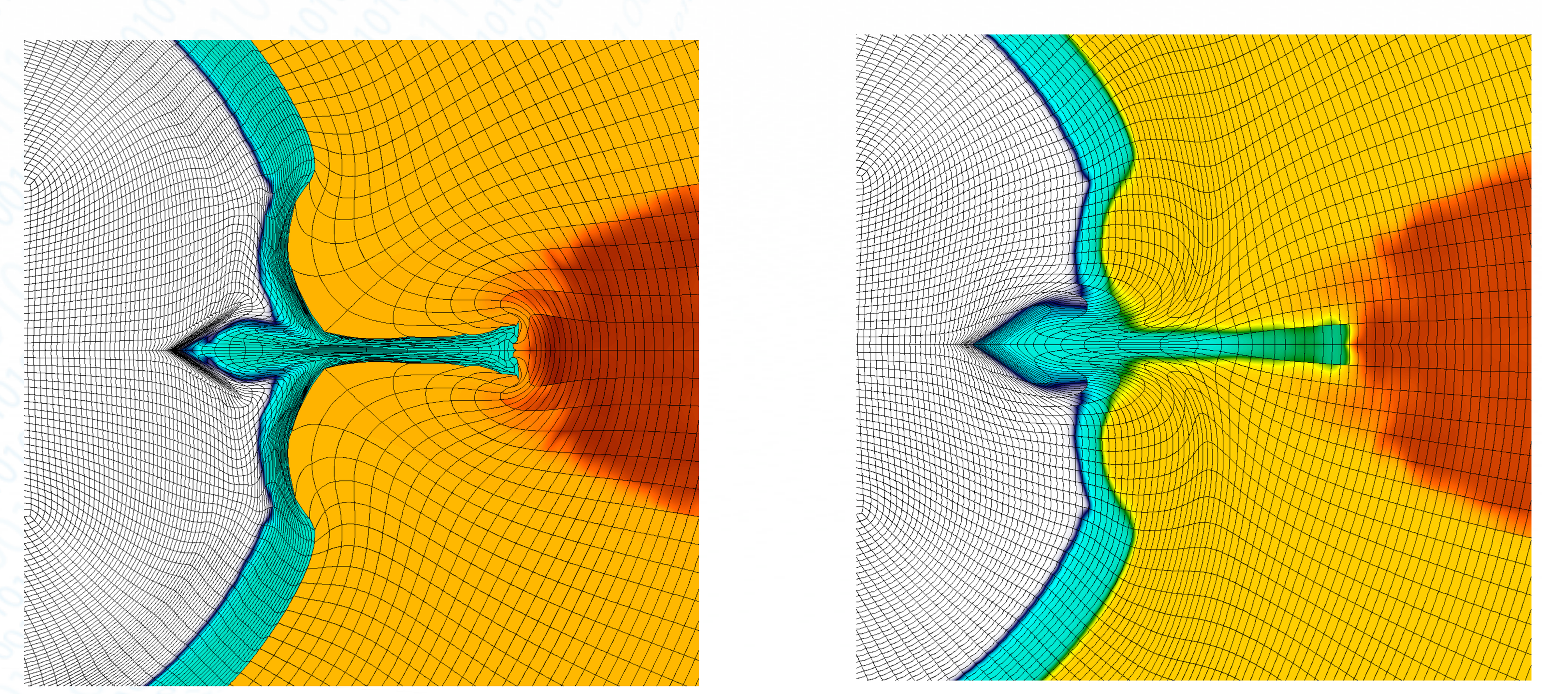
Initial μ_2 shape quality on a 2nd order mesh



ICF-like mesh optimized by various ETHOS methods



3D shape optimization of a perturbed sphere



Shape + size optimization in High-Order ALE simulation

Conclusions and Future Work

Our **rigorous theory for high-order mesh quality** provides a general & flexible mesh optimization tool for next-gen applications. Next steps:

- Incorporation of **general adaptivity tensor** in TMOP with application to **r-adaptivity**.
- Investigate metrics based on **higher-order derivatives** (beyond Jacobians) and **velocity-based ODE evolution** solution algorithms.
- **Demonstrations** in large-scale simulations.

Areas in which we can help

- **Freely available, easy-to-use mesh optimization algorithms:** powerful and general (any order), open-source [5].
- **Tailor the TMOP algorithms to your needs:** specific targets, quality metrics, etc.
- **High-quality interpolation/transfer** of fields between original and optimized mesh.
- High-order **visualization**, **mesh format**, unstructured **finite element discretizations.**

Areas in which we need help

- **High-order applications needs/requirements:** *what is a "good" mesh for your simulation?*
- **Optimization solvers for the global problem:** *derivative-free, Newton-like, constrained (valid mesh/positive Jacobian).*

References

[1] P. Knupp, *"Algebraic mesh quality metrics"*, SISC, 2001
[2] P. Knupp, *"Introducing the target-matrix paradigm for mesh optimization via node-movement"*, Eng.Comp., 2012
[3] V. Dobrev, P. Knupp, T. Kolev, V. Tomov, K. Mittal, *"The target-matrix optimization paradigm for high-order meshes"*, preprint
[4] V. Dobrev, T. Kolev, R. Rieben, *"High-order curvilinear finite element methods for Lagrangian hydrodynamics"*, SISC, 2012
[5] *MFEM library*, http://mfem.org

This work performed under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory under Contract DE-AC52-07NA27344. LLNL-POST-738230