High-Order Mesh Optimization

Motivation

High-Order methods are increasingly important for HPC simulations. High-order meshes can be very beneficial but are difficult to control due to their rich sub-zonal properties [4]. ETHOS develops rigorous theory for high-order mesh quality based on TMOP and VM [1,2,3] and uses it to produce mesh optimization algorithms for a wide spectrum of users. Synergistic with ECP: work is partially motivated by and benefits the CEED co-design center and its apps. The algorithms are easy-to-use and freely available through open-source software [5].

Approach

Extend Variation Minimization and the Target-Matrix Optimization Paradigm [1,2,3] to HO meshes.

2. Point-based mesh quality metric $\mu(T)$
   
   Can measure shape / size / alignment:
   
   $$\mu^\text{shape} = \frac{|T|^2}{2 \det(T)} - 1$$
   
   $$\mu^\text{size} = 0.5 (\det(T) - 1)^2$$

3. Minimize a variational objective function

   $$\frac{\partial F(x)}{\partial x} = 0,$$ where $F(x) = \sum K_i \mu(T(x))$

   Constraints: $\det(T) > 0$ at all integration pts. Hessian-based methods require $\partial^2 \mu/\partial T^2$.

4. Additional capabilities: tangential relaxation, space-dependent compositions of metrics, limited node movement, mesh untangling.

Conclusions and Future Work

Our rigorous theory for high-order mesh quality provides a general & flexible mesh optimization tool for next-gen applications. Next steps:

- Incorporation of general adaptivity tensor in TMOP with application to r-adaptivity.
- Investigate metrics based on higher-order derivatives (beyond Jacobians) and velocity-based ODE evolution solution algorithms.
- Demonstrations in large-scale simulations.

Areas in which we can help

- Freely available, easy-to-use mesh optimization algorithms: powerful and general (any order), open-source [5].
- Tailor the TMOP algorithms to your needs: specific targets, quality metrics, etc.
- High-quality interpolation/transfer of fields between original and optimized mesh.
- High-order visualization, mesh format, unstructured finite element discretizations.

Areas in which we need help

- High-order applications needs/requirements: what is a “good” mesh for your simulation?
- Optimization solvers for the global problem: derivative-free, Newton-like, constrained (valid mesh/positive Jacobian).

References