User Documentation for IDAS v4.1.0
(SUNDIALS v5.1.0)

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Chapter 1

Introduction

IDAS is part of a software family called SUNDIALS: SUite of Nonlinear and DIfferential/ALgebraic equation Solvers [29]. This suite consists of CVODE, ARKODE, KINSOL, and IDA, and variants of these with sensitivity analysis capabilities, CVODES and IDAS.

IDAS is a general purpose solver for the initial value problem (IVP) for systems of differential-algebraic equations (DAEs). The name IDAS stands for Implicit Differential-Algebraic solver with Sensitivity capabilities. IDAS is an extension of the IDA solver within SUNDIALS, itself based on DASPK [9, 10]; however, like all SUNDIALS solvers, IDAS is written in ANSI-standard C rather than FORTRAN77. Its most notable features are that, (1) in the solution of the underlying nonlinear system at each time step, it offers a choice of Newton/direct methods and a choice of Inexact Newton/Krylov (iterative) methods; (2) it is written in a data-independent manner in that it acts on generic vectors and matrices without any assumptions on the underlying organization of the data; and (3) it provides a flexible, extensible framework for sensitivity analysis, using either forward or adjoint methods. Thus IDAS shares significant modules previously written within CASC at LLNL to support the ordinary differential equation (ODE) solvers CVODE [30, 17] and PVODE [13, 14], the DAE solver IDA [33] on which IDAS is based, the sensitivity-enabled ODE solver CVODES [31, 47], and also the nonlinear system solver KINSOL [18].

At present, IDAS may utilize a variety of Krylov methods provided in SUNDIALS that can be used in conjunction with Newton iteration: these include the GMRES (Generalized Minimal RESidual) [46], FGMRES (Flexible Generalized Minimum RESidual) [45], Bi-CGStab (Bi-Conjugate Gradient Stabilized) [49], TFQMR (Transpose-Free Quasi-Minimal Residual) [24], and PCG (Preconditioned Conjugate Gradient) [26] linear iterative methods. As Krylov methods, these require little matrix storage for solving the Newton equations as compared to direct methods. However, the algorithms allow for a user-supplied preconditioner matrix, and, for most problems, preconditioning is essential for an efficient solution.

For very large DAE systems, the Krylov methods are preferable over direct linear solver methods, and are often the only feasible choice. Among the Krylov methods in SUNDIALS, we recommend GMRES as the best overall choice. However, users are encouraged to compare all options, especially if encountering convergence failures with GMRES. Bi-CGFSktab and TFQMR have an advantage in storage requirements, in that the number of workspace vectors they require is fixed, while that number for GMRES depends on the desired Krylov subspace size. FGMRES has an advantage in that it is designed to support preconditioners that vary between iterations (e.g. iterative methods). PCG exhibits rapid convergence and minimal workspace vectors, but only works for symmetric linear systems.

IDAS is written with a functionality that is a superset of that of IDA. Sensitivity analysis capabilities, both forward and adjoint, have been added to the main integrator. Enabling forward sensitivity computations in IDAS will result in the code integrating the so-called sensitivity equations simultaneously with the original IVP, yielding both the solution and its sensitivity with respect to parameters in the model. Adjoint sensitivity analysis, most useful when the gradients of relatively few functionals of the solution with respect to many parameters are sought, involves integration of the original IVP
forward in time followed by the integration of the so-called adjoint equations backward in time. IDAS provides the infrastructure needed to integrate any final-condition ODE dependent on the solution of the original IVP (in particular the adjoint system).

There are several motivations for choosing the C language for IDAS. First, a general movement away from Fortran and toward C in scientific computing was apparent. Second, the pointer, structure, and dynamic memory allocation features in C are extremely useful in software of this complexity, with the great variety of method options offered. Finally, we prefer C over C++ for IDAS because of the wider availability of C compilers, the potentially greater efficiency of C, and the greater ease of interfacing the solver to applications written in extended Fortran.

1.1 Changes from previous versions

Changes in v4.1.0

Fixed a build system bug related to finding LAPACK/BLAS.

Fixed a build system bug related to checking if the KLU library works.

Added a new build system option, CUDA_ARCH, that can be used to specify the CUDA architecture to compile for.

Fixed a build system bug related to finding PETSc when using the CMake variables PETSC_INCLUDES and PETSC_LIBRARIES instead of PETSC_DIR.

Added two utility functions, SUNDIALSFileOpen and SUNDIALSFileClose for creating/destroying file pointers that are useful when using the Fortran 2003 interfaces.

Changes in v4.0.0

Build system changes

- Increased the minimum required CMake version to 3.5 for most SUNDIALS configurations, and 3.10 when CUDA or OpenMP with device offloading are enabled.

- The CMake option BLAS_ENABLE and the variable BLAS_LIBRARIES have been removed to simplify builds as SUNDIALS packages do not use BLAS directly. For third party libraries that require linking to BLAS, the path to the BLAS library should be included in the _LIBRARIES variable for the third party library e.g., SUPERLUDIST_LIBRARIES when enabling SuperLU_DIST.

- Fixed a bug in the build system that prevented the nvector_pthreads module from being built.

NVECTOR module changes

- Two new functions were added to aid in creating custom nVECTOR objects. The constructor N_VNewEmpty allocates an “empty” generic NVECTOR with the object’s content pointer and the function pointers in the operations structure initialized to NULL. When used in the constructor for custom objects this function will ease the introduction of any new optional operations to the nVECTOR API by ensuring only required operations need to be set. Additionally, the function N_VCopyOps(ν, ν) has been added to copy the operation function pointers between vector objects. When used in clone routines for custom vector objects these functions also will ease the introduction of any new optional operations to the nVECTOR API by ensuring all operations are copied when cloning objects. See §8.1.5 for more details.

- Two new nVECTOR implementations, nVECTOR_MANYVECTOR and nVECTOR_MPMANYVECTOR, have been created to support flexible partitioning of solution data among different processing elements (e.g., CPU + GPU) or for multi-physics problems that couple distinct MPI-based simulations together. This implementation is accompanied by additions to user documentation and SUNDIALS examples. See §8.13 and §8.14 for more details.
1.1 Changes from previous versions

- One new required vector operation and ten new optional vector operations have been added to the nVECTOR API. The new required operation, `N_VGetLength`, returns the global length of an N_Vector. The optional operations have been added to support the new nVECTOR_MPIVMANYVECTOR implementation. The operation `N_VGetCommunicator` must be implemented by subvectors that are combined to create an nVECTOR_MPIVMANYVECTOR, but is not used outside of this context. The remaining nine operations are optional local reduction operations intended to eliminate unnecessary latency when performing vector reduction operations (norms, etc.) on distributed memory systems. The optional local reduction vector operations are `N_VDotProdLocal`, `N_VMaxNormLocal`, `N_VMinLocal`, `N_VL1NormLocal`, `N_VWSqrSumLocal`, `N_VWSqrSumMaskLocal`, `N_VInvTestLocal`, `N_VConstrMaskLocal`, and `N_VMinQuotientLocal`. If an nVECTOR implementation defines any of the local operations as NULL, then the nVECTOR_MPIVMANYVECTOR will call standard nVECTOR operations to complete the computation. See §8.1.4 for more details.

- An additional nVECTOR implementation, nVECTOR_MPIPLUSX, has been created to support the MPI+X paradigm where X is a type of on-node parallelism (e.g., OpenMP, CUDA). The implementation is accompanied by additions to user documentation and SUNDIALS examples. See §8.15 for more details.

- The *_MPICuda and *_MPIRaja functions have been removed from the nVECTOR_CUDA and nVECTOR_RAJA implementations respectively. Accordingly, the nvector_mpicuda.h, nvector_mpiraja.h, libsundials_nvecmpicuda.lib, and libsundials_nvecmpicudaraja.lib files have been removed. Users should use the nVECTOR_MPIPLUSX module coupled in conjunction with the nVECTOR_CUDA or nVECTOR_RAJA modules to replace the functionality. The necessary changes are minimal and should require few code modifications. See the programs in examples/ida/mpicuda and examples/ida/mpiraja for examples of how to use the nVECTOR_MPIPLUSX module with the nVECTOR_CUDA and nVECTOR_RAJA modules respectively.

- Fixed a memory leak in the nVECTOR_PETSC module clone function.

- Made performance improvements to the nVECTOR_CUDA module. Users who utilize a non-default stream should no longer see default stream synchronizations after memory transfers.

- Added a new constructor to the nVECTOR_CUDA module that allows a user to provide custom allocate and free functions for the vector data array and internal reduction buffer. See §8.9.1 for more details.

- Added new Fortran 2003 interfaces for most nVECTOR modules. See Chapter 8 for more details on how to use the interfaces.

- Added three new nVECTOR utility functions, `FN_VGetVecAtIndexVectorArray`, `FN_VSetVecAtIndexVectorArray`, and `FN_VNewVectorArray`, for working with N_Vector arrays when using the Fortran 2003 interfaces. See §8.1.5 for more details.

**SUNMatrix module changes**

- Two new functions were added to aid in creating custom SUNMATRIX objects. The constructor SUNMatNewEmpty allocates an “empty” generic SUNMATRIX with the object’s content pointer and the function pointers in the operations structure initialized to NULL. When used in the constructor for custom objects this function will ease the introduction of any new optional operations to the SUNMATRIX API by ensuring only required operations need to be set. Additionally, the function SUNMatCopyOps(A, B) has been added to copy the operation function pointers between matrix objects. When used in clone routines for custom matrix objects these functions also will ease the introduction of any new optional operations to the SUNMATRIX API by ensuring all operations are copied when cloning objects. See §9.1.2 for more details.
• A new operation, SUNMatMatvecSetup, was added to the SUNMATRIX API to perform any setup necessary for computing a matrix-vector product. This operation is useful for SUNMATRIX implementations which need to prepare the matrix itself, or communication structures before performing the matrix-vector product. Users who have implemented custom SUNMATRIX modules will need to at least update their code to set the corresponding ops structure member, matvecsetup, to NULL. See §9.1.1 for more details.

• The generic SUNMATRIX API now defines error codes to be returned by SUNMATRIX operations. Operations which return an integer flag indicating success/failure may return different values than previously. See §9.1.3 for more details.

• A new SUNMATRIX (and SUNLINSOL) implementation was added to facilitate the use of the SuperLU_DIST library with SUNDIALS. See §9.6 for more details.

• Added new Fortran 2003 interfaces for most SUNMATRIX modules. See Chapter 9 for more details on how to use the interfaces.

SUNLinearSolver module changes

• A new function was added to aid in creating custom SUNLINSOL objects. The constructor SUNLinSolNewEmpty allocates an “empty” generic SUNLINSOL with the object’s content pointer and the function pointers in the operations structure initialized to NULL. When used in the constructor for custom objects this function will ease the introduction of any new optional operations to the SUNLINSOL API by ensuring only required operations need to be set. See §10.3 for more details.

• The return type of the SUNLINSOL API function SUNLinSolLastFlag has changed from long int to sunindextype to be consistent with the type used to store row indices in dense and banded linear solver modules.

• Added a new optional operation to the SUNLINSOL API, SUNLinSolGetID, that returns a SUNLinearSolver_ID for identifying the linear solver module.

• The SUNLINSOL API has been updated to make the initialize and setup functions optional.

• A new SUNLINSOL (and SUNMATRIX) implementation was added to facilitate the use of the SuperLU_DIST library with SUNDIALS. See §10.10 for more details.

• Added a new SUNLINSOL implementation, SUNLinearSolver_cUSOLVER_Sp_batchQR, which leverages the NVIDIA cuSOLVER sparse batched QR method for efficiently solving block diagonal linear systems on NVIDIA GPUs. See §10.12 for more details.

• Added three new accessor functions to the SUNLINSOL_KLU module, SUNLinSol_KLUGetSymbolic, SUNLinSol_KLUGetNumeric, and SUNLinSol_KLUGetCommon, to provide user access to the underlying KLU solver structures. See §10.9.2 for more details.

• Added new Fortran 2003 interfaces for most SUNLINSOL modules. See Chapter 10 for more details on how to use the interfaces.

SUNNonlinearSolver module changes

• A new function was added to aid in creating custom SUNNONLINSOL objects. The constructor SUNNonlinSolNewEmpty allocates an “empty” generic SUNNONLINSOL with the object’s content pointer and the function pointers in the operations structure initialized to NULL. When used in the constructor for custom objects this function will ease the introduction of any new optional operations to the SUNNONLINSOL API by ensuring only required operations need to be set. See §11.1.8 for more details.
1.1 Changes from previous versions

- To facilitate the use of user supplied nonlinear solver convergence test functions the SUNNonlinSolSetConvTestFn function in the SUNNONLINSOL API has been updated to take a void* data pointer as input. The supplied data pointer will be passed to the nonlinear solver convergence test function on each call.

- The inputs values passed to the first two inputs of the SUNNonlinSolSolve function in the SUNNONLINSOL have been changed to be the predicted state and the initial guess for the correction to that state. Additionally, the definitions of SUNNonlinSolLSetupFn and SUNNonlinSolLSolveFn in the SUNNONLINSOL API have been updated to remove unused input parameters. For more information on the nonlinear system formulation see §11.2 and for more details on the API functions see Chapter 11.

- Added a new SUNNONLINSOL implementation, SUNNONLINSOL_PETSCSNES, which interfaces to the PETSc SNES nonlinear solver API. See §11.4 for more details.

- Added new Fortran 2003 interfaces for most SUNNONLINSOL modules. See Chapter 11 for more details on how to use the interfaces.

IDAS changes

- A bug was fixed in the IDAS linear solver interface where an incorrect Jacobian-vector product increment was used with iterative solvers other than SUNLINSOL_SPGMR and SUNLINSOL_SPFGM.

- Fixed a bug where the IDASolveF function would not return a root in IDA_NORMAL_STEP mode if the root occurred after the desired output time.

- Fixed a bug where the IDASolveF function would return the wrong flag under certain circumstances.

- Fixed a bug in IDAQuadReInitB where an incorrect memory structure was passed to IDAQuadReInit.

- Removed extraneous calls to N_VMin for simulations where the scalar valued absolute tolerance, or all entries of the vector-valued absolute tolerance array, are strictly positive. In this scenario, IDAS will remove at least one global reduction per time step.

- The IDALS interface has been updated to only zero the Jacobian matrix before calling a user-supplied Jacobian evaluation function when the attached linear solver has type SUNLINEARSOLVER_DIRECT.

- Added the new functions, IDAGetCurrentCj, IDAGetCurrentY, IDAGetCurrentYp, IDAComputeCurrentY, IDAComputeCurrentYp, IDAComputeCurrentYSens, IDAComputeCurrentYpSens, IDAComputeCurrentYSens, and IDAComputeCurrentYpSens, which may be useful to users who choose to provide their own nonlinear solver implementations.

- Added a Fortran 2003 interface to IDAS. See Chapter 7 for more details.

Changes in v3.1.0

An additional nvector implementation was added for the Tpetra vector from the Trilinos library to facilitate interoperability between Sundials and Trilinos. This implementation is accompanied by additions to user documentation and Sundials examples.

A bug was fixed where a nonlinear solver object could be freed twice in some use cases. The EXAMPLES_ENABLE_RAJA CMake option has been removed. The option EXAMPLES_ENABLE_CUDA enables all examples that use CUDA including the RAJA examples with a CUDA back end (if the RAJA nvector is enabled).

The implementation header file idas_impl.h is no longer installed. This means users who are directly manipulating the IDAMem structure will need to update their code to use IDAS’s public API.

Python is no longer required to run make test and make test_install.
Changes in v3.0.2

Added information on how to contribute to SUNDIALS and a contributing agreement.

Moved definitions of DLS and SPILS backwards compatibility functions to a source file. The symbols are now included in the IDAS library, libsundials_idas.

Changes in v3.0.1

No changes were made in this release.

Changes in v3.0.0

IDAS’ previous direct and iterative linear solver interfaces, IDADLS and IDASPILS, have been merged into a single unified linear solver interface, IDALS, to support any valid SUNLINSOL module. This includes the “DIRECT” and “ITERATIVE” types as well as the new “MATRIX ITERATIVE” type. Details regarding how IDALS utilizes linear solvers of each type as well as discussion regarding intended use cases for user-supplied SUNLINSOL implementations are included in Chapter 10. All IDAS example programs and the standalone linear solver examples have been updated to use the unified linear solver interface.

The unified interface for the new IDALS module is very similar to the previous IDADLS and IDASPILS interfaces. To minimize challenges in user migration to the new names, the previous C routine names may still be used; these will be deprecated in future releases, so we recommend that users migrate to the new names soon.

The names of all constructor routines for SUNDIALS-provided SUNLINSOL implementations have been updated to follow the naming convention SUNLinSol_• where • is the name of the linear solver. The new names are SUNLinSol_Band, SUNLinSol_Dense, SUNLinSol_KLU, SUNLinSol_LapackBand, SUNLinSol_LapackDense, SUNLinSol_PCG, SUNLinSol_SPBCGS, SUNLinSol_SPGMR, SUNLinSol_SPGMRES, SUNLinSol_SPTFQMR, and SUNLinSol_SuperLUMT. Solver-specific “set” routine names have been similarly standardized. To minimize challenges in user migration to the new names, the previous routine names may still be used; these will be deprecated in future releases, so we recommend that users migrate to the new names soon. All IDAS example programs and the standalone linear solver examples have been updated to use the new naming convention.

The SUNBandMatrix constructor has been simplified to remove the storage upper bandwidth argument.

SUNDIALS integrators have been updated to utilize generic nonlinear solver modules defined through the SUNNONLINSOL API. This API will ease the addition of new nonlinear solver options and allow for external or user-supplied nonlinear solvers. The SUNNONLINSOL API and SUNDIALS provided modules are described in Chapter 11 and follow the same object oriented design and implementation used by the NVECTOR, SUNMATRIX, and SUNLINSOL modules. Currently two SUNNONLINSOL implementations are provided, SUNNONLINSOL_NEWTON and SUNNONLINSOL_FIXEDPOINT. These replicate the previous integrator specific implementations of a Newton iteration and a fixed-point iteration (previously referred to as a functional iteration), respectively. Note the SUNNONLINSOL_FIXEDPOINT module can optionally utilize Anderson’s method to accelerate convergence. Example programs using each of these nonlinear solver modules in a standalone manner have been added and all IDAS example programs have been updated to use generic SUNNONLINSOL modules.

By default IDAS uses the SUNNONLINSOL_NEWTON module. Since IDAS previously only used an internal implementation of a Newton iteration no changes are required to user programs and functions for setting the nonlinear solver options (e.g., IDASGetMaxNonlinIters) or getting nonlinear solver statistics (e.g., IDASGetNumNonlinSolvIters) remain unchanged and internally call generic SUNNONLINSOL functions as needed. While SUNDIALS includes a fixed-point nonlinear solver module, it is not currently supported in IDAS. For details on attaching a user-supplied nonlinear solver to IDAS see Chapter 4, 5, and 6.

Three fused vector operations and seven vector array operations have been added to the NVECTOR API. These optional operations are disabled by default and may be activated by calling vector specific routines after creating an NVECTOR (see Chapter 8 for more details). The new operations are
1.1 Changes from previous versions

intended to increase data reuse in vector operations, reduce parallel communication on distributed memory systems, and lower the number of kernel launches on systems with accelerators. The fused operations are \texttt{N \_VLin}ear\texttt{Combination}, \texttt{N \_VScaleAddMulti}, and \texttt{N \_VDotProdMulti} and the vector array operations are \texttt{N \_VLin}ear\texttt{CombinationVectorArray}, \texttt{N \_VScaleVectorArray}, \texttt{N \_VConstVectorArray}, \texttt{N \_VWrmsNormVectorArray}, \texttt{N \_VWrmsNormMaskVectorArray}, \texttt{N \_VScaleAddMultiVectorArray}, and \texttt{N \_VLin}ear\texttt{CombinationVectorArray}. If an \texttt{NVVECTOR} implementation defines any of these operations as \texttt{NULL}, then standard \texttt{NVVECTOR} operations will automatically be called as necessary to complete the computation.

Multiple updates to \texttt{NVVECTOR} \texttt{CUDA} were made:

- Changed \texttt{N \_VGetLength} \texttt{Cuda} to return the global vector length instead of the local vector length.
- Added \texttt{N \_VGetLocalLength} \texttt{Cuda} to return the local vector length.
- Added \texttt{N \_VGetMPIComm} \texttt{Cuda} to return the MPI communicator used.
- Removed the accessor functions in the namespace suncudavec.
- Changed the \texttt{N \_VMake} \texttt{Cuda} function to take a host data pointer and a device data pointer instead of an \texttt{N \_Vectorizer} \texttt{Content} \texttt{Cuda} object.
- Added the ability to set the \texttt{cudaStream} \texttt{t} used for execution of the \texttt{NVVECTOR} \texttt{CUDA} kernels. See the function \texttt{N \_VSetCudaStreams} \texttt{Cuda}.
- Added \texttt{N \_VNewManaged} \texttt{Cuda}, \texttt{N \_VMakeManaged} \texttt{Cuda}, and \texttt{N \_VIsManagedMemory} \texttt{Cuda} functions to accommodate using managed memory with the \texttt{NVVECTOR} \texttt{CUDA}.

Multiple changes to \texttt{NVVECTOR} \texttt{RAJA} were made:

- Changed \texttt{N \_VGetLength} \texttt{Raja} to return the global vector length instead of the local vector length.
- Added \texttt{N \_VGetLocalLength} \texttt{Raja} to return the local vector length.
- Added \texttt{N \_VGetMPIComm} \texttt{Raja} to return the MPI communicator used.
- Removed the accessor functions in the namespace suncudavec.

A new \texttt{NVVECTOR} implementation for leveraging OpenMP 4.5+ device offloading has been added, \texttt{NVVECTOR} \texttt{OPENMPDEV}. See §8.11 for more details.

Changes in v2.2.1

The changes in this minor release include the following:

- Fixed a bug in the CUDA \texttt{NVVECTOR} where the \texttt{N \_VINvTest} operation could write beyond the allocated vector data.
- Fixed library installation path for multiarch systems. This fix changes the default library installation path to \texttt{CMAKE\_INSTALL\_PREFIX}/\texttt{CMAKE\_INSTALL\_LIBDIR} from \texttt{CMAKE\_INSTALL\_PREFIX}/lib. \texttt{CMAKE\_INSTALL\_LIBDIR} is automatically set, but is available as a CMake option that can modified.

Changes in v2.2.0

Fixed a bug in \texttt{IDAS} where the saved residual value used in the nonlinear solve for consistent initial conditions was passed as temporary workspace and could be overwritten.

Fixed a thread-safety issue when using adjoint sensitivity analysis.
Fixed a problem with setting `sunindextype` which would occur with some compilers (e.g. arm-clang) that did not define `__STDC_VERSION__`.

Added hybrid MPI/CUDA and MPI/RAJA vectors to allow use of more than one MPI rank when using a GPU system. The vectors assume one GPU device per MPI rank.

Changed the name of the RAJA NVVECTOR library to `libsundials_nvecudaraja.lib` from `libsundials_nvecraja.lib` to better reflect that we only support CUDA as a backend for RAJA currently.

Several changes were made to the build system:

- CMake 3.1.3 is now the minimum required CMake version.
- Deprecate the behavior of the `SUNDIALS_INDEX_TYPE` CMake option and added the `SUNDIALS_INDEX_SIZE` CMake option to select the `sunindextype` integer size.
- The native CMake FindMPI module is now used to locate an MPI installation.
- If MPI is enabled and MPI compiler wrappers are not set, the build system will check if `CMAKE_<language>_COMPILER` can compile MPI programs before trying to locate and use an MPI installation.
- The previous options for setting MPI compiler wrappers and the executable for running MPI programs have been have been deprecated. The new options that align with those used in native CMake FindMPI module are `MPI_C_COMPILER`, `MPI_CXX_COMPILER`, `MPI_Fortran_COMPILER`, and `MPIEXEC_EXECUTABLE`.
- When a Fortran name-mangling scheme is needed (e.g., `LAPACK_ENABLE` is ON) the build system will infer the scheme from the Fortran compiler. If a Fortran compiler is not available or the inferred or default scheme needs to be overridden, the advanced options `SUNDIALS_F77_FUNC_CASE` and `SUNDIALS_F77_FUNC_UNDERSCORES` can be used to manually set the name-mangling scheme and bypass trying to infer the scheme.
- Parts of the main CMakeLists.txt file were moved to new files in the `src` and `example` directories to make the CMake configuration file structure more modular.

### Changes in v2.1.2

The changes in this minor release include the following:

- Updated the minimum required version of CMake to 2.8.12 and enabled using rpath by default to locate shared libraries on OSX.
- Fixed Windows specific problem where `sunindextype` was not correctly defined when using 64-bit integers for the `sundials` index type. On Windows `sunindextype` is now defined as the MSVC basic type `int64`.
- Added sparse SUNMatrix “Reallocate” routine to allow specification of the nonzero storage.
- Updated the KLU SUNLINSOL module to set constants for the two reinitialization types, and fixed a bug in the full reinitialization approach where the sparse SUNMatrix pointer would go out of scope on some architectures.
- Updated the “ScaleAdd” and “ScaleAddI” implementations in the sparse SUNMatrix module to more optimally handle the case where the target matrix contained sufficient storage for the sum, but had the wrong sparsity pattern. The sum now occurs in-place, by performing the sum backwards in the existing storage. However, it is still more efficient if the user-supplied Jacobian routine allocates storage for the sum $I + \gamma J$ manually (with zero entries if needed).
- Changed the LICENSE install path to `instdir/include/sundials`.
1.1 Changes from previous versions

Changes in v2.1.1

The changes in this minor release include the following:

- Fixed a potential memory leak in the SPGMR and SPFGMR linear solvers: if “Initialize” was called multiple times then the solver memory was reallocated (without being freed).
- Updated KLU SUNLinearSolver module to use a typedef for the precision-specific solve function to be used (to avoid compiler warnings).
- Added missing typecasts for some (void*) pointers (again, to avoid compiler warnings).
- Bugfix in summatrix_sparse.c where we had used int instead of sunindextype in one location.
- Added missing #include <stdio.h> in NVVECTOR and SUNMATRIX header files.
- Added missing prototype for IDASpilsGetNumJTEvals.
- Fixed an indexing bug in the CUDA NVVECTOR implementation of N_VWrhsNormMask and revised the RAJA NVVECTOR implementation of N_VWrhsNormMask to work with mask arrays using values other than zero or one. Replaced double with realtype in the RAJA vector test functions.

In addition to the changes above, minor corrections were also made to the example programs, build system, and user documentation.

Changes in v2.1.0

Added NVVECTOR print functions that write vector data to a specified file (e.g., N_VPrintFile_Serial). Added make test and make test_install options to the build system for testing SUNDIALS after building with make and installing with make install respectively.

Changes in v2.0.0

All interfaces to matrix structures and linear solvers have been reworked, and all example programs have been updated. The goal of the redesign of these interfaces was to provide more encapsulation and to ease interfacing of custom linear solvers and interoperability with linear solver libraries. Specific changes include:

- Added generic SUNMATRIX module with three provided implementations: dense, banded and sparse. These replicate previous SUNDIALS Dls and Sls matrix structures in a single object-oriented API.
- Added example problems demonstrating use of generic SUNMATRIX modules.
- Added generic SUNLinearSolver module with eleven provided implementations: SUNDIALS native dense, SUNDIALS native banded, LAPACK dense, LAPACK band, KLU, SuperLU_MT, SPGMR, SPBCGS, SPTFQMR, SPFGMR, and PCG. These replicate previous SUNDIALS generic linear solvers in a single object-oriented API.
- Added example problems demonstrating use of generic SUNLinearSolver modules.
- Expanded package-provided direct linear solver (Dls) interfaces and scaled, preconditioned, iterative linear solver (Spils) interfaces to utilize generic SUNMATRIX and SUNLinearSolver objects.
- Removed package-specific, linear solver-specific, solver modules (e.g. CVDENSE, KINBAND, IDAKLU, ARKSFGMR) since their functionality is entirely replicated by the generic Dls/Spils interfaces and SUNLinearSolver/SUNMATRIX modules. The exception is CVDIAG, a diagonal approximate Jacobian solver available to CVODE and CVODES.
- Converted all SUNDIALS example problems and files to utilize the new generic SUNMATRIX and SUNLinearSolver objects, along with updated Dls and Spils linear solver interfaces.
- Added Spils interface routines to ARKODE, CVODE, CVODES, IDA, and IDAS to allow specification of a user-provided "JTSetup" routine. This change supports users who wish to set up data structures for the user-provided Jacobian-times-vector ("JTimes") routine, and where the cost of one JTSetup setup per Newton iteration can be amortized between multiple JTimes calls.

Two additional NVECTOR implementations were added – one for CUDA and one for RAJA vectors. These vectors are supplied to provide very basic support for running on GPU architectures. Users are advised that these vectors both move all data to the GPU device upon construction, and speedup will only be realized if the user also conducts the right-hand-side function evaluation on the device. In addition, these vectors assume the problem fits on one GPU. Further information about RAJA, users are referred to the web site, https://software.llnl.gov/RAJA/. These additions are accompanied by additions to various interface functions and to user documentation.

All indices for data structures were updated to a new sunindextype that can be configured to be a 32- or 64-bit integer data index type. sunindextype is defined to be int32_t or int64_t when portable types are supported, otherwise it is defined as int or long int. The Fortran interfaces continue to use long int for indices, except for their sparse matrix interface that now uses the new sunindextype. This new flexible capability for index types includes interfaces to PETSc, hypre, SuperLU_MT, and KLU with either 32-bit or 64-bit capabilities depending how the user configures SUNDIALS.

To avoid potential namespace conflicts, the macros defining boolean type values TRUE and FALSE have been changed to SUNTRUE and SUNFALSE respectively.

Temporary vectors were removed from preconditioner setup and solve routines for all packages. It is assumed that all necessary data for user-provided preconditioner operations will be allocated and stored in user-provided data structures.

The file include/sundials_fconfig.h was added. This file contains SUNDIALS type information for use in Fortran programs.

The build system was expanded to support many of the xSDK-compliant keys. The xSDK is a movement in scientific software to provide a foundation for the rapid and efficient production of high-quality, sustainable extreme-scale scientific applications. More information can be found at, https://xsdk.info.

Added functions SUNDIALSGetVersion and SUNDIALSGetVersionNumber to get SUNDIALS release version information at runtime.

In addition, numerous changes were made to the build system. These include the addition of separate BLAS_ENABLE and BLAS_LIBRARIES CMake variables, additional error checking during CMake configuration, minor bug fixes, and renaming CMake options to enable/disable examples for greater clarity and an added option to enable/disable Fortran 77 examples. These changes included changing EXAMPLES_ENABLE to EXAMPLES_ENABLE_C, changing CXX_ENABLE to EXAMPLES_ENABLE_CXX, changing F90_ENABLE to EXAMPLES_ENABLE_F90, and adding an EXAMPLES_ENABLE_F77 option.

A bug fix was done to add a missing prototype for IDASetMaxBacksIC in ida.h.

Corrections and additions were made to the examples, to installation-related files, and to the user documentation.

Changes in v1.3.0

Two additional NVECTOR implementations were added – one for Hypre (parallel) ParVector vectors, and one for PETSc vectors. These additions are accompanied by additions to various interface functions and to user documentation.

Each NVECTOR module now includes a function, NVGetVectorID, that returns the NVECTOR module name.

An optional input function was added to set a maximum number of linesearch backtracks in the initial condition calculation, and four user-callable functions were added to support the use of LAPACK linear solvers in solving backward problems for adjoint sensitivity analysis.

For each linear solver, the various solver performance counters are now initialized to 0 in both the solver specification function and in solver limit function. This ensures that these solver counters are initialized upon linear solver instantiation as well as at the beginning of the problem solution.
1.1 Changes from previous versions

A bug in for-loop indices was fixed in **IDAAckpntAllocVectors**. A bug was fixed in the interpolation functions used in solving backward problems.

A memory leak was fixed in the banded preconditioner interface. In addition, updates were done to return integers from linear solver and preconditioner 'free' functions.

In interpolation routines for backward problems, added logic to bypass sensitivity interpolation if input sensitivity argument is NULL.

The Krylov linear solver Bi-CGstab was enhanced by removing a redundant dot product. Various additions and corrections were made to the interfaces to the sparse solvers KLU and SuperLU_MT, including support for CSR format when using KLU.

New examples were added for use of the OpenMP vector and for use of sparse direct solvers within sensitivity integrations.

Minor corrections and additions were made to the **idas** solver, to the examples, to installation-related files, and to the user documentation.

**Changes in v1.2.0**

Two major additions were made to the linear system solvers that are available for use with the **idas** solver. First, in the serial case, an interface to the sparse direct solver KLU was added. Second, an interface to SuperLU_MT, the multi-threaded version of SuperLU, was added as a thread-parallel sparse direct solver option, to be used with the serial version of the **nvector** module. As part of these additions, a sparse matrix (CSC format) structure was added to **idas**.

Otherwise, only relatively minor modifications were made to **idas**:

In **IDARootfind**, a minor bug was corrected, where the input array **rootdir** was ignored, and a line was added to break out of root-search loop if the initial interval size is below the tolerance **ttol**.

In **IDALapackBand**, the line

\[ \text{smu} = \text{MIN}(N-1, \text{mu} + \text{ml}) \]

was changed to

\[ \text{smu} = \text{mu} + \text{ml} \]

correcting an illegal input error for **DGBTRF/DGBTRS**.

An option was added in the case of Adjoint Sensitivity Analysis with dense or banded Jacobian: With a call to **IDADlsSetDenseJacFnBS** or **IDADlsSetBandJacFnBS**, the user can specify a user-supplied Jacobian function of type **IDADls***JacFnBS**, for the case where the backward problem depends on the forward sensitivities.

A minor bug was fixed regarding the testing of the input **tstop** on the first call to **IDASolve**.

For the Adjoint Sensitivity Analysis case in which the backward problem depends on the forward sensitivities, options have been added to allow for user-supplied **pset**, **psolve**, and **jtimes** functions.

In order to avoid possible name conflicts, the mathematical macro and function names **MIN**, **MAX**, **SQR**, **RAbs**, **RSqrt**, **RExp**, **RPowerI**, and **RPowerR** were changed to **SUNMIN**, **SUNMAX**, **SUNSQR**, **SUNRabs**, **SUNRsqrt**, **SUNRexp**, **SRpowerI**, and **SUNRpowerR**, respectively. These names occur in both the solver and in various example programs.

In the User Guide, a paragraph was added in Section 6.2.1 on **IDAAdjReInit**, and a paragraph was added in Section 6.2.9 on **IDAGetAdjY**.

Two new **nvector** modules have been added for thread-parallel computing environments — one for OpenMP, denoted **NVECTOR_OPENMP**, and one for Pthreads, denoted **NVECTOR_PTHREADS**.

With this version of SUNDIALS, support and documentation of the Autotools mode of installation is being dropped, in favor of the CMake mode, which is considered more widely portable.

**Changes in v1.1.0**

One significant design change was made with this release: The problem size and its relatives, bandwidth parameters, related internal indices, pivot arrays, and the optional output **lsflag** have all been changed from type **int** to type **long int**, except for the problem size and bandwidths in user calls to routines specifying BLAS/LAPACK routines for the dense/band linear solvers. The function **NewIntArray** is replaced by a pair **NewIntArray/NewLIntArray**, for **int** and **long int** arrays, respectively. In a minor change to the user interface, the type of the index which in **idas** was changed from **long int** to **int**.

Errors in the logic for the integration of backward problems were identified and fixed.
A large number of minor errors have been fixed. Among these are the following: A missing vector pointer setting was added in IDASensLineSrch. In IDACompleteStep, conditionals around lines loading a new column of three auxiliary divided difference arrays, for a possible order increase, were fixed. After the solver memory is created, it is set to zero before being filled. In each linear solver interface function, the linear solver memory is freed on an error return, and the **Free function now includes a line setting to NULL the main memory pointer to the linear solver memory. A memory leak was fixed in two of the IDASp***Free functions. In the rootfinding functions IDARcheck1/IDARcheck2, when an exact zero is found, the array glo of g values at the left endpoint is adjusted, instead of shifting the t location tlo slightly. In the installation files, we modified the treatment of the macro SUNDIALS_USE_GENERIC_MATH, so that the parameter GENERIC_MATH_LIB is either defined (with no value) or not defined.

1.2 Reading this User Guide

The structure of this document is as follows:

- In Chapter 2, we give short descriptions of the numerical methods implemented by idas for the solution of initial value problems for systems of DAES, continue with short descriptions of preconditioning (§2.2) and rootfinding (§2.3), and then give an overview of the mathematical aspects of sensitivity analysis, both forward (§2.5) and adjoint (§2.6).

- The following chapter describes the structure of the SUNDIALS suite of solvers (§3.1) and the software organization of the idas solver (§3.2).

- Chapter 4 is the main usage document for idas for simulation applications. It includes a complete description of the user interface for the integration of DAE initial value problems. Readers that are not interested in using idas for sensitivity analysis can then skip the next two chapters.

- Chapter 5 describes the usage of idas for forward sensitivity analysis as an extension of its IVP integration capabilities. We begin with a skeleton of the user main program, with emphasis on the steps that are required in addition to those already described in Chapter 4. Following that we provide detailed descriptions of the user-callable interface routines specific to forward sensitivity analysis and of the additional optional user-defined routines.

- Chapter 6 describes the usage of idas for adjoint sensitivity analysis. We begin by describing the idas checkpointing implementation for interpolation of the original IVP solution during integration of the adjoint system backward in time, and with an overview of a user’s main program. Following that we provide complete descriptions of the user-callable interface routines for adjoint sensitivity analysis as well as descriptions of the required additional user-defined routines.

- Chapter 8 gives a brief overview of the generic NVECTOR module shared amongst the various components of SUNDIALS, as well as details on the NVECTOR implementations provided with SUNDIALS.

- Chapter 9 gives a brief overview of the generic SUNMATRIX module shared among the various components of SUNDIALS, and details on the SUNMATRIX implementations provided with SUNDIALS: a dense implementation (§9.3), a banded implementation (§9.4) and a sparse implementation (§9.5).

- Chapter 10 gives a brief overview of the generic SUNLINSOL module shared among the various components of SUNDIALS. This chapter contains details on the SUNLINSOL implementations provided with SUNDIALS. The chapter also contains details on the SUNLINSOL implementations provided with SUNDIALS that interface with external linear solver libraries.

- Chapter 11 describes the SUNNONLINSOL API and nonlinear solver implementations shared among the various components of SUNDIALS.
Finally, in the appendices, we provide detailed instructions for the installation of IDAS, within the structure of SUNDIALS (Appendix A), as well as a list of all the constants used for input to and output from IDAS functions (Appendix B).

Finally, the reader should be aware of the following notational conventions in this user guide: program listings and identifiers (such as IDAInit) within textual explanations appear in typewriter type style; fields in C structures (such as content) appear in italics; and packages or modules, such as IDALS, are written in all capitals. Usage and installation instructions that constitute important warnings are marked with a triangular symbol in the margin.

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1.3.3 SUNDIALS Release Numbers

- LLNL-CODE-667205 (ARKODE)
- UCRL-CODE-155951 (CVODE)
- UCRL-CODE-155950 (CVODES)
- UCRL-CODE-155952 (IDA)
- UCRL-CODE-237203 (IDAS)
- LLNL-CODE-665877 (KINSOL)
Chapter 2

Mathematical Considerations

IDAS solves the initial-value problem (IVP) for a DAE system of the general form

\[ F(t, y, \dot{y}) = 0, \quad y(t_0) = y_0, \quad \dot{y}(t_0) = \dot{y}_0, \]  

(2.1)

where \( y, \dot{y}, \) and \( F \) are vectors in \( \mathbb{R}^N, \) \( t \) is the independent variable, \( \dot{y} = dy/dt, \) and initial values \( y_0, \) \( \dot{y}_0 \) are given. (Often \( t \) is time, but it certainly need not be.)

Additionally, if (2.1) depends on some parameters \( p \in \mathbb{R}^{N_p}, \) i.e.

\[ F(t, y, \dot{y}, p) = 0 \]
\[ y(t_0) = y_0(p), \quad \dot{y}(t_0) = \dot{y}_0(p), \]  

(2.2)

IDAS can also compute first order derivative information, performing either forward sensitivity analysis or adjoint sensitivity analysis. In the first case, IDAS computes the sensitivities of the solution with respect to the parameters \( p, \) while in the second case, IDAS computes the gradient of a derived function with respect to the parameters \( p. \)

2.1 IVP solution

Prior to integrating a DAE initial-value problem, an important requirement is that the pair of vectors \( y_0 \) and \( \dot{y}_0 \) are both initialized to satisfy the DAE residual \( F(t_0, y_0, \dot{y}_0) = 0. \) For a class of problems that includes so-called semi-explicit index-one systems, IDAS provides a routine that computes consistent initial conditions from a user’s initial guess [10]. For this, the user must identify sub-vectors of \( y \) (not necessarily contiguous), denoted \( y_d \) and \( y_a, \) which are its differential and algebraic parts, respectively, such that \( F \) depends on \( \dot{y}_d \) but not on any components of \( \dot{y}_a. \) The assumption that the system is “index one” means that for a given \( t \) and \( y_d, \) the system \( F(t, y, \dot{y}) = 0 \) defines \( y_a \) uniquely. In this case, a solver within IDAS computes \( y_a \) and \( \dot{y}_a \) at \( t = t_0, \) given \( y_d \) and an initial guess for \( y_a. \) A second available option with this solver also computes all of \( y(t_0) \) given \( \dot{y}(t_0); \) this is intended mainly for quasi-steady-state problems, where \( \dot{y}(t_0) = 0 \) is given. In both cases, IDA solves the system \( F(t_0, y_0, \dot{y}_0) = 0 \) for the unknown components of \( y_0 \) and \( \dot{y}_0, \) using Newton iteration augmented with a line search global strategy. In doing this, it makes use of the existing machinery that is to be used for solving the linear systems during the integration, in combination with certain tricks involving the step size (which is set artificially for this calculation). For problems that do not fall into either of these categories, the user is responsible for passing consistent values, or risks failure in the numerical integration.

The integration method used in IDAS is the variable-order, variable-coefficient BDF (Backward Differentiation Formula), in fixed-leading-coefficient form [6]. The method order ranges from 1 to 5, with the BDF of order \( q \) given by the multistep formula

\[ \sum_{i=0}^{q} \alpha_{n,i} y_{n-i} = h_n \dot{y}_n, \]  

(2.3)
where $y_n$ and $\dot{y}_n$ are the computed approximations to $y(t_n)$ and $\dot{y}(t_n)$, respectively, and the step size is $h_n = t_n - t_{n-1}$. The coefficients $\alpha_{n,i}$ are uniquely determined by the order $q$, and the history of the step sizes. The application of the BDF (2.3) to the DAE system (2.1) results in a nonlinear algebraic system to be solved at each step:

$$G(y_n) \equiv F(t_n, y_n, h_n^{-1} \sum_{i=0}^{q} \alpha_{n,i} y_{n-i}) = 0.$$  \hspace{1cm} (2.4)

By default IDAS solves (2.4) with a Newton iteration but IDAS also allows for user-defined nonlinear solvers (see Chapter 11). Each Newton iteration requires the solution of a linear system of the form

$$J[y_{n(m+1)} - y_{n(m)}] = -G(y_{n(m)}),$$  \hspace{1cm} (2.5)

where $y_{n(m)}$ is the $m$-th approximation to $y_n$. Here $J$ is some approximation to the system Jacobian

$$J = \frac{\partial G}{\partial y} = \frac{\partial F}{\partial y} + \alpha \frac{\partial F}{\partial \dot{y}},$$  \hspace{1cm} (2.6)

where $\alpha = \alpha_{n,0}/h_n$. The scalar $\alpha$ changes whenever the step size or method order changes.

For the solution of the linear systems within the Newton iteration, IDAS provides several choices, including the option of a user-supplied linear solver module (see Chapter 10). The linear solver modules distributed with SUNDIALS are organized in two families, a direct family comprising direct linear solvers for dense, banded, or sparse matrices and a spils family comprising scaled preconditioned iterative (Krylov) linear solvers. The methods offered through these modules are as follows:

- dense direct solvers, using either an internal implementation or a BLAS/LAPACK implementation (serial or threaded vector modules only),
- band direct solvers, using either an internal implementation or a BLAS/LAPACK implementation (serial or threaded vector modules only),
- sparse direct solver interfaces, using either the KLU sparse solver library [19, 1], or the thread-enabled SuperLU_MT sparse solver library [39, 21, 4] (serial or threaded vector modules only) [Note that users will need to download and install the KLU or SUPERLUMT packages independent of IDAS],
- SPGMR, a scaled preconditioned GMRES (Generalized Minimal Residual method) solver with or without restarts,
- SPFGMRES, a scaled preconditioned FGMRES (Flexible Generalized Minimal Residual method) solver with or without restarts,
- SPBCGS, a scaled preconditioned Bi-CGSTab (Bi-Conjugate Gradient Stable method) solver,
- SPTFQMR, a scaled preconditioned TFQMR (Transpose-Free Quasi-Minimal Residual method) solver, or
- PCG, a scaled preconditioned CG (Conjugate Gradient method) solver.

For large stiff systems, where direct methods are not feasible, the combination of a BDF integrator and a preconditioned Krylov method yields a powerful tool because it combines established methods for stiff integration, nonlinear iteration, and Krylov (linear) iteration with a problem-specific treatment of the dominant source of stiffness, in the form of the user-supplied preconditioner matrix [8]. For the spils linear solvers with IDAS, preconditioning is allowed only on the left (see §2.2). Note that the dense, band, and sparse direct linear solvers can only be used with serial and threaded vector representations.
In the process of controlling errors at various levels, IDAS uses a weighted root-mean-square norm, denoted \( || \cdot ||_{\text{WRMS}} \), for all error-like quantities. The multiplicative weights used are based on the current solution and on the relative and absolute tolerances input by the user, namely

\[
W_i = 1/[\text{RTOL} \cdot |y_i| + \text{ATOL}_i].
\]

Because \( 1/W_i \) represents a tolerance in the component \( y_i \), a vector whose norm is 1 is regarded as “small.” For brevity, we will usually drop the subscript WRMS on norms in what follows.

In the case of a matrix-based linear solver, the default Newton iteration is a Modified Newton iteration, in that the Jacobian \( J \) is fixed (and usually out of date) throughout the nonlinear iterations, with a coefficient \( \alpha \) in place of \( \alpha \) in \( J \). However, in the case that a matrix-free iterative linear solver is used, the default Newton iteration is an Inexact Newton iteration, in which \( J \) is applied in a matrix-free manner, with matrix-vector products \( Jv \) obtained by either difference quotients or a user-supplied routine. In this case, the linear residual \( J\Delta y + G \) is nonzero but controlled.

The default stopping test for nonlinear solver iterations in IDAS ensures that the iteration error \( y_n - y_{n(m)} \) is small relative to \( y \) itself. For this, we estimate the linear convergence rate at all iterations \( m > 1 \) as

\[
R = \left( \frac{\delta_m}{\delta_1} \right)^{1/3},
\]

where the \( \delta_m = y_{n(m)} - y_{n(m-1)} \) is the correction at iteration \( m = 1, 2, \ldots \). The nonlinear solver iteration is halted if \( R > 0.9 \). The convergence test at the \( m \)-th iteration is then

\[
S||\delta_m|| < 0.33,
\]

where \( S = R/(R - 1) \) whenever \( m > 1 \) and \( R \leq 0.9 \). The user has the option of changing the constant in the convergence test from its default value of 0.33. The quantity \( S \) is set to \( S = 20 \) initially and whenever \( J \) or \( P \) is updated, and it is reset to \( S = 100 \) on a step with \( \alpha = \tilde{\alpha} \). Note that at \( m = 1 \), the convergence test \((2.8)\) uses an old value for \( S \). Therefore, at the first nonlinear solver iteration, we make an additional test and stop the iteration if \( ||\delta_1|| < 0.33 \cdot 10^{-4} \) (since such a \( \delta_1 \) is probably just noise and therefore not appropriate for use in evaluating \( R \)). We allow only a small number (default value 4) of nonlinear iterations. If convergence fails with \( J \) or \( P \) current, we are forced to reduce the step size \( h_n \), and we replace \( h_n \) by \( h_n/4 \). The integration is halted after a preset number (default value 10) of convergence failures. Both the maximum number of allowable nonlinear iterations and the maximum number of nonlinear convergence failures can be changed by the user from their default values.

When an iterative method is used to solve the linear system, to minimize the effect of linear iteration errors on the nonlinear and local integration error controls, we require the preconditioned linear residual to be small relative to the allowed error in the nonlinear iteration, i.e., \( ||P^{-1}(Jx + G)|| < 0.05 \cdot 0.33 \). The safety factor 0.05 can be changed by the user.

When the Jacobian is stored using either dense or band SUNMATRIX objects, the Jacobian \( J \) defined in \((2.6)\) can be either supplied by the user or have IDAS compute one internally by difference quotients. In the latter case, we use the approximation

\[
J_{ij} = [F_i(t, y + \sigma_j e_j, \dot{y} + \alpha \sigma_j e_j) - F_i(t, y, \dot{y})]/\sigma_j, \quad \text{with}
\]

\[
\sigma_j = \sqrt{U} \max \{ |y_j|, |h\dot{y}_j|, 1/W_j \} \text{sign}(h\dot{y}_j),
\]
where $U$ is the unit roundoff, $h$ is the current step size, and $W_y$ is the error weight for the component $y_j$ defined by (2.7). We note that with sparse and user-supplied SUNMATRIX objects, the Jacobian must be supplied by a user routine.

In the case of an iterative linear solver, if a routine for $Jv$ is not supplied, such products are approximated by

$$Jv = [F(t, y + \sigma v, \dot{y} + \alpha \sigma v) - F(t, y, \dot{y})]/\sigma,$$

where the increment $\sigma = 1/\|v\|$. As an option, the user can specify a constant factor that is inserted into this expression for $\sigma$.

During the course of integrating the system, IDAS computes an estimate of the local truncation error, LTE, at the $n$-th time step, and requires this to satisfy the inequality

$$\|LTE\|_{WRMS} \leq 1.$$

Asymptotically, LTE varies as $h^{q+1}$ at step size $h$ and order $q$, as does the predictor-corrector difference $\Delta_n \equiv y_n - y_n(0)$. Thus there is a constant $C$ such that

$$LTE = C\Delta_n + O(h^{q+2}),$$

and so the norm of LTE is estimated as $|C| \cdot \|\Delta_n\|$. In addition, IDAS requires that the error in the associated polynomial interpolant over the current step be bounded by 1 in norm. The leading term of the norm of this error is bounded by $\widetilde{C}\|\Delta_n\|$ for another constant $\widetilde{C}$. Thus the local error test in IDAS is

$$\max\{|C|, \widetilde{C}\|\Delta_n\| \leq 1. \quad (2.9)$$

A user option is available by which the algebraic components of the error vector are omitted from the test (2.9), if these have been so identified.

In IDAS, the local error test is tightly coupled with the logic for selecting the step size and order. First, there is an initial phase that is treated specially: for the first few steps, the step size is doubled and the order raised (from its initial value of 1) on every step, until (a) the local error test (2.9) fails, (b) the order is reduced (by the rules given below), or (c) the order reaches 5 (the maximum). For step and order selection on the general step, IDAS uses a different set of local error estimates, based on the asymptotic behavior of the local error in the case of fixed step sizes. At each of the orders $q'$ equal to $q$, $q - 1$ (if $q > 1$), $q - 2$ (if $q > 2$), or $q + 1$ (if $q < 5$), there are constants $C(q')$ such that the norm of the local truncation error at order $q'$ satisfies

$$LTE(q') = C(q')\|\phi(q' + 1)\| + O(h^{q' + 2}),$$

where $\phi(k)$ is a modified divided difference of order $k$ that is retained by IDAS (and behaves asymptotically as $h^k$). Thus the local truncation errors are estimated as $ELTE(q') = C(q')\|\phi(q' + 1)\|$ to select step sizes. But the choice of order in IDAS is based on the requirement that the scaled derivative norms, $\|h^ky(k)\|$, are monotonically decreasing with $k$, for $k$ near $q$. These norms are again estimated using the $\phi(k)$, and in fact

$$\|h^{q' + 1}y(q' + 1)\| \approx T(q') \equiv (q' + 1)ELTE(q').$$

The step/order selection begins with a test for monotonicity that is made even before the local error test is performed. Namely, the order is reset to $q' = q - 1$ if (a) $q = 2$ and $T(1) \leq T(2)/2$, or (b) $q > 2$ and $\max\{T(q - 1), T(q - 2)\} \leq T(q)$; otherwise $q' = q$. Next the local error test (2.9) is performed, and if it fails, the step is redone at order $q' \leftarrow q'$ and a new step size $h'$. The latter is based on the $h^{q+1}$ asymptotic behavior of $ELTE(q)$, and, with safety factors, is given by

$$\eta = h'/h = 0.9/[2ELTE(q)]^{1/(q+1)}.$$

The value of $\eta$ is adjusted so that $0.25 \leq \eta < 0.9$ before setting $h \leftarrow h' = \eta h$. If the local error test fails a second time, IDAS uses $\eta = 0.25$, and on the third and subsequent failures it uses $q = 1$ and $\eta = 0.25$. After 10 failures, IDAS returns with a give-up message.
As soon as the local error test has passed, the step and order for the next step may be adjusted. No such change is made if \( q' = q - 1 \) from the prior test, if \( q = 5 \), or if \( q \) was increased on the previous step. Otherwise, if the last \( q + 1 \) steps were taken at a constant order \( q < 5 \) and a constant step size, IDAS considers raising the order to \( q + 1 \). The logic is as follows: (a) If \( q = 1 \), then reset \( q = 2 \) if \( T(2) < T(1)/2 \). (b) If \( q > 1 \) then

- reset \( q \leftarrow q - 1 \) if \( T(q - 1) \leq \min\{T(q), T(q + 1)\} \);
- else reset \( q \leftarrow q + 1 \) if \( T(q + 1) < T(q) \);
- leave \( q \) unchanged otherwise [then \( T(q - 1) > T(q) \leq T(q + 1) \)].

In any case, the new step size \( h' \) is set much as before:

\[
\eta = h'/h = 1/[2 \text{ ELTE}(q)]^{1/(q+1)}.
\]

The value of \( \eta \) is adjusted such that (a) if \( \eta > 2 \), \( \eta \) is reset to 2; (b) if \( \eta \leq 1 \), \( \eta \) is restricted to \( 0.5 \leq \eta \leq 0.9 \); and (c) if \( 1 < \eta < 2 \) we use \( \eta = 1 \). Finally \( h \) is reset to \( h' = \eta h \). Thus we do not increase the step size unless it can be doubled. See [6] for details.

IDAS permits the user to impose optional inequality constraints on individual components of the solution vector \( y \). Any of the following four constraints can be imposed: \( y_i > 0 \), \( y_i < 0 \), \( y_i \geq 0 \), or \( y_i \leq 0 \). The constraint satisfaction is tested after a successful nonlinear system solution. If any constraint fails, we declare a convergence failure of the nonlinear iteration and reduce the step size. Rather than cutting the step size by some arbitrary factor, IDAS estimates a new step size \( h' \) using a linear approximation of the components in \( y \) that failed the constraint test (including a safety factor of 0.9 to cover the strict inequality case). These additional constraints are also imposed during the calculation of consistent initial conditions. If a step fails to satisfy the constraints repeatedly within a step attempt then the integration is halted and an error is returned. In this case the user may need to employ other strategies as discussed in §4.5.2 to satisfy the inequality constraints.

Normally, IDAS takes steps until a user-defined output value \( t = t_{\text{out}} \) is overtaken, and then computes \( y(t_{\text{out}}) \) by interpolation. However, a “one step” mode option is available, where control returns to the calling program after each step. There are also options to force IDAS not to integrate past a given stopping point \( t = t_{\text{stop}} \).

## 2.2 Preconditioning

When using a nonlinear solver that requires the solution of a linear system of the form \( J \Delta y = -G \) (e.g., the default Newton iteration), IDAS makes repeated use of a linear solver. If this linear system solve is done with one of the scaled preconditioned iterative linear solvers supplied with SUNDIALS, these solvers are rarely successful if used without preconditioning; it is generally necessary to precondition the system in order to obtain acceptable efficiency. A system \( Ax = b \) can be preconditioned on the left, on the right, or on both sides. The Krylov method is then applied to a system with the matrix \( P^{-1}A \), or \( AP^{-1} \), or \( P_L^{-1}AP_R^{-1} \) instead of \( A \). However, within IDAS, preconditioning is allowed only on the left, so that the iterative method is applied to systems \( (P^{-1}J) \Delta y = -P^{-1}G \). Left preconditioning is required to make the norm of the linear residual in the nonlinear iteration meaningful; in general, \( \|J \Delta y + G\| \) is meaningless, since the weights used in the WRMS-norm correspond to \( y \).

In order to improve the convergence of the Krylov iteration, the preconditioner matrix \( P \) should in some sense approximate the system matrix \( A \). Yet at the same time, in order to be cost-effective, the matrix \( P \) should be reasonably efficient to evaluate and solve. Finding a good point in this tradeoff between rapid convergence and low cost can be very difficult. Good choices are often problem-dependent (for example, see [8] for an extensive study of preconditioners for reaction-transport systems).

Typical preconditioners used with IDAS are based on approximations to the iteration matrix of the systems involved; in other words, \( P \approx \alpha \frac{\partial F}{\partial y} + \alpha \frac{\partial F}{\partial y} \), where \( \alpha \) is a scalar inversely proportional to the integration step size \( h \). Because the Krylov iteration occurs within a nonlinear solver iteration and further also within a time integration, and since each of these iterations has its own test for
convergence, the preconditioner may use a very crude approximation, as long as it captures the dominant numerical feature(s) of the system. We have found that the combination of a preconditioner with the Newton-Krylov iteration, using even a fairly poor approximation to the Jacobian, can be surprisingly superior to using the same matrix without Krylov acceleration (i.e., a modified Newton iteration), as well as to using the Newton-Krylov method with no preconditioning.

### 2.3 Rootfinding

The IDAS solver has been augmented to include a rootfinding feature. This means that, while integrating the Initial Value Problem (2.1), IDAS can also find the roots of a set of user-defined functions \( g_i(t, y, \dot{y}) \) that depend on \( t \), the solution vector \( y = y(t) \), and its \( t \)-derivative \( \dot{y}(t) \). The number of these root functions is arbitrary, and if more than one \( g_i \) is found to have a root in any given interval, the various root locations are found and reported in the order that they occur on the \( t \) axis, in the direction of integration.

Generally, this rootfinding feature finds only roots of odd multiplicity, corresponding to changes in sign of \( g_i(t, y(t), \dot{y}(t)) \), denoted \( g_i(t) \) for short. If a user root function has a root of even multiplicity (no sign change), it will probably be missed by IDAS. If such a root is desired, the user should reformulate the root function so that it changes sign at the desired root.

The basic scheme used is to check for sign changes of any \( g_i(t) \) over each time step taken, and then (when a sign change is found) to home in on the root (or roots) with a modified secant method [27]. In addition, each time \( g \) is computed, IDAS checks to see if \( g_i(t) = 0 \) exactly, and if so it reports this as a root. However, if an exact zero of any \( g_i \) is found at a point \( t \), IDAS computes \( g \) at \( t + \delta \) for a small increment \( \delta \), slightly further in the direction of integration, and if any \( g_i(t + \delta) = 0 \) also, IDAS stops and reports an error. This way, each time IDAS takes a time step, it is guaranteed that the values of all \( g_i \) are nonzero at some past value of \( t \), beyond which a search for roots is to be done.

At any given time in the course of the time-stepping, after suitable checking and adjusting has been done, IDAS has an interval \([t_{lo}, t_{hi}]\) in which roots of the \( g_i(t) \) are to be sought, such that \( t_{hi} \) is further ahead in the direction of integration, and all \( g_i(t_{lo}) \neq 0 \). The endpoint \( t_{hi} \) is either \( t_n \), the end of the time step last taken, or the next requested output time \( t_{out} \) if this comes sooner. The endpoint \( t_{lo} \) is either \( t_{n-1} \), or the last output time \( t_{out} \) (if this occurred within the last step), or the last root location (if a root was just located within this step), possibly adjusted slightly toward \( t_n \) if an exact zero was found. The algorithm checks \( g \) at \( t_{hi} \) for zeros and for sign changes in \((t_{lo}, t_{hi})\). If no sign changes are found, then either a root is reported (if some \( g_i(t_{hi}) = 0 \)) or we proceed to the next time interval (starting at \( t_{hi} \)). If one or more sign changes were found, then a loop is entered to locate the root to within a rather tight tolerance, given by

\[
\tau = 100 \times U \times (|t_n| + |h|) \quad (U = \text{unit roundoff})
\]

Whenever sign changes are seen in two or more root functions, the one deemed most likely to have its root occur first is the one with the largest value of \(|g_i(t_{hi})|/|g_i(t_{hi}) - g_i(t_{lo})|\), corresponding to the closest to \( t_{lo} \) of the secant method values. At each pass through the loop, a new value \( t_{mid} \) is set, strictly within the search interval, and the values of \( g_i(t_{mid}) \) are checked. Then either \( t_{lo} \) or \( t_{hi} \) is reset to \( t_{mid} \) according to which subinterval is found to have the sign change. If there is none in \((t_{lo}, t_{mid})\) but some \( g_i(t_{mid}) = 0 \), then that root is reported. The loop continues until \(|t_{hi} - t_{lo}| < \tau \), and then the reported root location is \( t_{hi} \).

In the loop to locate the root of \( g_i(t) \), the formula for \( t_{mid} \) is

\[
t_{mid} = t_{hi} - (t_{hi} - t_{lo})g_i(t_{hi})/[g_i(t_{hi}) - \alpha g_i(t_{lo})],
\]

where \( \alpha \) a weight parameter. On the first two passes through the loop, \( \alpha \) is set to 1, making \( t_{mid} \) the secant method value. Thereafter, \( \alpha \) is reset according to the side of the subinterval (low vs high, i.e. toward \( t_{lo} \) vs toward \( t_{hi} \)) in which the sign change was found in the previous two passes. If the two sides were opposite, \( \alpha \) is set to 1. If the two sides were the same, \( \alpha \) is halved (if on the low side) or doubled (if on the high side). The value of \( t_{mid} \) is closer to \( t_{lo} \) when \( \alpha < 1 \) and closer to \( t_{hi} \).
when $\alpha > 1$. If the above value of $t_{mid}$ is within $\tau/2$ of $t_{lo}$ or $t_{hi}$, it is adjusted inward, such that its fractional distance from the endpoint (relative to the interval size) is between .1 and .5 (.5 being the midpoint), and the actual distance from the endpoint is at least $\tau/2$.

### 2.4 Pure quadrature integration

In many applications, and most notably during the backward integration phase of an adjoint sensitivity analysis run (see §2.6) it is of interest to compute integral quantities of the form

$$z(t) = \int_{t_0}^{t} q(\tau, y(\tau), \dot{y}(\tau), p) \, d\tau. \quad (2.10)$$

The most effective approach to compute $z(t)$ is to extend the original problem with the additional ODEs (obtained by applying Leibnitz’s differentiation rule):

$$\dot{z} = q(t, y, \dot{y}, p), \quad z(t_0) = 0. \quad (2.11)$$

Note that this is equivalent to using a quadrature method based on the underlying linear multistep polynomial representation for $y(t)$. This can be done at the “user level” by simply exposing to IDAS the extended DAE system (2.2)+(2.10). However, in the context of an implicit integration solver, this approach is not desirable since the nonlinear solver module will require the Jacobian (or Jacobian-vector product) of this extended DAE. Moreover, since the additional states, $z$, do not enter the right-hand side of the ODE (2.10) and therefore the residual of the extended DAE system does not depend on $z$, it is much more efficient to treat the ODE system (2.10) separately from the original DAE system (2.2) by “taking out” the additional states $z$ from the nonlinear system (2.4) that must be solved in the correction step of the LMM. Instead, “corrected” values $z_n$ are computed explicitly as

$$z_n = \frac{1}{\alpha_n,0} \left( h_n q(t_n, y_n, \dot{y}_n, p) - \sum_{i=1}^{q} \alpha_{n,i} z_{n-i} \right),$$

once the new approximation $y_n$ is available.

The quadrature variables $z$ can be optionally included in the error test, in which case corresponding relative and absolute tolerances must be provided.

### 2.5 Forward sensitivity analysis

Typically, the governing equations of complex, large-scale models depend on various parameters, through the right-hand side vector and/or through the vector of initial conditions, as in (2.2). In addition to numerically solving the DAEs, it may be desirable to determine the sensitivity of the results with respect to the model parameters. Such sensitivity information can be used to estimate which parameters are most influential in affecting the behavior of the simulation or to evaluate optimization gradients (in the setting of dynamic optimization, parameter estimation, optimal control, etc.).

The solution sensitivity with respect to the model parameter $p_i$ is defined as the vector $s_i(t) = \partial y(t)/\partial p_i$ and satisfies the following forward sensitivity equations (or sensitivity equations for short):

$$\frac{\partial F}{\partial y} s_i + \frac{\partial F}{\partial y} \dot{s}_i + \frac{\partial F}{\partial p_i} = 0$$

$$s_i(t_0) = \frac{\partial y_0(p)}{\partial p_i}, \quad \dot{s}_i(t_0) = \frac{\partial \dot{y}_0(p)}{\partial p_i}, \quad (2.12)$$

obtained by applying the chain rule of differentiation to the original DAEs (2.2).

When performing forward sensitivity analysis, IDAS carries out the time integration of the combined system, (2.2) and (2.12), by viewing it as a DAE system of size $N(N_s + 1)$, where $N_s$ is the number
of model parameters \( p_i \), with respect to which sensitivities are desired \( (N_s \leq N_p) \). However, major improvements in efficiency can be made by taking advantage of the special form of the sensitivity equations as linearizations of the original DAEs. In particular, the original DAE system and all sensitivity systems share the same Jacobian matrix \( J \) in (2.6).

The sensitivity equations are solved with the same linear multistep formula that was selected for the original DAEs and the same linear solver is used in the correction phase for both state and sensitivity variables. In addition, IDAS offers the option of including (full error control) or excluding (partial error control) the sensitivity variables from the local error test.

### 2.5.1 Forward sensitivity methods

In what follows we briefly describe three methods that have been proposed for the solution of the combined DAE and sensitivity system for the vector \( \hat{y} = [y, s_1, \ldots, s_{N_s}] \).

- **Staggered Direct** In this approach \([16]\), the nonlinear system (2.4) is first solved and, once an acceptable numerical solution is obtained, the sensitivity variables at the new step are found by solving (2.12) after the BDF discretization is used to eliminate \( \dot{s}_i \). Although the system matrix of the above linear system is based on exactly the same information as the matrix \( J \) in (2.6), it must be updated and factored at every step of the integration, in contrast to an evaluation of \( J \) which is updated only occasionally. For problems with many parameters (relative to the problem size), the staggered direct method can outperform the methods described below \([38]\). However, the computational cost associated with matrix updates and factorizations makes this method unattractive for problems with many more states than parameters (such as those arising from semidiscretization of PDEs) and is therefore not implemented in IDAS.

- **Simultaneous Corrector** In this method \([42]\), the discretization is applied simultaneously to both the original equations (2.2) and the sensitivity systems (2.12) resulting in an “extended” nonlinear system \( \hat{G}(\hat{y}_n) = 0 \) where \( \hat{y}_n = [y_n, \ldots, s_i, \ldots] \). This combined nonlinear system can be solved using a modified Newton method as in (2.5) by solving the corrector equation

\[
\hat{J}[\hat{y}_{n(m+1)} - \hat{y}_{n(m)}] = -\hat{G}(\hat{y}_{n(m)})
\]

at each iteration, where

\[
\hat{J} = \begin{bmatrix}
J & J \\
J_1 & 0 & J \\
\vdots & \ddots & \ddots \\
J_{Ns} & 0 & \ldots & 0 \\
\end{bmatrix},
\]

\( J \) is defined as in (2.6), and \( J_i = \langle \partial / \partial y \rangle [F_y s_i + F_y \dot{s}_i + F_{p_i}] \). It can be shown that 2-step quadratic convergence can be retained by using only the block-diagonal portion of \( \hat{J} \) in the corrector equation (2.13). This results in a decoupling that allows the reuse of \( \hat{J} \) without additional matrix factorizations. However, the sum \( F_y s_i + F_y \dot{s}_i + F_{p_i} \) must still be reevaluated at each step of the iterative process (2.13) to update the sensitivity portions of the residual \( \hat{G} \).

- **Staggered corrector** In this approach \([23]\), as in the staggered direct method, the nonlinear system (2.4) is solved first using the Newton iteration (2.5). Then, for each sensitivity vector \( \xi \equiv s_i \), a separate Newton iteration is used to solve the sensitivity system (2.12):

\[
J[\xi_{n(m+1)} - \xi_{n(m)}] = - \left[ F_y(t_n, y_n, \dot{y}_n) \xi_{n(m)} + F_y(t_n, y_n, \dot{y}_n) \cdot h_n^{-1} \left( \alpha_{n,0} \xi_{n(m)} + \sum_{i=1}^{q} \alpha_{n,i} \xi_{n-i} \right) + F_{p}(t_n, y_n, \dot{y}_n) \right].
\]

(2.14)
2.5 Forward sensitivity analysis

In other words, a modified Newton iteration is used to solve a linear system. In this approach, the matrices \( \partial F/\partial y, \partial F/\partial \dot{y} \) and vectors \( \partial F/\partial p_i \) need be updated only once per integration step, after the state correction phase (2.5) has converged.

IDAS implements both the simultaneous corrector method and the staggered corrector method.

An important observation is that the staggered corrector method, combined with a Krylov linear solver, effectively results in a staggered direct method. Indeed, the Krylov solver requires only the action of the matrix \( J \) on a vector, and this can be provided with the current Jacobian information. Therefore, the modified Newton procedure (2.14) will theoretically converge after one iteration.

2.5.2 Selection of the absolute tolerances for sensitivity variables

If the sensitivities are included in the error test, IDAS provides an automated estimation of absolute tolerances for the sensitivity variables based on the absolute tolerance for the corresponding state variable. The relative tolerance for sensitivity variables is set to be the same as for the state variables. However, this choice of tolerances for the sensitivity variables based on the absolute tolerance for the corresponding state variables and \( \bar{p} \), is a vector of scaling factors that are dimensionally consistent with the model parameters \( p \) and give an indication of their order of magnitude. This choice of relative and absolute tolerances is equivalent to requiring that the weighted root-mean-square norm of the sensitivity vector \( \bar{s}_i \) with weights based on \( s_i \) be the same as the weighted root-mean-square norm of the vector of scaled sensitivities \( \bar{s}_i = [\bar{p}_i] s_i \) with weights based on the state variables (the scaled sensitivities \( \bar{s}_i \) being dimensionally consistent with the state variables). However, this choice of tolerances for the \( s_i \) may be a poor one, and the user of IDAS can provide different values as an option.

2.5.3 Evaluation of the sensitivity right-hand side

There are several methods for evaluating the residual functions in the sensitivity systems (2.12): analytic evaluation, automatic differentiation, complex-step approximation, and finite differences (or directional derivatives). IDAS provides all the software hooks for implementing interfaces to automatic differentiation (AD) or complex-step approximation; future versions will include a generic interface to AD-generated functions. At the present time, besides the option for analytical sensitivity right-hand sides (user-provided), IDAS can evaluate these quantities using various finite difference-based approximations to evaluate the terms \( (\partial F/\partial y)s_i + (\partial F/\partial \dot{y})\dot{s}_i \) and \( (\partial F/\partial p_i) \), or using directional derivatives to evaluate \( [(\partial F/\partial y)s_i + (\partial F/\partial \dot{y})\dot{s}_i + (\partial F/\partial p_i)] \). As is typical for finite differences, the proper choice of perturbations is a delicate matter. IDAS takes into account several problem-related features: the relative DAE error tolerance \( \text{RTOL} \), the machine unit roundoff \( U \), the scale factor \( \bar{p}_i \), and the weighted root-mean-square norm of the sensitivity vector \( s_i \).

Using central finite differences as an example, the two terms \( (\partial F/\partial y)s_i + (\partial F/\partial \dot{y})\dot{s}_i \) and \( \partial F/\partial p_i \) in (2.12) can be evaluated either separately:

\[
\frac{\partial F}{\partial y} \, s_i + \frac{\partial F}{\partial \dot{y}} \, \dot{s}_i \approx \frac{F(t, y + \sigma s_i, \dot{y} + \sigma \dot{s}_i, p) - F(t, y - \sigma s_i, \dot{y} - \sigma \dot{s}_i, p)}{2 \sigma_y},
\]

\[
\frac{\partial F}{\partial p_i} \approx \frac{F(t, y, \dot{y}, p + \sigma e_i) - F(t, y, \dot{y}, p - \sigma e_i)}{2 \sigma_t},
\]

\[
\sigma_t = |\bar{p}_i| \sqrt{\max(\text{RTOL}, U)}, \quad \sigma_y = \frac{1}{\max(1/\sigma_t, \|s_i\|_{\text{WRMS}}/|\bar{p}_i|)},
\]

or simultaneously:

\[
\frac{\partial F}{\partial y} \, s_i + \frac{\partial F}{\partial \dot{y}} \, \dot{s}_i + \frac{\partial F}{\partial p_i} \approx \frac{F(t, y + \sigma s_i, \dot{y} + \sigma \dot{s}_i, p + \sigma e_i) - F(t, y - \sigma s_i, \dot{y} - \sigma \dot{s}_i, p - \sigma e_i)}{2 \sigma},
\]

\[
\sigma = \min(\sigma_t, \sigma_y),
\]
or by adaptively switching between \((2.15)+(2.15')\) and \(2.16\), depending on the relative size of the two finite difference increments \(\sigma_i\) and \(\sigma_y\). In the adaptive scheme, if \(\rho = \max(\sigma_i/\sigma_y, \sigma_y/\sigma_i)\), we use separate evaluations if \(\rho > \rho_{\text{max}}\) (an input value), and simultaneous evaluations otherwise.

These procedures for choosing the perturbations \((\sigma_i, \sigma_y, \sigma)\) and switching between derivative formulas have also been implemented for one-sided difference formulas. Forward finite differences can be applied to \((\partial F/\partial y) s_i + (\partial F/\partial \dot{y}) \dot{s}_i\) and \(\partial F/\partial p_i\) separately, or the single directional derivative formula

\[
\frac{\partial F}{\partial y} s_i + \frac{\partial F}{\partial \dot{y}} \dot{s}_i + \frac{\partial F}{\partial p_i} \approx \frac{F(t, y + \sigma s_i, \dot{y} + \sigma \dot{s}_i, p + \sigma e_i) - F(t, y, \dot{y}, p)}{\sigma}
\]

can be used. In IDAS, the default value of \(\rho_{\text{max}} = 0\) indicates the use of the second-order centered directional derivative formula \(2.16\) exclusively. Otherwise, the magnitude of \(\rho_{\text{max}}\) and its sign (positive or negative) indicates whether this switching is done with regard to (centered or forward) finite differences, respectively.

### 2.5.4 Quadratures depending on forward sensitivities

If pure quadrature variables are also included in the problem definition (see §2.4), IDAS does not carry their sensitivities automatically. Instead, we provide a more general feature through which integrals depending on both the states \(y\) of \((2.2)\) and the state sensitivities \(s_i\) of \((2.12)\) can be evaluated. In other words, IDAS provides support for computing integrals of the form:

\[
\bar{z}(t) = \int_{t_0}^{t} \bar{q}(\tau, y(\tau), \dot{y}(\tau), s_1(\tau), \ldots, s_{N_p}(\tau), p) d\tau.
\]

If the sensitivities of the quadrature variables \(z\) of \((2.10)\) are desired, these can then be computed by using:

\[
\bar{q}_i = q_y s_i + q_\dot{y} \dot{s}_i + q_p, \quad i = 1, \ldots, N_p,
\]

as integrands for \(\bar{z}\), where \(q_y\), \(q_\dot{y}\), and \(q_p\) are the partial derivatives of the integrand function \(q\) of \((2.10)\).

As with the quadrature variables \(z\), the new variables \(\bar{z}\) are also excluded from any nonlinear solver phase and “corrected” values \(\bar{z}_n\) are obtained through explicit formulas.

### 2.6 Adjoint sensitivity analysis

In the forward sensitivity approach described in the previous section, obtaining sensitivities with respect to \(N_s\) parameters is roughly equivalent to solving an DAE system of size \((1 + N_s)N\). This can become prohibitively expensive, especially for large-scale problems, if sensitivities with respect to many parameters are desired. In this situation, the adjoint sensitivity method is a very attractive alternative, provided that we do not need the solution sensitivities \(s_i\), but rather the gradients with respect to model parameters of a relatively few derived functionals of the solution. In other words, if \(y(t)\) is the solution of \((2.2)\), we wish to evaluate the gradient \(dG/dp\) of

\[
G(p) = \int_{t_0}^{T} g(t, y, p) dt,
\]

or, alternatively, the gradient \(dg/dp\) of the function \(g(t, y, p)\) at the final time \(t = T\). The function \(g\) must be smooth enough that \(\partial g/\partial y\) and \(\partial g/\partial p\) exist and are bounded.

In what follows, we only sketch the analysis for the sensitivity problem for both \(G\) and \(g\). For details on the derivation see [15].
2.6 Adjoint sensitivity analysis

2.6.1 Sensitivity of \( G(p) \)

We focus first on solving the sensitivity problem for \( G(p) \) defined by \((2.17)\). Introducing a Lagrange multiplier \( \lambda \), we form the augmented objective function

\[
I(p) = G(p) - \int_{t_0}^{T} \lambda^* F(t, y, \dot{y}, p) dt.
\]

Since \( F(t, y, \dot{y}, p) = 0 \), the sensitivity of \( G \) with respect to \( p \)

\[
\frac{dG}{dp} = \frac{dI}{dp} = \int_{t_0}^{T} (g_p + g_y y_p) dt - \int_{t_0}^{T} \lambda^* (F_p + F_y y_p + \dot{F}_y y_p) dt,
\]

where subscripts on functions such as \( F \) or \( g \) are used to denote partial derivatives. By integration by parts, we have

\[
\int_{t_0}^{T} \lambda^* \dot{g} y_p dt = (\lambda^* F_y y_p)|_{t_0}^{T} - \int_{t_0}^{T} (\lambda^* F_y)' y_p dt,
\]

where \((\cdots)'\) denotes the \( t \)-derivative. Thus equation \((2.18)\) becomes

\[
\frac{dG}{dp} = \int_{t_0}^{T} (g_p - \lambda^* F_p) dt - \int_{t_0}^{T} [-g_y + \lambda^* F_y - (\lambda^* F_y)'] y_p dt - (\lambda^* F_y y_p)|_{t_0}^{T}.
\]

Now by requiring \( \lambda \) to satisfy

\[
(\lambda^* F_y)' - \lambda^* F_y = -g_y,
\]

we obtain

\[
\frac{dG}{dp} = \int_{t_0}^{T} (g_p - \lambda^* F_p) dt - (\lambda^* F_y y_p)|_{t_0}.
\]

Note that \( y_p \) at \( t = t_0 \) is the sensitivity of the initial conditions with respect to \( p \), which is easily obtained. To find the initial conditions (at \( t = T \)) for the adjoint system, we must take into consideration the structure of the DAE system.

For index-0 and index-1 DAE systems, we can simply take

\[
\lambda^* F_y|_{t=T} = 0,
\]

yielding the sensitivity equation for \( dG/dp \)

\[
\frac{dG}{dp} = \int_{t_0}^{T} (g_p - \lambda^* F_p) dt + (\lambda^* F_y y_p)|_{t=t_0}.
\]

This choice will not suffice for a Hessenberg index-2 DAE system. For a derivation of proper final conditions in such cases, see [15].

The first thing to notice about the adjoint system \((2.20)\) is that there is no explicit specification of the parameters \( p \); this implies that, once the solution \( \lambda \) is found, the formula \((2.21)\) can then be used to find the gradient of \( G \) with respect to any of the parameters \( p \). The second important remark is that the adjoint system \((2.20)\) is a terminal value problem which depends on the solution \( y(t) \) of the original IVP \((2.2)\). Therefore, a procedure is needed for providing the states \( y \) obtained during a forward integration phase of \((2.2)\) to IDAS during the backward integration phase of \((2.20)\). The approach adopted in IDAS, based on checkpointing, is described in §2.6.3 below.

2.6.2 Sensitivity of \( g(T, p) \)

Now let us consider the computation of \( dg/dp(T) \). From \( dg/dp(T) = (d/dT)(dG/dp) \) and equation \((2.21)\), we have

\[
\frac{dg}{dp} = (g_p - \lambda^* F_p)(T) - \int_{t_0}^{T} \lambda^* F_p dt + (\lambda^* F_y y_p)|_{t=t_0} - \frac{d(\lambda^* F_y y_p)}{dT}.
\]
where $\lambda_T$ denotes $\partial \lambda / \partial T$. For index-0 and index-1 DAEs, we obtain

$$\frac{d(\lambda^* F_y y_p)}{dT}|_{t=T} = 0,$$

while for a Hessenberg index-2 DAE system we have

$$\frac{d(\lambda^* F_y y_p)}{dT}|_{t=T} = -\frac{d(g y (CB)^{-1} f^2_p)}{dt}|_{t=T}.$$

The corresponding adjoint equations are

$$(\lambda_T^* F_y)' - \lambda_T^* F_y y_p|_{t=T} = 0.$$ \tag{2.25}

For index-0 and index-1 DAEs (as shown above, the index-2 case is different), to find the boundary condition for this equation we write $\lambda$ as $\lambda(t, T)$ because it depends on both $t$ and $T$. Then

$$\lambda^*(T, T) F_y|_{t=T} = 0.$$

Taking the total derivative, we obtain

$$(\lambda_t + \lambda_T)^*(T, T) F_y|_{t=T} + \lambda^*(T, T) \frac{dF_y}{dt}|_{t=T} = 0.$$

Since $\lambda_t$ is just $\dot{\lambda}$, we have the boundary condition

$$(\lambda_T^* F_y)|_{t=T} = -\left[\lambda^*(T, T) \frac{dF_y}{dt} + \dot{\lambda}^* F_y\right]|_{t=T}.$$

For the index-one DAE case, the above relation and (2.20) yield

$$(\lambda_T^* F_y)|_{t=T} = [g y - \lambda^* F_y]|_{t=T}.$$ \tag{2.26}

For the regular implicit ODE case, $F_y$ is invertible; thus we have $\lambda(T, T) = 0$, which leads to $\lambda_T(T) = -\lambda(T)$. As with the final conditions for $\lambda(T)$ in (2.20), the above selection for $\lambda_T(T)$ is not sufficient for index-two Hessenberg DAEs (see [15] for details).

### 2.6.3 Checkpointing scheme

During the backward integration, the evaluation of the right-hand side of the adjoint system requires, at the current time, the states $y$ which were computed during the forward integration phase. Since IDAS implements variable-step integration formulas, it is unlikely that the states will be available at the desired time and so some form of interpolation is needed. The IDAS implementation being also variable-order, it is possible that during the forward integration phase the order may be reduced as low as first order, which means that there may be points in time where only $y$ and $\dot{y}$ are available. These requirements therefore limit the choices for possible interpolation schemes. IDAS implements two interpolation methods: a cubic Hermite interpolation algorithm and a variable-degree polynomial interpolation method which attempts to mimic the BDF interpolant for the forward integration.

However, especially for large-scale problems and long integration intervals, the number and size of the vectors $y$ and $\dot{y}$ that would need to be stored make this approach computationally intractable. Thus, IDAS settles for a compromise between storage space and execution time by implementing a so-called checkpointing scheme. At the cost of at most one additional forward integration, this approach offers the best possible estimate of memory requirements for adjoint sensitivity analysis. To begin with, based on the problem size $N$ and the available memory, the user decides on the number $N_d$ of data pairs $(y, \dot{y})$ if cubic Hermite interpolation is selected, or on the number $N_d$ of $y$ vectors in the case of variable-degree polynomial interpolation, that can be kept in memory for the purpose of interpolation. Then, during the first forward integration stage, after every $N_d$ integration steps a
2.7 Second-order sensitivity analysis

Figure 2.1: Illustration of the checkpointing algorithm for generation of the forward solution during the integration of the adjoint system.

checkpoint is formed by saving enough information (either in memory or on disk) to allow for a hot restart, that is a restart which will exactly reproduce the forward integration. In order to avoid storing Jacobian-related data at each checkpoint, a reevaluation of the iteration matrix is forced before each checkpoint. At the end of this stage, we are left with \( N_c \) checkpoints, including one at \( t_0 \). During the backward integration stage, the adjoint variables are integrated backwards from \( T \) to \( t_0 \), going from one checkpoint to the previous one. The backward integration from checkpoint \( i + 1 \) to checkpoint \( i \) is preceded by a forward integration from \( i \) to \( i + 1 \) during which the \( N_d \) vectors \( y \) (and, if necessary \( \dot{y} \)) are generated and stored in memory for interpolation.

This approach transfers the uncertainty in the number of integration steps in the forward integration phase to uncertainty in the final number of checkpoints. However, \( N_c \) is much smaller than the number of steps taken during the forward integration, and there is no major penalty for writing/reading the checkpoint data to/from a temporary file. Note that, at the end of the first forward integration stage, interpolation data are available from the last checkpoint to the end of the interval of integration. If no checkpoints are necessary (\( N_d \) is larger than the number of integration steps taken in the solution of (2.2)), the total cost of an adjoint sensitivity computation can be as low as one forward plus one backward integration. In addition, IDAS provides the capability of reusing a set of checkpoints for multiple backward integrations, thus allowing for efficient computation of gradients of several functionals (2.17).

Finally, we note that the adjoint sensitivity module in IDAS provides the necessary infrastructure to integrate backwards in time any DAE terminal value problem dependent on the solution of the IVP (2.2), including adjoint systems (2.20) or (2.25), as well as any other quadrature ODEs that may be needed in evaluating the integrals in (2.21). In particular, for DAE systems arising from semi-discretization of time-dependent PDEs, this feature allows for integration of either the discretized adjoint PDE system or the adjoint of the discretized PDE.

2.7 Second-order sensitivity analysis

In some applications (e.g., dynamically-constrained optimization) it may be desirable to compute second-order derivative information. Considering the DAE problem (2.2) and some model output functional\(^1\) \( g(y) \), the Hessian \( \frac{d^2 g}{dp^2} \) can be obtained in a forward sensitivity analysis setting as

\[
\frac{d^2 g}{dp^2} = \left( g_y \otimes I_{N_p} \right) y_{pp} + y_p^T g_{yy} y_p ,
\]

\(^1\)The degree of the interpolation polynomial is always that of the current BDF order for the forward interpolation at the first point to the right of the time at which the interpolated value is sought (unless too close to the \( i \)-th checkpoint, in which case it uses the BDF order at the right-most relevant point). However, because of the FLC BDF implementation (see §2.1), the resulting interpolation polynomial is only an approximation to the underlying BDF interpolant.

The Hermite cubic interpolation option is present because it was implemented chronologically first and it is also used by other adjoint solvers (e.g. DASPKADJOINT). The variable-degree polynomial is more memory-efficient (it requires only half of the memory storage of the cubic Hermite interpolation) and is more accurate.

\(^2\)For the sake of simplifity in presentation, we do not include explicit dependencies of \( g \) on time \( t \) or parameters \( p \). Moreover, we only consider the case in which the dependency of the original DAE (2.2) on the parameters \( p \) is through its initial conditions only. For details on the derivation in the general case, see [43].
where \( \otimes \) is the Kronecker product. The second-order sensitivities are solution of the matrix DAE system:

\[
(F_y \otimes I_{N_p}) \cdot \dot{y}_{pp} + (F_y \otimes I_{N_p}) \cdot y_{pp} + (I_N \otimes y_p^T) \cdot (F_y \dot{y}_p + F_{yy} y_p) + (I_N \otimes y_p^T) \cdot (F_{yy} \dot{y}_p + F_{yy} y_p) = 0
\]

\[
y_{pp}(t_0) = \frac{\partial^2 y_0}{\partial p^2}, \quad \dot{y}_{pp}(t_0) = \frac{\partial^2 y_0}{\partial p^2},
\]

where \( y_p \) denotes the first-order sensitivity matrix, the solution of \( N_p \) systems (2.12), and \( y_{pp} \) is a third-order tensor. It is easy to see that, except for situations in which the number of parameters \( N_p \) is very small, the computational cost of this so-called \textit{forward-over-forward} approach is exorbitant as it requires the solution of \( N_p + N_p^2 \) additional DAE systems of the same dimension as (2.2).

A much more efficient alternative is to compute Hessian-vector products using a so-called \textit{forward-over-adjoint} approach. This method is based on using the same “trick” as the one used in computing gradients of pointwise functionals with the adjoint method, namely applying a formal directional forward derivation to the gradient of (2.21) (or the equivalent one for a pointwise functional \( g(T, y(T)) \)). With that, the cost of computing a full Hessian is roughly equivalent to the cost of computing the gradient with forward sensitivity analysis. However, Hessian-vector products can be cheaply computed with one additional adjoint solve.

As an illustration\(^3\), consider the ODE problem

\[
\dot{y} = f(t, y), \quad y(t_0) = y_0(p),
\]

depending on some parameters \( p \) through the initial conditions only and consider the model functional output \( G(p) = \int_{t_0}^{T} g(t, y) \, dt \). It can be shown that the product between the Hessian of \( G \) (with respect to the parameters \( p \)) and some vector \( u \) can be computed as

\[
\frac{\partial^2 G}{\partial p^2} u = \left[ (\lambda^T \otimes I_{N_p}) y_{pp} u + y_p^T \mu \right]_{t = t_0},
\]

where \( \lambda \) and \( \mu \) are solutions of

\[
-\lambda = f_y \mu + \left( \lambda^T \otimes I_{n} \right) f_{yy} s; \quad \mu(t_f) = 0
\]

\[
-\dot{\lambda} = f_y \lambda + g_{yy}^T s; \quad \lambda(t_f) = 0
\]

\[
\dot{s} = f_y s; \quad s(t_0) = y_{pp} u.
\]

In the above equation, \( s = y_p u \) is a linear combination of the columns of the sensitivity matrix \( y_p \). The \textit{forward-over-adjoint} approach hinges crucially on the fact that \( s \) can be computed at the cost of a forward sensitivity analysis with respect to a single parameter (the last ODE problem above) which is possible due to the linearity of the forward sensitivity equations (2.12).

Therefore (and this is also valid for the DAE case), the cost of computing the Hessian-vector product is roughly that of two forward and two backward integrations of a system of DAEs of size \( N \). For more details, including the corresponding formulas for a pointwise model functional output, see the work by Ozyurt and Barton [43] who discuss this problem for ODE initial value problems. As far as we know, there is no published equivalent work on DAE problems. However, the derivations given in [43] for ODE problems can be extended to DAEs with some careful consideration given to the derivation of proper final conditions on the adjoint systems, following the ideas presented in [15].

To allow the \textit{forward-over-adjoint} approach described above, IDAS provides support for:

- the integration of multiple backward problems depending on the same underlying forward problem (2.2), and

- the integration of backward problems and computation of backward quadratures depending on both the states \( y \) and forward sensitivities (for this particular application, \( s \)) of the original problem (2.2).

\(^3\)The derivation for the general DAE case is too involved for the purposes of this discussion.
Chapter 3

Code Organization

3.1 SUNDIALS organization

The family of solvers referred to as SUNDIALS consists of the solvers CVODE and ARKODE (for ODE systems), KINSOL (for nonlinear algebraic systems), and IDA (for differential-algebraic systems). In addition, SUNDIALS also includes variants of CVODE and IDA with sensitivity analysis capabilities (using either forward or adjoint methods), called CVODES and IDAS, respectively.

The various solvers of this family share many subordinate modules. For this reason, it is organized as a family, with a directory structure that exploits that sharing (see Figures 3.1 and 3.2).

Figure 3.1: High-level diagram of the SUNDIALS suite.
following is a list of the solver packages presently available, and the basic functionality of each:

- **cvode**, a solver for stiff and nonstiff ODE systems $dy/dt = f(t, y)$ based on Adams and BDF methods;
- **cvodes**, a solver for stiff and nonstiff ODE systems with sensitivity analysis capabilities;
- **arkode**, a solver for stiff, nonstiff, mixed stiff-nonstiff, and multirate ODE systems $Mdy/dt = f_1(t, y) + f_2(t, y)$ based on Runge-Kutta methods;
- **ida**, a solver for differential-algebraic systems $F(t, y, \dot{y}) = 0$ based on BDF methods;
- **idas**, a solver for differential-algebraic systems with sensitivity analysis capabilities;
- **kinsol**, a solver for nonlinear algebraic systems $F(u) = 0$.

Note for modules that provide interfaces to third-party libraries (i.e., LAPACK, KLU, SUPERLUMT, SuperLU_DIST, hypre, PETSc, Trilinos, and RAJA) users will need to download and compile those packages independently.

### 3.2 IDAS organization

The **idas** package is written in the ANSI C language. The following summarizes the basic structure of the package, although knowledge of this structure is not necessary for its use.

The overall organization of the **idas** package is shown in Figure 3.3. The central integration module, implemented in the files **idas.h**, **idas_impl.h**, and **idas.c**, deals with the evaluation of
3.2 IDAS organization

IDAS utilizes generic linear and nonlinear solver modules defined by the SUNLINSOL API (see Chapter 10) and SUNNONLINSOL API (see Chapter 11) respectively. As such, IDAS has no knowledge of the method being used to solve the linear and nonlinear systems that arise in each time step. For any given user problem, there exists a single nonlinear solver interface and, if necessary, one of the linear system solver interfaces is specified, and invoked as needed during the integration. While SUNDIALS includes a fixed-point nonlinear solver module, it is not currently supported in IDAS (note the fixed-point module is listed in Figure 3.1 but not Figure 3.3).

In addition, if forward sensitivity analysis is turned on, the main module will integrate the forward sensitivity equations simultaneously with the original IVP. The sensitivity variables may be included in the local error control mechanism of the main integrator. IDAS provides two different strategies for dealing with the correction stage for the sensitivity variables: IDA_SIMULTANEOUS_IDA_STAGGERED (see §2.5). The IDAS package includes an algorithm for the approximation of the sensitivity equations residuals by difference quotients, but the user has the option of supplying these residual functions directly.

The adjoint sensitivity module (file idaa.c) provides the infrastructure needed for the backward integration of any system of DAEs which depends on the solution of the original IVP, in particular the adjoint system and any quadratures required in evaluating the gradient of the objective functional. This module deals with the setup of the checkpoints, the interpolation of the forward solution during the backward integration, and the backward integration of the adjoint equations.

IDAS now has a single unified linear solver interface, IDALS, supporting both direct and iterative linear solvers built using the generic SUNLINSOL API (see Chapter 10). These solvers may utilize a SUNMATRIX object (see Chapter 9) for storing Jacobian information, or they may be matrix-free. Since IDAS can operate on any valid SUNLINSOL implementation, the set of linear solver modules available to IDAS will expand as new SUNLINSOL modules are developed.

For users employing dense or banded Jacobian matrices, IDALS includes algorithms for their approximation through difference quotients, but the user also has the option of supplying the Jacobian (or an approximation to it) directly. This user-supplied routine is required when using sparse or user-supplied Jacobian matrices.

For users employing matrix-free iterative linear solvers, IDALS includes an algorithm for the approx-
imation by difference quotients of the product between the Jacobian matrix and a vector, $Jv$. Again, the user has the option of providing routines for this operation, in two phases: setup (preprocessing of Jacobian data) and multiplication.

For preconditioned iterative methods, the preconditioning must be supplied by the user, again in two phases: setup and solve. While there is no default choice of preconditioner analogous to the difference-quotient approximation in the direct case, the references [8, 12], together with the example and demonstration programs included with IDAS, offer considerable assistance in building preconditioners.

IDAS’ linear solver interface consists of four primary routines, devoted to (1) memory allocation and initialization, (2) setup of the matrix data involved, (3) solution of the system, and (4) freeing of memory. The setup and solution phases are separate because the evaluation of Jacobians and preconditioners is done only periodically during the integration, as required to achieve convergence. The call list within the central IDAS module to each of the four associated functions is fixed, thus allowing the central module to be completely independent of the linear system method.

IDAS also provides a preconditioner module, IDABBDPRE, for use with any of the Krylov iterative linear solvers. It works in conjunction with NVECTOR_PARALLEL and generates a preconditioner that is a block-diagonal matrix with each block being a banded matrix.

All state information used by IDAS to solve a given problem is saved in a structure, and a pointer to that structure is returned to the user. There is no global data in the IDAS package, and so, in this respect, it is reentrant. State information specific to the linear solver is saved in a separate structure, a pointer to which resides in the IDAS memory structure. The reentrancy of IDAS was motivated by the situation where two or more problems are solved by intermixed calls to the package from one user program.
Chapter 4

Using IDAS for IVP Solution

This chapter is concerned with the use of IDAS for the integration of DAEs in a C language setting. The following sections treat the header files, the layout of the user’s main program, description of the IDAS user-callable functions, and description of user-supplied functions. This usage is essentially equivalent to using IDA [33].

The sample programs described in the companion document [48] may also be helpful. Those codes may be used as templates (with the removal of some lines involved in testing), and are included in the IDAS package.

The user should be aware that not all SUNLINSOL and SUNMATRIX modules are compatible with all NVECTOR implementations. Details on compatibility are given in the documentation for each SUNMATRIX module (Chapter 9) and each SUNLINSOL module (Chapter 10). For example, NVECTOR_PARALLEL is not compatible with the dense, banded, or sparse SUNMATRIX types, or with the corresponding dense, banded, or sparse SUNLINSOL modules. Please check Chapters 9 and 10 to verify compatibility between these modules. In addition to that documentation, we note that the preconditioner module IDABBDPRE can only be used with NVECTOR_PARALLEL. It is not recommended to use a threaded vector module with SuperLU_MT unless it is the NVECTOR_OPENMP module, and SuperLU_MT is also compiled with OpenMP.

IDAS uses various constants for both input and output. These are defined as needed in this chapter, but for convenience are also listed separately in Appendix B.

4.1 Access to library and header files

At this point, it is assumed that the installation of IDAS, following the procedure described in Appendix A, has been completed successfully.

Regardless of where the user’s application program resides, its associated compilation and load commands must make reference to the appropriate locations for the library and header files required by IDAS. The relevant library files are

- \texttt{libdir/libsundials_idas\.lib},
- \texttt{libdir/libsundials_nvec\.lib},

where the file extension \texttt{.lib} is typically \texttt{.so} for shared libraries and \texttt{.a} for static libraries. The relevant header files are located in the subdirectories

- \texttt{incdir/include/idas}
- \texttt{incdir/include/sundials}
- \texttt{incdir/include/nvector}
- \texttt{incdir/include/sunmatrix}
• `incdir/include/sunlinsol`
• `incdir/include/sunnonlinsol`

The directories `libdir` and `incdir` are the install library and include directories, respectively. For a default installation, these are `instdir/lib` and `instdir/include`, respectively, where `instdir` is the directory where SUNDIALS was installed (see Appendix A).

Note that an application cannot link to both the IDA and IDAS libraries because both contain user-callable functions with the same names (to ensure that IDAS is backward compatible with IDA). Therefore, applications that contain both DAE problems and DAEs with sensitivity analysis, should use IDAS.

### 4.2 Data types

The `sundials_types.h` file contains the definition of the type `realtype`, which is used by the SUNDIALS solvers for all floating-point data, the definition of the integer type `sunindextype`, which is used for vector and matrix indices, and `booleantype`, which is used for certain logic operations within SUNDIALS.

#### 4.2.1 Floating point types

The type `realtype` can be `float`, `double`, or `long double`, with the default being `double`. The user can change the precision of the SUNDIALS solvers arithmetic at the configuration stage (see §A.1.2).

Additionally, based on the current precision, `sundials_types.h` defines `BIG_REAL` to be the largest value representable as a `realtype`, `SMALL_REAL` to be the smallest value representable as a `realtype`, and `UNIT_ROUNDOFF` to be the difference between 1.0 and the minimum `realtype` greater than 1.0.

Within SUNDIALS, real constants are set by way of a macro called `RCONST`. It is this macro that needs the ability to branch on the definition `realtype`. In ANSI C, a floating-point constant with no suffix is stored as a `double`. Placing the suffix “F” at the end of a floating point constant makes it a `float`, whereas using the suffix “L” makes it a `long double`. For example,

```c
define A 1.0
#define B 1.0F
#define C 1.0L
```

defines A to be a `double` constant equal to 1.0, B to be a `float` constant equal to 1.0, and C to be a `long double` constant equal to 1.0. The macro call `RCONST(1.0)` automatically expands to 1.0 if `realtype` is `double`, to `1.0F` if `realtype` is `float`, or to `1.0L` if `realtype` is `long double`. SUNDIALS uses the `RCONST` macro internally to declare all of its floating-point constants.

Additionally, SUNDIALS defines several macros for common mathematical functions `e.g., fabs, sqrt, exp, etc.` in `sundials_math.h`. The macros are prefixed with `SUNR` and expand to the appropriate C function based on the `realtype`. For example, the macro `SUNRabs` expands to the C function `fabs` when `realtype` is `double`, `fabsf` when `realtype` is `float`, and `fabsl` when `realtype` is `long double`.

A user program which uses the type `realtype`, the `RCONST` macro, and the `SUNR` mathematical function macros is precision-independent except for any calls to precision-specific library functions. Our example programs use `realtype`, `RCONST`, and the `SUNR` macros. Users can, however, use the type `double`, `float`, or `long double` in their code (assuming that this usage is consistent with the typedef for `realtype`) and call the appropriate math library functions directly. Thus, a previously existing piece of ANSI C code can use SUNDIALS without modifying the code to use `realtype`, `RCONST`, or the `SUNR` macros so long as the SUNDIALS libraries use the correct precision (for details see §A.1.2).

#### 4.2.2 Integer types used for indexing

The type `sunindextype` is used for indexing array entries in SUNDIALS modules (`e.g., vectors lengths and matrix sizes`) as well as for storing the total problem size. During configuration `sunindextype`
may be selected to be either a 32- or 64-bit signed integer with the default being 64-bit. See §A.1.2 for the configuration option to select the desired size of sunindextype. When using a 32-bit integer the total problem size is limited to $2^{31} - 1$ and with 64-bit integers the limit is $2^{63} - 1$. For users with problem sizes that exceed the 64-bit limit an advanced configuration option is available to specify the type used for sunindextype.

A user program which uses sunindextype to handle indices will work with both index storage types except for any calls to index storage-specific external libraries. Our C and C++ example programs use sunindextype. Users can, however, use any compatible type (e.g., int, long int, int32_t, int64_t, or long long int) in their code, assuming that this usage is consistent with the typedef for sunindextype on their architecture. Thus, a previously existing piece of ANSI C code can use SUNDIALS without modifying the code to use sunindextype, so long as the SUNDIALS libraries use the appropriate index storage type (for details see §A.1.2).

### 4.3 Header files

The calling program must include several header files so that various macros and data types can be used. The header file that is always required is:

- **idas/idas.h**, the header file for IDAS, which defines the several types and various constants, and includes function prototypes. This includes the header file for IDALS, ida/ida_ls.h.

Note that idas.h includes sundials_types.h, which defines the types realtype, sunindextype, and booleantype and the constants SUNFALSE and SUNTRUE.

The calling program must also include an NVECTOR implementation header file, of the form nvector/nvector_***.h. See Chapter 8 for the appropriate name. This file in turn includes the header file sundials_nvector.h which defines the abstract N_Vector data type.

If using a non-default nonlinear solver module, or when interacting with a SUNNONLINSOL module directly, the calling program must also include a SUNNONLINSOL implementation header file, of the form sundials/nonlinsol_***.h where *** is the name of the nonlinear solver module (see Chapter 11 for more information). This file in turn includes the header file sundials_nonlinear_solver.h which defines the abstract SUNNonlinearSolver data type.

If using a nonlinear solver that requires the solution of a linear system of the form (2.5) (e.g., the default Newton iteration), a linear solver module header file is also required. The header files corresponding to the various SUNDIALS-provided linear solver modules available for use with IDAS are:

- **Direct linear solvers:**
  - sundials/sundials_dense.h, which is used with the dense linear solver module, SUNLINSOL_DENSE;
  - sundials/sundials_band.h, which is used with the banded linear solver module, SUNLINSOL_BAND;
  - sundials/sundials_lapack_dense.h, which is used with the LAPACK dense linear solver module, SUNLINSOL_LAPACKDENSE;
  - sundials/sundials_lapack_band.h, which is used with the LAPACK banded linear solver module, SUNLINSOL_LAPACKBAND;
  - sundials/sundials_klu.h, which is used with the KLU sparse linear solver module, SUNLINSOL_KLU;
  - sundials/sundials_superlumt.h, which is used with the SUPERLUMT sparse linear solver module, SUNLINSOL_SUPERLUMT;

- **Iterative linear solvers:**
  - sundials/sundials_spqmr.h, which is used with the scaled, preconditioned GMRES Krylov linear solver module, SUNLINSOL_SPGMR;
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- `sunlinsol/sunlinsol_spgmr.h`, which is used with the scaled, preconditioned FGMRES Krylov linear solver module, SUNLINSOL_SPFGMR;
- `sunlinsol/sunlinsol_spbcgs.h`, which is used with the scaled, preconditioned Bi-CGStab Krylov linear solver module, SUNLINSOL_SPBCGS;
- `sunlinsol/sunlinsol_sptfqmr.h`, which is used with the scaled, preconditioned TFQMR Krylov linear solver module, SUNLINSOL_SPTFQMR;
- `sunlinsol/sunlinsol_pcg.h`, which is used with the scaled, preconditioned CG Krylov linear solver module, SUNLINSOL_PCG;

The header files for the SUNLINSOL_DENSE and SUNLINSOL_LAPACKDENSE linear solver modules include the file `sunmatrix/sunmatrix_dense.h`, which defines the SUNMATRIX_DENSE matrix module, as well as various functions and macros acting on such matrices.

The header files for the SUNLINSOL_BAND and SUNLINSOL_LAPACKBAND linear solver modules include the file `sunmatrix/sunmatrix_band.h`, which defines the SUNMATRIX_BAND matrix module, as well as various functions and macros acting on such matrices.

The header files for the SUNLINSOL_KLU and SUNLINSOL_SUPERLUMT sparse linear solvers include the file `sunmatrix/sunmatrix_sparse.h`, which defines the SUNMATRIX_SPARSE matrix module, as well as various functions and macros acting on such matrices.

The header files for the Krylov iterative solvers include the file `sundials/sundials_iterative.h`, which enumerates the kind of preconditioning, and (for the SPGMR and SPFGMR solvers) the choices for the Gram-Schmidt process.

Other headers may be needed, according to the choice of preconditioner, etc. For example, in the `idasFoodWeb_kryp` example (see [48]), preconditioning is done with a block-diagonal matrix. For this, even though the SUNLINSOL_SPGMR linear solver is used, the header `sundials/sundials_dense.h` is included for access to the underlying generic dense matrix arithmetic routines.

### 4.4 A skeleton of the user’s main program

The following is a skeleton of the user’s main program (or calling program) for the integration of a DAE IVP. Most of the steps are independent of the NVECTOR, SUNMATRIX, SUNLINSOL, and SUNNONLINSOL implementations used. For the steps that are not, refer to Chapter 8, 9, 10, and 11 for the specific name of the function to be called or macro to be referenced.

1. **Initialize parallel or multi-threaded environment, if appropriate**

   For example, call `MPI_Init` to initialize MPI if used, or set `num_threads`, the number of threads to use within the threaded vector functions, if used.

2. **Set problem dimensions etc.**

   This generally includes the problem size \( N \), and may include the local vector length \( N_{\text{local}} \).

   Note: The variables \( N \) and \( N_{\text{local}} \) should be of type `sunindextype`.

3. **Set vectors of initial values**

   To set the vectors \( y_0 \) and \( y_{p0} \) to initial values for \( y \) and \( \dot{y} \), use the appropriate functions defined by the particular NVECTOR implementation.

   For native SUNDIALS vector implementations (except the CUDA and RAJA-based ones), use a call of the form \( y_0 = \text{NVMake}_* \) if the `realtype` array `ydata` containing the initial values of \( y \) already exists. Otherwise, create a new vector by making a call of the form \( y_0 = \text{NVNew}_* \), and then set its elements by accessing the underlying data with a call of the form \( \text{ydata} = \text{NVGetArrayPointer}(y_0) \). See §8.3-8.6 for details.

   For the hypre and PETSc vector wrappers, first create and initialize the underlying vector and then create an NVECTOR wrapper with a call of the form \( y_0 = \text{NMake}_* \) (yvec), where `yvec`
4.4 A skeleton of the user’s main program

is a hypre or PETSc vector. Note that calls like NVNew***(...) and NVGetArrayPointer(...) are not available for these vector wrappers. See §8.7 and §8.8 for details.

If using either the CUDA- or RAJA-based vector implementations use a call of the form y0 = NVMake***(..., c) where c is a pointer to a suncudavec or sunrajavec vector class if this class already exists. Otherwise, create a new vector by making a call of the form y0 = NVNew***(...), and then set its elements by accessing the underlying data where it is located with a call of the form NVGetDeviceArrayPointer*** or NVGetHostArrayPointer***. Note that the vector class will allocate memory on both the host and device when instantiated. See §8.9-8.10 for details.

Set the vector yp0 of initial conditions for ˙\(y\) similarly.

4. Create IDAS object

Call ida_mem = IDACreate() to create the IDAS memory block. IDACreate returns a pointer to the IDAS memory structure. See §4.5.1 for details. This void * pointer must then be passed as the first argument to all subsequent IDAS function calls.

5. Initialize IDAS solver

Call IDAInit(...) to provide required problem specifications (residual function, initial time, and initial conditions), allocate internal memory for IDAS, and initialize IDAS. IDAInit returns an error flag to indicate success or an illegal argument value. See §4.5.1 for details.

6. Specify integration tolerances

Call IDASSetTolerances(...) or IDASVTolerances(...) to specify, respectively, a scalar relative tolerance and scalar absolute tolerance, or a scalar relative tolerance and a vector of absolute tolerances. Alternatively, call IDAWFTolerances to specify a function which sets directly the weights used in evaluating WRMS vector norms. See §4.5.2 for details.

7. Create matrix object

If a nonlinear solver requiring a linear solver will be used (e.g., the default Newton iteration) and the linear solver will be a matrix-based linear solver, then a template Jacobian matrix must be created by using the appropriate constructor function defined by the particular SUNMATRIX implementation.

For the SUNDIALS-supplied SUNMATRIX implementations, the matrix object may be created using a call of the form

SUNMatrix J = SUNBandMatrix(...);

or

SUNMatrix J = SUNDenseMatrix(...);

or

SUNMatrix J = SUNSparseMatrix(...);

NOTE: The dense, banded, and sparse matrix objects are usable only in a serial or threaded environment.

8. Create linear solver object

If a nonlinear solver requiring a linear solver is chosen (e.g., the default Newton iteration), then the desired linear solver object must be created by calling the appropriate constructor function defined by the particular SUNLINSOL implementation.

For any of the SUNDIALS-supplied SUNLINSOL implementations, the linear solver object may be created using a call of the form

SUNLinearSolver LS = SUNLinSol_*(...);
38 Using IDAS for IVP Solution

where * can be replaced with “Dense”, “SPGMR”, or other options, as discussed in §4.5.3 and Chapter 10.

9. **Set linear solver optional inputs**
   
   Call *Set* functions from the selected linear solver module to change optional inputs specific to that linear solver. See the documentation for each SUNLINSOL module in Chapter 10 for details.

10. **Attach linear solver module**
    
    If a nonlinear solver requiring a linear solver is chosen (e.g., the default Newton iteration), then initialize the IDALS linear solver interface by attaching the linear solver object (and matrix object, if applicable) with the following call (for details see §4.5.3):
    
    ```c
    ier = IDASetLinearSolver(...);
    ```

11. **Set optional inputs**
    
    Optionally, call IDASet* functions to change from their default values any optional inputs that control the behavior of IDAS. See §4.5.8.1 and §4.5.8 for details.

12. **Create nonlinear solver object** *(optional)*
    
    If using a non-default nonlinear solver (see §4.5.4), then create the desired nonlinear solver object by calling the appropriate constructor function defined by the particular SUNNONLINSOL implementation (e.g., NLS = SUNNonlinSol_***(...); where *** is the name of the nonlinear solver (see Chapter 11 for details).

13. **Attach nonlinear solver module** *(optional)*
    
    If using a non-default nonlinear solver, then initialize the nonlinear solver interface by attaching the nonlinear solver object by calling `ier = IDASetNonlinearSolver(ida_mem, NLS);` (see §4.5.4 for details).

14. **Set nonlinear solver optional inputs** *(optional)*
    
    Call the appropriate set functions for the selected nonlinear solver module to change optional inputs specific to that nonlinear solver. These must be called after IDAInit if using the default nonlinear solver or after attaching a new nonlinear solver to IDAS, otherwise the optional inputs will be overridden by IDAS defaults. See Chapter 11 for more information on optional inputs.

15. **Correct initial values**
    
    Optionally, call IDACalcIC to correct the initial values y0 and yp0 passed to IDAInit. See §4.5.5. Also see §4.5.8.3 for relevant optional input calls.

16. **Specify rootfinding problem**
    
    Optionally, call IDARootInit to initialize a rootfinding problem to be solved during the integration of the DAE system. See §4.5.6 for details, and see §4.5.8.4 for relevant optional input calls.

17. **Advance solution in time**
    
    For each point at which output is desired, call `flag = IDASolve(ida_mem, tout, &tret, yret, ypret, itask).` Here `itask` specifies the return mode. The vector `yret` (which can be the same as the vector `y0` above) will contain \( y(t) \), while the vector `ypret` (which can be the same as the vector `yp0` above) will contain \( \dot{y}(t) \). See §4.5.7 for details.

18. **Get optional outputs**
    
    Call IDA*Get* functions to obtain optional output. See §4.5.10 for details.

19. **Deallocate memory for solution vectors**
Upon completion of the integration, deallocate memory for the vectors \( y_{\text{ret}} \) and \( y_{\text{pret}} \) (or \( y \) and \( y_{p} \)) by calling the appropriate destructor function defined by the NVECTOR implementation:

\[
\text{NVDestroy}(y_{\text{ret}});
\]

and similarly for \( y_{\text{pret}} \).

20. **Free solver memory**

IDAFree(&ida_mem) to free the memory allocated for IDAS.

21. **Free nonlinear solver memory** *(optional)*

If a non-default nonlinear solver was used, then call SUNNonlinSolFree(NLS) to free any memory allocated for the SUNNONLINSOL object.

22. **Free linear solver and matrix memory**

Call SUNLinSolFree and SUNMatDestroy to free any memory allocated for the linear solver and matrix objects created above.

23. **Finalize MPI, if used**

Call MPI_Finalize() to terminate MPI.

SUNDIALS provides some linear solvers only as a means for users to get problems running and not as highly efficient solvers. For example, if solving a dense system, we suggest using the LAPACK solvers if the size of the linear system is \( > 50,000 \). (Thanks to A. Nicolai for his testing and recommendation.) Table 4.1 shows the linear solver interfaces available as SUNLINSOL modules and the vector implementations required for use. As an example, one cannot use the dense direct solver interfaces with the MPI-based vector implementation. However, as discussed in Chapter 10 the SUNDIALS packages operate on generic SUNLINSOL objects, allowing a user to develop their own solvers should they so desire.

<table>
<thead>
<tr>
<th>Linear Solver</th>
<th>Serial</th>
<th>Parallel (MPI)</th>
<th>OpenMP</th>
<th>pThreads</th>
<th>Krylov</th>
<th>PETSc</th>
<th>CUDA</th>
<th>RAJA</th>
<th>User Supp.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dense</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Band</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>LapackDense</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>LapackBand</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>KLU</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>SUPERLUMT</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>SPGMR</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>SPFQMR</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>SPBCGS</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>SPTFQMR</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>PCG</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>User Supp.</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>

**Table 4.1:** SUNDIALS linear solver interfaces and vector implementations that can be used for each.

4.5 **User-callable functions**

This section describes the IDAS functions that are called by the user to set up and solve a DAE. Some of these are required. However, starting with §4.5.8, the functions listed involve optional inputs/outputs
or restarting, and those paragraphs can be skipped for a casual use of IDAS. In any case, refer to §4.4 for the correct order of these calls.

On an error, each user-callable function returns a negative value and sends an error message to the error handler routine, which prints the message on stderr by default. However, the user can set a file as error output or can provide his own error handler function (see §4.5.8.1).

### 4.5.1 IDAS initialization and deallocation functions

The following three functions must be called in the order listed. The last one is to be called only after the DAE solution is complete, as it frees the IDAS memory block created and allocated by the first two calls.

#### IDACreate

Call

\[
\text{ida_mem} = \text{IDACreate}();
\]

Description The function IDACreate instantiates an IDAS solver object.

Arguments IDACreate has no arguments.

Return value If successful, IDACreate returns a pointer to the newly created IDAS memory block (of type void *). Otherwise it returns NULL.

F2003 Name FIDACreate

#### IDAInit

Call

\[
\text{flag} = \text{IDAInit(ida_mem, res, t0, y0, yp0)};
\]

Description The function IDAInit provides required problem and solution specifications, allocates internal memory, and initializes IDAS.

Arguments

- **ida_mem** (void *) pointer to the IDAS memory block returned by IDACreate.
- **res** (IDAResFn) is the C function which computes the residual function \( F \) in the DAE. This function has the form \( \text{res}(t, yy, yp, \text{resval}, \text{user_data}) \). For full details see §4.6.1.
- **t0** (realttype) is the initial value of \( t \).
- **y0** (N_Vector) is the initial value of \( y \).
- **yp0** (N_Vector) is the initial value of \( \dot{y} \).

Return value The return value **flag** (of type int) will be one of the following:

- **IDA_SUCCESS** The call to IDAInit was successful.
- **IDA_MEM_NULL** The IDAS memory block was not initialized through a previous call to IDACreate.
- **IDA_MEM_FAIL** A memory allocation request has failed.
- **IDA_Ill_INPUT** An input argument to IDAInit has an illegal value.

Notes If an error occurred, IDAInit also sends an error message to the error handler function.

F2003 Name FIDAInit

#### IDAFree

Call

\[
\text{IDAFree(&ida_mem)};
\]

Description The function IDAFree frees the pointer allocated by a previous call to IDACreate.

Arguments The argument is the pointer to the IDAS memory block (of type void *).

Return value The function IDAFree has no return value.

F2003 Name FIDAFree
4.5 User-callable functions

4.5.2 IDAS tolerance specification functions

One of the following three functions must be called to specify the integration tolerances (or directly specify the weights used in evaluating WRMS vector norms). Note that this call must be made after the call to IDAInit.

**IDASStolerances**

Call

```c
flag = IDASStolerances(ida_mem, reltol, abstol);
```

Description

The function IDASStolerances specifies scalar relative and absolute tolerances.

Arguments

- `ida_mem` (void *) pointer to the IDAS memory block returned by IDACreate.
- `reltol` (realtype) is the scalar relative error tolerance.
- `abstol` (realtype) is the scalar absolute error tolerance.

Return value

The return value `flag` (of type `int`) will be one of the following:

<table>
<thead>
<tr>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>IDA_SUCCESS</td>
<td>The call to IDASStolerances was successful.</td>
</tr>
<tr>
<td>IDA_MEM_NULL</td>
<td>The IDAS memory block was not initialized through a previous call to IDACreate.</td>
</tr>
<tr>
<td>IDA_NO_MALLOC</td>
<td>The allocation function IDAInit has not been called.</td>
</tr>
<tr>
<td>IDA_ILL_INPUT</td>
<td>One of the input tolerances was negative.</td>
</tr>
</tbody>
</table>

F2003 Name FIDASStolerances

**IDASVtolerances**

Call

```c
flag = IDASVtolerances(ida_mem, reltol, abstol);
```

Description

The function IDASVtolerances specifies scalar relative tolerance and vector absolute tolerances.

Arguments

- `ida_mem` (void *) pointer to the IDAS memory block returned by IDACreate.
- `reltol` (realtype) is the scalar relative error tolerance.
- `abstol` (N_Vector) is the vector of absolute error tolerances.

Return value

The return value `flag` (of type `int`) will be one of the following:

<table>
<thead>
<tr>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>IDA_SUCCESS</td>
<td>The call to IDASVtolerances was successful.</td>
</tr>
<tr>
<td>IDA_MEM_NULL</td>
<td>The IDAS memory block was not initialized through a previous call to IDACreate.</td>
</tr>
<tr>
<td>IDA_NO_MALLOC</td>
<td>The allocation function IDAInit has not been called.</td>
</tr>
<tr>
<td>IDA_ILL_INPUT</td>
<td>The relative error tolerance was negative or the absolute tolerance had a negative component.</td>
</tr>
</tbody>
</table>

Notes

This choice of tolerances is important when the absolute error tolerance needs to be different for each component of the state vector $y$.

F2003 Name FIDASVtolerances

**IDAWFtolerances**

Call

```c
flag = IDAWFtolerances(ida_mem, efun);
```

Description

The function IDAWFtolerances specifies a user-supplied function `efun` that sets the multiplicative error weights $W_i$ for use in the weighted RMS norm, which are normally defined by Eq. (2.7).

Arguments

- `ida_mem` (void *) pointer to the IDAS memory block returned by IDACreate.
- `efun` (IDAEwtFn) is the C function which defines the $\text{ewt}$ vector (see §4.6.3).

Return value

The return value `flag` (of type `int`) will be one of the following:
IDA_SUCCESS The call to IDAWFtolerances was successful.
IDA_MEM_NULL The IDAS memory block was not initialized through a previous call to IDACreate.
IDA_NO_MALLOC The allocation function IDAInit has not been called.

F2003 Name FIDAWFtolerances

General advice on choice of tolerances. For many users, the appropriate choices for tolerance values in reltol and abstol are a concern. The following pieces of advice are relevant.

(1) The scalar relative tolerance reltol is to be set to control relative errors. So \( \text{reltol} = 10^{-4} \) means that errors are controlled to .01%. We do not recommend using reltol larger than \( 10^{-3} \). On the other hand, reltol should not be so small that it is comparable to the unit roundoff of the machine arithmetic (generally around \( 10^{-15} \)).

(2) The absolute tolerances abstol (whether scalar or vector) need to be set to control absolute errors when any components of the solution vector \( y \) may be so small that pure relative error control is meaningless. For example, if \( y[i] \) starts at some nonzero value, but in time decays to zero, then pure relative error control on \( y[i] \) makes no sense (and is overly costly) after \( y[i] \) is below some noise level. Then abstol (if scalar) or abstol[i] (if a vector) needs to be set to that noise level. If the different components have different noise levels, then abstol should be a vector. See the example idasRoberts_dns in the IDAS package, and the discussion of it in the IDAS Examples document [48]. In that problem, the three components vary between 0 and 1, and have different noise levels; hence the abstol vector. It is impossible to give any general advice on abstol values, because the appropriate noise levels are completely problem-dependent. The user or modeler hopefully has some idea as to what those noise levels are.

(3) Finally, it is important to pick all the tolerance values conservatively, because they control the error committed on each individual time step. The final (global) errors are a sort of accumulation of those per-step errors. A good rule of thumb is to reduce the tolerances by a factor of .01 from the actual desired limits on errors. So if you want .01% accuracy (globally), a good choice is \( \text{reltol} = 10^{-6} \). But in any case, it is a good idea to do a few experiments with the tolerances to see how the computed solution values vary as tolerances are reduced.

Advice on controlling unphysical negative values. In many applications, some components in the true solution are always positive or non-negative, though at times very small. In the numerical solution, however, small negative (hence unphysical) values can then occur. In most cases, these values are harmless, and simply need to be controlled, not eliminated. The following pieces of advice are relevant.

(1) The way to control the size of unwanted negative computed values is with tighter absolute tolerances. Again this requires some knowledge of the noise level of these components, which may or may not be different for different components. Some experimentation may be needed.

(2) If output plots or tables are being generated, and it is important to avoid having negative numbers appear there (for the sake of avoiding a long explanation of them, if nothing else), then eliminate them, but only in the context of the output medium. Then the internal values carried by the solver are unaffected. Remember that a small negative value in \( y_{\text{ret}} \) returned by IDAS, with magnitude comparable to abstol or less, is equivalent to zero as far as the computation is concerned.

(3) The user’s residual routine \( \text{res} \) should never change a negative value in the solution vector \( yy \) to a non-negative value, as a ”solution” to this problem. This can cause instability. If the \( \text{res} \) routine cannot tolerate a zero or negative value (e.g., because there is a square root or log of it), then the offending value should be changed to zero or a tiny positive number in a temporary variable (not in the input \( yy \) vector) for the purposes of computing \( F(t,y,\dot{y}) \).

(4) IDAS provides the option of enforcing positivity or non-negativity on components. Also, such constraints can be enforced by use of the recoverable error return feature in the user-supplied residual function. However, because these options involve some extra overhead cost, they should only be exercised if the use of absolute tolerances to control the computed values is unsuccessful.
4.5 User-callable functions

4.5.3 Linear solver interface functions

As previously explained, if the nonlinear solver requires the solution of linear systems of the form (2.5) (e.g., the default Newton iteration, then solution of these linear systems is handled with the IDALS linear solver interface. This interface supports all valid SUNLINSOL modules. Here, matrix-based SUNLINSOL modules utilize SUNMATRIX objects to store the Jacobian matrix $J = \partial F/\partial y + \alpha \partial F/\partial \dot{y}$ and factorizations used throughout the solution process. Conversely, matrix-free SUNLINSOL modules instead use iterative methods to solve the linear systems of equations, and only require the action of the Jacobian on a vector, $Jv$.

With most iterative linear solvers, preconditioning can be done on the left only, on the right only, on both the left and the right, or not at all. The exceptions to this rule are SPFGMR that supports right preconditioning only and PCG that performs symmetric preconditioning. However, in IDAS only left preconditioning is supported. For the specification of a preconditioner, see the iterative linear solver sections in §4.5.8 and §4.6. A preconditioner matrix $P$ must approximate the Jacobian $J$, at least crudely.

To specify a generic linear solver to IDAS, after the call to IDACreate but before any calls to IDASolve, the user’s program must create the appropriate SUNLINSOL object and call the function IDASetLinearSolver, as documented below. To create the SUNLinearSolver object, the user may call one of the SUNDIALS-packaged SUNLINSOL module constructor routines via a call of the form

$$\text{SUNLinearSolver \ LS = SUNLinSol_*(...);}$$

The current list of such constructor routines includes SUNLinSol_Dense, SUNLinSol_Band, SUNLinSol_LapackDense, SUNLinSol_LapackBand, SUNLinSol_KLU, SUNLinSol_SuperLUMT, SUNLinSol_SPGMR, SUNLinSol_SPFGMR, SUNLinSol_SPBCGS, SUNLinSol_SPTFQMR, and SUNLinSol_PCG.

Alternately, a user-supplied SUNLinearSolver module may be created and used instead. The use of each of the generic linear solvers involves certain constants, functions and possibly some macros, that are likely to be needed in the user code. These are available in the corresponding header file associated with the specific SUNMATRIX or SUNLINSOL module in question, as described in Chapters 9 and 10.

Once this solver object has been constructed, the user should attach it to IDAS via a call to IDASetLinearSolver. The first argument passed to this function is the IDAS memory pointer returned by IDACreate; the second argument is the desired SUNLINSOL object to use for solving systems. The third argument is an optional SUNMATRIX object to accompany matrix-based SUNLINSOL inputs (for matrix-free linear solvers, the third argument should be NULL). A call to this function initializes the IDALS linear solver interface, linking it to the main IDAS integrator, and allows the user to specify additional parameters and routines pertinent to their choice of linear solver.

```c
int IDASetLinearSolver(void *ida_mem, SUNLinearSolver LS, SUNMatrix J);
```

Call $\text{flag = IDASetLinearSolver(ida_mem, LS, J);}$$

Description The function IDASetLinearSolver attaches a generic SUNLINSOL object LS and corresponding template Jacobian SUNMATRIX object J (if applicable) to IDAS, initializing the IDALS linear solver interface.

Arguments

- `ida_mem` (void *) pointer to the IDAS memory block.
- `LS` (SUNLinearSolver) SUNLINSOL object to use for solving linear systems of the form (2.5).
- `J` (SUNMatrix) SUNMATRIX object for used as a template for the Jacobian (or NULL if not applicable).

Return value The return value `flag` (of type `int`) is one of

- **IDALS_SUCCESS** The IDALS initialization was successful.
- **IDALS_MEM_NULL** The `ida_mem` pointer is NULL.
- **IDALS_ILL_INPUT** The IDALS interface is not compatible with the LS or J input objects or is incompatible with the current NVECTOR module.
IDALS\_SUNLS\_FAIL A call to the LS object failed.
IDALS\_MEM\_FAIL A memory allocation request failed.

**Notes**

If LS is a matrix-based linear solver, then the template Jacobian matrix $J$ will be used in the solve process, so if additional storage is required within the SUNMATRIX object (e.g., for factorization of a banded matrix), ensure that the input object is allocated with sufficient size (see the documentation of the particular SUNMATRIX type in Chapter 9 for further information).

The previous routines IDADlsSetLinearSolver and IDASpilsSetLinearSolver are now wrappers for this routine, and may still be used for backward-compatibility. However, these will be deprecated in future releases, so we recommend that users transition to the new routine name soon.

F2003 Name FIDASetLinearSolver

### 4.5.4 Nonlinear solver interface function

By default IDAS uses the SUNNONLINSOL implementation of Newton's method defined by the SUNNONLINSOL\_NEWTON module (see §11.3). To specify a different nonlinear solver in IDAS, the user's program must create a SUNNONLINSOL object by calling the appropriate constructor routine. The user must then attach the SUNNONLINSOL object to IDAS by calling IDASetNonlinearSolver, as documented below.

When changing the nonlinear solver in IDAS, IDASetNonlinearSolver must be called after IDAInit. If any calls to IDASolve have been made, then IDAS will need to be reinitialized by calling IDAREInit to ensure that the nonlinear solver is initialized correctly before any subsequent calls to IDASolve.

The first argument passed to the routine IDASetNonlinearSolver is the IDAS memory pointer returned by IDACreate and the second argument is the SUNNONLINSOL object to use for solving the nonlinear system 2.4. A call to this function attaches the nonlinear solver to the main IDAS integrator. We note that at present, the SUNNONLINSOL object must be of type SUNNONLINEARSOLVER\_ROOTFIND.

**IDASetNonlinearSolver**

**Call**

```c
flag = IDASetNonlinearSolver(ida_mem, NLS);
```

**Description**
The function IDASetNonlinearSolver attaches a SUNNONLINSOL object (NLS) to IDAS.

**Arguments**
- **ida_mem** (void *) pointer to the IDAS memory block.
- **NLS** (SUNNonlinearSolver) SUNNONLINSOL object to use for solving nonlinear systems.

**Return value**
The return value `flag` (of type `int`) is one of

- **IDA\_SUCCESS** The nonlinear solver was successfully attached.
- **IDA\_MEM\_NULL** The `ida_mem` pointer is NULL.
- **IDA\_ILL\_INPUT** The SUNNONLINSOL object is NULL, does not implement the required nonlinear solver operations, is not of the correct type, or the residual function, convergence test function, or maximum number of nonlinear iterations could not be set.

**Notes**

When forward sensitivity analysis capabilities are enabled and the `IDA\_STAGGERED` corrector method is used this function sets the nonlinear solver method for correcting state variables (see §5.2.3 for more details).

F2003 Name FIDASetNonlinearSolver

### 4.5.5 Initial condition calculation function

IDACalcIC calculates corrected initial conditions for the DAE system for certain index-one problems including a class of systems of semi-implicit form. (See §2.1 and Ref. [10].) It uses Newton iteration
combined with a linesearch algorithm. Calling IDACalcIC is optional. It is only necessary when the initial conditions do not satisfy the given system. Thus if $y_0$ and $\dot{y}_0$ are known to satisfy $F(t_0, y_0, \dot{y}_0) = 0$, then a call to IDACalcIC is generally not necessary.

A call to the function IDACalcIC must be preceded by successful calls to IDACreate and IDAInit (or IDAREInit), and by a successful call to the linear system solver specification function. The call to IDACalcIC should precede the call(s) to IDASolve for the given problem.

**IDACalcIC**

Call

```c
flag = IDACalcIC(ida_mem, icopt, tout1);
```

**Description** The function IDACalcIC corrects the initial values $y_0$ and $\dot{y}_0$ at time $t_0$.

**Arguments**

- `ida_mem` (void *) pointer to the IDAS memory block.
- `icopt` (int) is one of the following two options for the initial condition calculation.
  - `icopt=IDA_YA_YDP_INIT` directs IDACalcIC to compute the algebraic components of $y$ and differential components of $\dot{y}$, given the differential components of $y$. This option requires that the N_Vector `id` was set through IDASetId, specifying the differential and algebraic components.
  - `icopt=IDA_Y_INIT` directs IDACalcIC to compute all components of $y$, given $\dot{y}$. In this case, `id` is not required.
- `tout1` (realtype) is the first value of $t$ at which a solution will be requested (from IDASolve). This value is needed here only to determine the direction of integration and rough scale in the independent variable $t$.

**Return value** The return value `flag` (of type `int`) will be one of the following:

- `IDA_SUCCESS` IDASolve succeeded.
- `IDA_MEM_NULL` The argument `ida_mem` was NULL.
- `IDA_NO_MALLOC` The allocation function IDAInit has not been called.
- `IDA_IILL_INPUT` One of the input arguments was illegal.
- `IDA_LSETUP_FAIL` The linear solver’s setup function failed in an unrecoverable manner.
- `IDA_LINIT_FAIL` The linear solver’s initialization function failed.
- `IDA_LSOLVE_FAIL` The linear solver’s solve function failed in an unrecoverable manner.
- `IDA_BAD_EWT` Some component of the error weight vector is zero (illegal), either for the input value of $y_0$ or a corrected value.
- `IDA_FIRST_RES_FAIL` The user’s residual function returned a recoverable error flag on the first call, but IDACalcIC was unable to recover.
- `IDA_RES_FAIL` The user’s residual function returned a nonrecoverable error flag.
- `IDA_NO_RECOVERY` The user’s residual function, or the linear solver’s setup or solve function had a recoverable error, but IDACalcIC was unable to recover.
- `IDA_CONSTR_FAIL` IDACalcIC was unable to find a solution satisfying the inequality constraints.
- `IDA_LINESearch_FAIL` The linesearch algorithm failed to find a solution with a step larger than `steptol` in weighted RMS norm, and within the allowed number of backtracks.
- `IDA_CONV_FAIL` IDACalcIC failed to get convergence of the Newton iterations.

**Notes** All failure return values are negative and therefore a test `flag < 0` will trap all IDACalcIC failures.

Note that IDACalcIC will correct the values of $y(t_0)$ and $\dot{y}(t_0)$ which were specified in the previous call to IDAInit or IDAREInit. To obtain the corrected values, call IDAGetconsistentIC (see §4.5.10.3).
4.5.6 Rootfinding initialization function

While integrating the IVP, IDAS has the capability of finding the roots of a set of user-defined functions. To activate the rootfinding algorithm, call the following function. This is normally called only once, prior to the first call to IDASolve, but if the rootfinding problem is to be changed during the solution, IDARootInit can also be called prior to a continuation call to IDASolve.

\[
\text{Call} \quad \text{flag} = \text{IDARootInit}(\text{ida\_mem}, \text{nrtfn}, g);
\]

**Description**
The function IDARootInit specifies that the roots of a set of functions \( g_i(t,y,\dot{y}) \) are to be found while the IVP is being solved.

**Arguments**
- \( \text{ida\_mem} \) (void *) pointer to the idas memory block returned by IDACreate.
- \( \text{nrtfn} \) (int) is the number of root functions \( g_i \).
- \( g \) (IDARootFn) is the C function which defines the \( \text{nrtfn} \) functions \( g_i(t,y,\dot{y}) \) whose roots are sought. See §4.6.4 for details.

**Return value**
The return value \( \text{flag} \) (of type int) is one of
- **IDA_SUCCESS** The call to IDARootInit was successful.
- **IDA_MEM_NULL** The \( \text{ida\_mem} \) argument was NULL.
- **IDA_MEM_FAIL** A memory allocation failed.
- **IDA_Ill_INPUT** The function \( g \) is NULL, but \( \text{nrtfn} > 0 \).

**Notes**
If a new IVP is to be solved with a call to IDAREInit, where the new IVP has no rootfinding problem but the prior one did, then call IDARootInit with \( \text{nrtfn} = 0 \).

4.5.7 IDAS solver function

This is the central step in the solution process, the call to perform the integration of the DAE. One of the input arguments (\( \text{itask} \)) specifies one of two modes as to where IDAS is to return a solution. But these modes are modified if the user has set a stop time (with IDASSetStopTime) or requested rootfinding.

\[
\text{Call} \quad \text{flag} = \text{IDASolve}(\text{ida\_mem}, \text{tout}, \&\text{tret}, \text{yret}, \text{ypret}, \text{itask});
\]

**Description**
The function IDASolve integrates the DAE over an interval in \( t \).

**Arguments**
- \( \text{ida\_mem} \) (void *) pointer to the idas memory block.
- \( \text{tout} \) (realtype) the next time at which a computed solution is desired.
- \( \text{tret} \) (realtype) the time reached by the solver (output).
- \( \text{yret} \) (N_Vector) the computed solution vector \( y \).
- \( \text{ypret} \) (N_Vector) the computed solution vector \( \dot{y} \).
- \( \text{itask} \) (int) a flag indicating the job of the solver for the next user step. The IDA_NORMAL task is to have the solver take internal steps until it has reached or just passed the user specified \( \text{tout} \) parameter. The solver then interpolates in order to return approximate values of \( y(t_{\text{out}}) \) and \( \dot{y}(t_{\text{out}}) \). The IDA_ONE_STEP option tells the solver to just take one internal step and return the solution at the point reached by that step.
Return value \texttt{IDASolve} returns vectors \texttt{yret} and \texttt{ypret} and a corresponding independent variable value \( t = \texttt{tret} \), such that \((\texttt{yret}, \texttt{ypret})\) are the computed values of \((y(t), \dot{y}(t))\).

In \texttt{IDA\_NORMAL} mode with no errors, \texttt{tret} will be equal to \texttt{tout} and \texttt{yret} = \( y(\texttt{tout}) \), \texttt{ypret} = \( \dot{y}(\texttt{tout}) \).

The return value \texttt{flag} (of type \texttt{int}) will be one of the following:

- **\texttt{IDA\_SUCCESS}** \texttt{IDASolve} succeeded.
- **\texttt{IDA\_TSTOP\_RETURN}** \texttt{IDASolve} succeeded by reaching the stop point specified through the optional input function \texttt{IDASetzStopTime}. See §4.5.8.1 for more information.
- **\texttt{IDA\_ROOT\_RETURN}** \texttt{IDASolve} succeeded and found one or more roots. In this case, \texttt{tret} is the location of the root. If \texttt{nrtfn} > 1, call \texttt{IDAGetRootInfo} to see which \( g_i \) were found to have a root. See §4.5.10.4 for more information.
- **\texttt{IDA\_MEM\_NULL}** The \texttt{ida\_mem} argument was \texttt{NULL}.
- **\texttt{IDA\_ILL\_INPUT}** One of the inputs to \texttt{IDASolve} was illegal, or some other input to the solver was either illegal or missing. The latter category includes the following situations: (a) The tolerances have not been set. (b) A component of the error weight vector became zero during internal time-stepping. (c) The linear solver initialization function (called by the user after calling \texttt{IDACreate}) failed to set the linear solver-specific \texttt{lsolve} field in \texttt{ida\_mem}. (d) A root of one of the root functions was found both at a point \( t \) and also very near \( t \). In any case, the user should see the printed error message for details.
- **\texttt{IDA\_TOO\_MUCH\_WORK}** The solver took \texttt{mxstep} internal steps but could not reach \texttt{tout}. The default value for \texttt{mxstep} is \texttt{MXSTEP\_DEFAULT} = 500.
- **\texttt{IDA\_TOO\_MUCH\_ACC}** The solver could not satisfy the accuracy demanded by the user for some internal step.
- **\texttt{IDA\_ERR\_FAIL}** Error test failures occurred too many times (\texttt{MXNEF} = 10) during one internal time step or occurred with \( |h| = h_{\text{min}} \).
- **\texttt{IDA\_CONV\_FAIL}** Convergence test failures occurred too many times (\texttt{MXNCF} = 10) during one internal time step or occurred with \( |h| = h_{\text{min}} \).
- **\texttt{IDA\_LINIT\_FAIL}** The linear solver’s initialization function failed.
- **\texttt{IDA\_LSETUP\_FAIL}** The linear solver’s setup function failed in an unrecoverable manner.
- **\texttt{IDA\_LSOLVE\_FAIL}** The linear solver’s solve function failed in an unrecoverable manner.
- **\texttt{IDA\_CONSTR\_FAIL}** The inequality constraints were violated and the solver was unable to recover.
- **\texttt{IDA\_REP\_RES\_ERR}** The user’s residual function repeatedly returned a recoverable error flag, but the solver was unable to recover.
- **\texttt{IDA\_RES\_FAIL}** The user’s residual function returned a nonrecoverable error flag.
- **\texttt{IDA\_RTFUNC\_FAIL}** The rootfinding function failed.

Notes

The vector \texttt{yret} can occupy the same space as the vector \texttt{y0} of initial conditions that was passed to \texttt{IDAInit}, and the vector \texttt{ypret} can occupy the same space as \texttt{yp0}.

In the \texttt{IDA\_ONE\_STEP} mode, \texttt{tout} is used on the first call only, and only to get the direction and rough scale of the independent variable.

If a stop time is enabled (through a call to \texttt{IDASetzStopTime}), then \texttt{IDASolve} returns the solution at \texttt{tstop}. Once the integrator returns at a stop time, any future testing for \texttt{tstop} is disabled (and can be reenabled only though a new call to \texttt{IDASetzStopTime}).

All failure return values are negative and therefore a test \texttt{flag < 0} will trap all \texttt{IDASolve} failures.
### Table 4.2: Optional inputs for IDAS and IDALS

<table>
<thead>
<tr>
<th>Optional input</th>
<th>Function name</th>
<th>Default</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pointer to an error file</td>
<td>IDASSetErrFile</td>
<td>stderr</td>
</tr>
<tr>
<td>Error handler function</td>
<td>IDASSetErrHandlerFn</td>
<td>internal fn.</td>
</tr>
<tr>
<td>User data</td>
<td>IDASSetUserData</td>
<td>NULL</td>
</tr>
<tr>
<td>Maximum order for BDF method</td>
<td>IDASMaxOrd</td>
<td>5</td>
</tr>
<tr>
<td>Maximum no. of internal steps before ( t_{\text{out}} )</td>
<td>IDASMaxNumSteps</td>
<td>500</td>
</tr>
<tr>
<td>Initial step size</td>
<td>IDASInitStep</td>
<td>estimated</td>
</tr>
<tr>
<td>Maximum absolute step size</td>
<td>IDASMaxStep</td>
<td>( \infty )</td>
</tr>
<tr>
<td>Value of ( t_{\text{stop}} )</td>
<td>IDASStopTime</td>
<td>( \infty )</td>
</tr>
<tr>
<td>Maximum no. of error test failures</td>
<td>IDASMaxErrTestFails</td>
<td>10</td>
</tr>
<tr>
<td>Maximum no. of nonlinear iterations</td>
<td>IDASMaxNonlinIters</td>
<td>4</td>
</tr>
<tr>
<td>Maximum no. of convergence failures</td>
<td>IDASMaxConvFails</td>
<td>10</td>
</tr>
<tr>
<td>Coeff. in the nonlinear convergence test</td>
<td>IDASNonlinConvCoef</td>
<td>0.33</td>
</tr>
<tr>
<td>Suppress alg. vars. from error test</td>
<td>IDASSetSuppressAlg</td>
<td>SUNFALSE</td>
</tr>
<tr>
<td>Variable types (differential/algebraic)</td>
<td>IDAId</td>
<td>NULL</td>
</tr>
<tr>
<td>Inequality constraints on solution</td>
<td>IDASSetConstraints</td>
<td>NULL</td>
</tr>
<tr>
<td>Direction of zero-crossing</td>
<td>IDASSetRootDirection</td>
<td>both</td>
</tr>
<tr>
<td>Disable rootfinding warnings</td>
<td>IDASSetNoInactiveRootWarn</td>
<td>none</td>
</tr>
<tr>
<td>IDAS initial conditions calculation</td>
<td>IDASSetNonlinConvCoefIC</td>
<td>0.0033</td>
</tr>
<tr>
<td>Maximum no. of steps</td>
<td>IDASMaxNumStepsIC</td>
<td>5</td>
</tr>
<tr>
<td>Maximum no. of Jacobian/precond. evals.</td>
<td>IDASMaxNumJacIC</td>
<td>4</td>
</tr>
<tr>
<td>Maximum no. of Newton iterations</td>
<td>IDASMaxNumItersIC</td>
<td>10</td>
</tr>
<tr>
<td>Max. linesearch backtracks per Newton iter.</td>
<td>IDASMaxBacksIC</td>
<td>100</td>
</tr>
<tr>
<td>Turn off linesearch</td>
<td>IDASSetLineSearchOffIC</td>
<td>SUNFALSE</td>
</tr>
<tr>
<td>Lower bound on Newton step</td>
<td>IDASSetStepToleranceIC</td>
<td>around ( 2/3 )</td>
</tr>
<tr>
<td>IDALS linear solver interface</td>
<td>IDASJacFnc</td>
<td>DQ</td>
</tr>
<tr>
<td>Jacobian-times-vector function</td>
<td>IDASJacTimes</td>
<td>NULL, DQ</td>
</tr>
<tr>
<td>Preconditioner functions</td>
<td>IDASSetPreconditioner</td>
<td>NULL, NULL</td>
</tr>
<tr>
<td>Ratio between linear and nonlinear tolerances</td>
<td>IDASSetEpsLin</td>
<td>0.05</td>
</tr>
<tr>
<td>Increment factor used in DQ ( Jv ) approx.</td>
<td>IDASSetIncrementFactor</td>
<td>1.0</td>
</tr>
</tbody>
</table>

On any error return in which one or more internal steps were taken by **IDASolve**, the returned values of \( t_{\text{ret}} \), \( y_{\text{ret}} \), and \( y_{\text{pret}} \) correspond to the farthest point reached in the integration. On all other error returns, these values are left unchanged from the previous **IDASolve** return.

F2003 Name **FIDASolve**

#### 4.5.8 Optional input functions

There are numerous optional input parameters that control the behavior of the IDAS solver. IDAS provides functions that can be used to change these optional input parameters from their default values. Table 4.2 lists all optional input functions in IDAS which are then described in detail in the remainder of this section. For the most casual use of IDAS, the reader can skip to §4.6.

We note that, on an error return, all these functions also send an error message to the error handler function. We also note that all error return values are negative, so a test flag < 0 will catch any error.
4.5.8.1 Main solver optional input functions

The calls listed here can be executed in any order. However, if the user’s program calls either `IDASetErrFile` or `IDASetErrHandlerFn`, then that call should appear first, in order to take effect for any later error message.

**IDASetErrFile**

Call  
```c
flag = IDASetErrFile(ida_mem, errfp);
```

Description  
The function `IDASetErrFile` specifies the pointer to the file where all IDAS messages should be directed when the default IDAS error handler function is used.

Arguments  
- `ida_mem` (void *) pointer to the IDAS memory block.
- `errfp` (FILE *) pointer to output file.

Return value  
The return value `flag` (of type `int`) is one of
- `IDA_SUCCESS` The optional value has been successfully set.
- `IDA_MEM_NULL` The `ida_mem` pointer is NULL.

Notes  
- The default value for `errfp` is `stderr`.
- Passing a value NULL disables all future error message output (except for the case in which the IDAS memory pointer is NULL). This use of `IDASetErrFile` is strongly discouraged.
- If `IDASetErrFile` is to be called, it should be called before any other optional input functions, in order to take effect for any later error message.

F2003 Name `FIDASetErrFile`

**IDASetErrHandlerFn**

Call  
```c
flag = IDASetErrHandlerFn(ida_mem, ehfun, eh_data);
```

Description  
The function `IDASetErrHandlerFn` specifies the optional user-defined function to be used in handling error messages.

Arguments  
- `ida_mem` (void *) pointer to the IDAS memory block.
- `ehfun` (IDAErrHandlerFn) is the user’s C error handler function (see §4.6.2).
- `eh_data` (void *) pointer to user data passed to `ehfun` every time it is called.

Return value  
The return value `flag` (of type `int`) is one of
- `IDA_SUCCESS` The function `ehfun` and data pointer `eh_data` have been successfully set.
- `IDA_MEM_NULL` The `ida_mem` pointer is NULL.

Notes  
- Error messages indicating that the IDAS solver memory is NULL will always be directed to `stderr`.

F2003 Name `FIDASetErrHandlerFn`

**IDASetUserData**

Call  
```c
flag = IDASetUserData(ida_mem, user_data);
```

Description  
The function `IDASetUserData` specifies the user data block `user_data` and attaches it to the main IDAS memory block.

Arguments  
- `ida_mem` (void *) pointer to the IDAS memory block.
- `user_data` (void *) pointer to the user data.

Return value  
The return value `flag` (of type `int`) is one of
- `IDA_SUCCESS` The optional value has been successfully set.
- `IDA_MEM_NULL` The `ida_mem` pointer is NULL.
Notes
If specified, the pointer to user data is passed to all user-supplied functions that have it as an argument. Otherwise, a NULL pointer is passed.

If user data is needed in user linear solver or preconditioner functions, the call to IDASetUserData must be made before the call to specify the linear solver.

F2003 Name FIDASetUserData

IDASetMaxOrd

Call
flag = IDASetMaxOrd(ida_mem, maxord);

Description
The function IDASetMaxOrd specifies the maximum order of the linear multistep method.

Arguments
ida_mem (void *) pointer to the IDAS memory block.
maxord (int) value of the maximum method order. This must be positive.

Return value
The return value flag (of type int) is one of
IDA_SUCCESS The optional value has been successfully set.
IDA_MEM_NULL The ida_mem pointer is NULL.
IDA_Ill_INPUT The input value maxord is ≤ 0, or larger than its previous value.

Notes
The default value is 5. If the input value exceeds 5, the value 5 will be used. Since maxord affects the memory requirements for the internal IDAS memory block, its value cannot be increased past its previous value.

F2003 Name FIDASetMaxOrd

IDASetMaxNumSteps

Call
flag = IDASetMaxNumSteps(ida_mem, mxsteps);

Description
The function IDASetMaxNumSteps specifies the maximum number of steps to be taken by the solver in its attempt to reach the next output time.

Arguments
ida_mem (void *) pointer to the IDAS memory block.
mxsteps (long int) maximum allowed number of steps.

Return value
The return value flag (of type int) is one of
IDA_SUCCESS The optional value has been successfully set.
IDA_MEM_NULL The ida_mem pointer is NULL.

Notes
Passing mxsteps = 0 results in IDAS using the default value (500).
Passing mxsteps < 0 disables the test (not recommended).

F2003 Name FIDASetMaxNumSteps

IDASetInitStep

Call
flag = IDASetInitStep(ida_mem, hin);

Description
The function IDASetInitStep specifies the initial step size.

Arguments
ida_mem (void *) pointer to the IDAS memory block.

hin (realtype) value of the initial step size to be attempted. Pass 0.0 to have IDAS use the default value.

Return value
The return value flag (of type int) is one of
IDA_SUCCESS The optional value has been successfully set.
IDA_MEM_NULL The ida_mem pointer is NULL.

Notes
By default, IDAS estimates the initial step as the solution of ∥h ˙y∥WRMS = 1/2, with an added restriction that |h| ≤ .001|tout - t0|.

F2003 Name FIDASetInitStep
4.5 User-callable functions

IDASetMaxStep

Call: flag = IDASetMaxStep(ida_mem, hmax);

Description: The function IDASetMaxStep specifies the maximum absolute value of the step size.

Arguments:
- ida_mem (void *) pointer to the IDAS memory block.
- hmax (realtype) maximum absolute value of the step size.

Return value: The return value flag (of type int) is one of:
- IDA_SUCCESS: The optional value has been successfully set.
- IDA_MEM_NULL: The ida_mem pointer is NULL.
- IDAillard_INPUT: Either hmax is not positive or it is smaller than the minimum allowable step.

Notes: Pass hmax = 0 to obtain the default value $\infty$.

F2003 Name: FIDASetMaxStep

IDASetStopTime

Call: flag = IDASetStopTime(ida_mem, tstop);

Description: The function IDASetStopTime specifies the value of the independent variable $t$ past which the solution is not to proceed.

Arguments:
- ida_mem (void *) pointer to the IDAS memory block.
- tstop (realtype) value of the independent variable past which the solution should not proceed.

Return value: The return value flag (of type int) is one of:
- IDA_SUCCESS: The optional value has been successfully set.
- IDA_MEM_NULL: The ida_mem pointer is NULL.
- IDAillard_INPUT: The value of tstop is not beyond the current t value, $t_n$.

Notes: The default, if this routine is not called, is that no stop time is imposed.

Once the integrator returns at a stop time, any future testing for tstop is disabled (and can be reenabled only though a new call to IDASetStopTime).

F2003 Name: FIDASetStopTime

IDASetMaxErrTestFails

Call: flag = IDASetMaxErrTestFails(ida_mem, maxnef);

Description: The function IDASetMaxErrTestFails specifies the maximum number of error test failures in attempting one step.

Arguments:
- ida_mem (void *) pointer to the IDAS memory block.
- maxnef (int) maximum number of error test failures allowed on one step (> 0).

Return value: The return value flag (of type int) is one of:
- IDA_SUCCESS: The optional value has been successfully set.
- IDA_MEM_NULL: The ida_mem pointer is NULL.

Notes: The default value is 10.

F2003 Name: FIDASetMaxErrTestFails
**IDASetMaxNonlinIters**

Call

```c
flag = IDASetMaxNonlinIters(ida_mem, maxcor);
```

Description The function IDASetMaxNonlinIters specifies the maximum number of nonlinear solver iterations at one step.

Arguments

- `ida_mem` (void *) pointer to the IDAS memory block.
- `maxcor` (int) maximum number of nonlinear solver iterations allowed on one step (> 0).

Return value The return value `flag` (of type `int`) is one of

- `IDA_SUCCESS` The optional value has been successfully set.
- `IDA_MEM_NULL` The `ida_mem` pointer is NULL.
- `IDA_MEM_FAIL` The `sunnonlinsol` module is NULL.

Notes

The default value is 4.

F2003 Name FIDASetMaxNonlinIters

**IDASetMaxConvFails**

Call

```c
flag = IDASetMaxConvFails(ida_mem, maxncf);
```

Description The function IDASetMaxConvFails specifies the maximum number of nonlinear solver convergence failures at one step.

Arguments

- `ida_mem` (void *) pointer to the IDAS memory block.
- `maxncf` (int) maximum number of allowable nonlinear solver convergence failures on one step (> 0).

Return value The return value `flag` (of type `int`) is one of

- `IDA_SUCCESS` The optional value has been successfully set.
- `IDA_MEM_NULL` The `ida_mem` pointer is NULL.

Notes

The default value is 10.

F2003 Name FIDASetMaxConvFails

**IDASetNonlinConvCoef**

Call

```c
flag = IDASetNonlinConvCoef(ida_mem, nlscoef);
```

Description The function IDASetNonlinConvCoef specifies the safety factor in the nonlinear convergence test; see Chapter 2, Eq. (2.8).

Arguments

- `ida_mem` (void *) pointer to the IDAS memory block.
- `nlscoef` (realtype) coefficient in nonlinear convergence test (> 0.0).

Return value The return value `flag` (of type `int`) is one of

- `IDA_SUCCESS` The optional value has been successfully set.
- `IDA_MEM_NULL` The `ida_mem` pointer is NULL.
- `IDA_Ill_INPUT` The value of `nlscoef` is <= 0.0.

Notes

The default value is 0.33.

F2003 Name FIDASetNonlinConvCoef
4.5 User-callable functions

**IDASetSuppressAlg**

Call: `flag = IDASetSuppressAlg(ida_mem, suppressalg);`

Description: The function `IDASetSuppressAlg` indicates whether or not to suppress algebraic variables in the local error test.

Arguments:
- `ida_mem`: (void *) pointer to the IDAS memory block.
- `suppressalg`: (boolean type) indicates whether to suppress (`SUNTRUE`) or not (`SUNFALSE`) the algebraic variables in the local error test.

Return value: The return value `flag` (of type `int`) is one of:
- `IDA_SUCCESS`: The optional value has been successfully set.
- `IDA_MEM_NULL`: The `ida_mem` pointer is NULL.

Notes:
- The default value is `SUNFALSE`.
- If `suppressalg = SUNTRUE` is selected, then the `id` vector must be set (through `IDASetId`) to specify the algebraic components.
- In general, the use of this option (with `suppressalg = SUNTRUE`) is discouraged when solving DAE systems of index 1, whereas it is generally encouraged for systems of index 2 or more. See pp. 146-147 of Ref. [6] for more on this issue.

F2003 Name: `FIDASetSuppressAlg`

**IDASetId**

Call: `flag = IDASetId(ida_mem, id);`

Description: The function `IDASetId` specifies algebraic/differential components in the `y` vector.

Arguments:
- `ida_mem`: (void *) pointer to the IDAS memory block.
- `id`: (N_Vector) state vector. A value of 1 indicates a differential variable, while 0 indicates an algebraic variable.

Return value: The return value `flag` (of type `int`) is one of:
- `IDA_SUCCESS`: The optional value has been successfully set.
- `IDA_MEM_NULL`: The `ida_mem` pointer is NULL.

Notes:
- The vector `id` is required if the algebraic variables are to be suppressed from the local error test (see `IDASetSuppressAlg`) or if `IDACalcIC` is to be called with `icopt = IDA_YA_YDP_INIT` (see §4.5.5).

F2003 Name: `FIDASetId`

**IDASetConstraints**

Call: `flag = IDASetConstraints(ida_mem, constraints);`

Description: The function `IDASetConstraints` specifies a vector defining inequality constraints for each component of the solution vector `y`.

Arguments:
- `ida_mem`: (void *) pointer to the IDAS memory block.
- `constraints`: (N_Vector) vector of constraint flags. If `constraints[i]` is:
  - 0.0 then no constraint is imposed on `y_i`.
  - 1.0 then `y_i` will be constrained to be `y_i >= 0.0`.
  - -1.0 then `y_i` will be constrained to be `y_i <= 0.0`.
  - 2.0 then `y_i` will be constrained to be `y_i > 0.0`.
  - -2.0 then `y_i` will be constrained to be `y_i < 0.0`.

Return value: The return value `flag` (of type `int`) is one of:
IDA_SUCCESS The optional value has been successfully set.
IDA_MEM_NULL The ida_mem pointer is NULL.
IDA_Ill_INPUT The constraints vector contains illegal values or the simultaneous corrector option has been selected when doing forward sensitivity analysis.

Notes
- The presence of a non-NULL constraints vector that is not 0.0 in all components will cause constraint checking to be performed. However, a call with 0.0 in all components of constraints will result in an illegal input return.
- Constraint checking when doing forward sensitivity analysis with the simultaneous corrector option is currently disallowed and will result in an illegal input return.

F2003 Name FIDASetConstraints

4.5.8.2 Linear solver interface optional input functions

The mathematical explanation of the linear solver methods available to IDAS is provided in §2.1. We group the user-callable routines into four categories: general routines concerning the overall IDALS linear solver interface, optional inputs for matrix-based linear solvers, optional inputs for matrix-free linear solvers, and optional inputs for iterative linear solvers. We note that the matrix-based and matrix-free groups are mutually exclusive, whereas the “iterative” tag can apply to either case.

When using matrix-based linear solver modules, the IDALS solver interface needs a function to compute an approximation to the Jacobian matrix \( J(t, y, \dot{y}) \). This function must be of type IDALSJacFn. The user can supply a Jacobian function, or if using a dense or banded matrix \( J \) can use the default internal difference quotient approximation that comes with the IDALS interface. To specify a user-supplied Jacobian function \( \text{jac} \), IDALS provides the function IDASSetJacFn. The IDALS interface passes the pointer user_data to the Jacobian function. This allows the user to create an arbitrary structure with relevant problem data and access it during the execution of the user-supplied Jacobian function, without using global data in the program. The pointer user_data may be specified through IDASSetUserData.

F2003 Name FIDASetJacFn

Call

\[
\text{flag} = \text{IDASSetJacFn}(\text{ida_mem}, \text{jac});
\]

Description The function IDASSetJacFn specifies the Jacobian approximation function to be used for a matrix-based solver within the IDALS interface.

Arguments
- ida_mem (void *) pointer to the IDAS memory block.
- jac (IDALSJacFn) user-defined Jacobian approximation function.

Return value The return value flag (of type int) is one of
- IDALS_SUCCESS The optional value has been successfully set.
- IDALS_MEM_NULL The ida_mem pointer is NULL.
- IDALS_MEM_NULL The IDALS linear solver interface has not been initialized.

Notes
- This function must be called after the IDALS linear solver interface has been initialized through a call to IDASSetLinearSolver.
- By default, IDALS uses an internal difference quotient function for dense and band matrices. If NULL is passed to jac, this default function is used. An error will occur if no jac is supplied when using other matrix types.
- The function type IDALSJacFn is described in §4.6.5.

The previous routine IDADlsSetJacFn is now a wrapper for this routine, and may still be used for backward-compatibility. However, this will be deprecated in future releases, so we recommend that users transition to the new routine name soon.

F2003 Name FIDASetJacFn
When using matrix-free linear solver modules, the IDALS solver interface requires a function to compute an approximation to the product between the Jacobian matrix $J(t,y)$ and a vector $v$. The user can supply a Jacobian-times-vector approximation function, or use the default internal difference quotient function that comes with the IDALS solver interface. A user-defined Jacobian-vector function must be of type IDALSJacTimesVecFn and can be specified through a call to IDASetJacTimes (see §4.6.6 for specification details). The evaluation and processing of any Jacobian-related data needed by the user's Jacobian-times-vector function may be done in the optional user-supplied function jtsetup (see §4.6.7 for specification details). The pointer user_data received through IDASetUserData (or a pointer to NULL if user_data was not specified) is passed to the Jacobian-times-vector setup and product functions, jtsetup and jtimes, each time they are called. This allows the user to create an arbitrary structure with relevant problem data and access it during the execution of the user-supplied preconditioner functions without using global data in the program.

### IDASetJacTimes

**Call**
```
flag = IDASetJacTimes(ida_mem, jsetup, jtimes);
```

**Description**
The function IDASetJacTimes specifies the Jacobian-vector setup and product functions.

**Arguments**
- `ida_mem` (void *) pointer to the IDAS memory block.
- `jsetup` (IDALSJacTimesSetupFn) user-defined function to set up the Jacobian-vector product. Pass NULL if no setup is necessary.
- `jtimes` (IDALSJacTimesVecFn) user-defined Jacobian-vector product function.

**Return value**
The return value `flag` (of type int) is one of

- IDALS_SUCCESS: The optional value has been successfully set.
- IDALS_MEM_NULL: The `ida_mem` pointer is NULL.
- IDALS_LMEM_NULL: The IDALS linear solver has not been initialized.
- IDALS_SUNLNS_FAIL: An error occurred when setting up the system matrix-times-vector routines in the SUNLINSOL object used by the IDALS interface.

**Notes**
The default is to use an internal finite difference quotient for jtimes and to omit jtsetup. If NULL is passed to jtimes, these defaults are used. A user may specify non-NULL jtimes and NULL jtsetup inputs.

This function must be called after the IDALS linear solver interface has been initialized through a call to IDASetLinearSolver.

The function type IDALSJacTimesSetupFn is described in §4.6.7.

The function type IDALSJacTimesVecFn is described in §4.6.6.

The previous routine IDASpilsSetJacTimes is now a wrapper for this routine, and may still be used for backward-compatibility. However, this will be deprecated in future releases, so we recommend that users transition to the new routine name soon.

F2003 Name: FIDASetJacTimes

Alternately, when using the default difference-quotient approximation to the Jacobian-vector product, the user may specify the factor to use in setting increments for the finite-difference approximation, via a call to IDASetIncrementFactor:

### IDASetIncrementFactor

**Call**
```
flag = IDASetIncrementFactor(ida_mem, dqincfac);
```

**Description**
The function IDASetIncrementFactor specifies the increment factor to be used in the difference-quotient approximation to the product $Jv$. Specifically, $Jv$ is approximated via the formula

$$Jv = \frac{1}{\sigma} \left[ F(t, \tilde{y}, \tilde{y}') - F(t, y, y') \right],$$

where $\sigma$ is the increment factor set by this function.
where \( \tilde{y} = y + \sigma v, \tilde{y}' = y' + c_j \sigma v, \) \( c_j \) is a BDF parameter proportional to the step size, \( \sigma = \sqrt{N \text{dqincfac}} \), and \( N \) is the number of equations in the DAE system.

**Arguments**  
ida_mem (void *) pointer to the IDAS memory block.  
dqincfac (realtype) user-specified increment factor (positive).

**Return value**  
The return value flag (of type int) is one of
- IDALS_SUCCESS: The optional value has been successfully set.  
- IDALS_MEM_NULL: The ida_mem pointer is NULL.  
- IDALS_LMEM_NULL: The IDALS linear solver has not been initialized.  
- IDALS_ILL_INPUT: The specified value of dqincfac is \( \leq 0 \).

**Notes**  
The default value is 1.0.  
This function must be called after the IDALS linear solver interface has been initialized through a call to IDASetLinearSolver.

The previous routine IDASpilsSetIncrementFactor is now a wrapper for this routine, and may still be used for backward-compatibility. However, this will be deprecated in future releases, so we recommend that users transition to the new routine name soon.

**F2003 Name**  
FIDASSetIncrementFactor

When using an iterative linear solver, the user may supply a preconditioning operator to aid in solution of the system. This operator consists of two user-supplied functions, psetup and psolve, that are supplied to IDA using the function IDASSetPreconditioner. The psetup function supplied to this routine should handle evaluation and preprocessing of any Jacobian data needed by the user’s preconditioner solve function, psolve. Both of these functions are fully specified in §4.6. The user data pointer received through IDASSetUserData (or a pointer to NULL if user data was not specified) is passed to the psetup and psolve functions. This allows the user to create an arbitrary structure with relevant problem data and access it during the execution of the user-supplied preconditioner functions without using global data in the program.

Also, as described in §2.1, the IDALS interface requires that iterative linear solvers stop when the norm of the preconditioned residual satisfies

\[
\|r\| \leq \frac{\epsilon_L \epsilon}{10}
\]

where \( \epsilon \) is the nonlinear solver tolerance, and the default \( \epsilon_L = 0.05 \); this value may be modified by the user through the IDASSetEpsLin function.

**IDASSetPreconditioner**

**Call**  
flag = IDASSetPreconditioner(ida_mem, psetup, psolve);

**Description**  
The function IDASSetPreconditioner specifies the preconditioner setup and solve functions.

**Arguments**  
ida_mem (void *) pointer to the IDAS memory block.  
psetup (IDALsPrecSetupFn) user-defined function to set up the preconditioner. Pass NULL if no setup is necessary.  
psolve (IDALsPrecSolveFn) user-defined preconditioner solve function.

**Return value**  
The return value flag (of type int) is one of
- IDALS_SUCCESS: The optional values have been successfully set.  
- IDALS_MEM_NULL: The ida_mem pointer is NULL.  
- IDALS_LMEM_NULL: The IDALS linear solver has not been initialized.  
- IDALS_SUNLS_FAIL: An error occurred when setting up preconditioning in the SUNLINSOL object used by the IDALS interface.
Notes The default is NULL for both arguments (i.e., no preconditioning). This function must be called after the IDALS linear solver interface has been initialized through a call to IDASpilsSetPreconditioner.

The function type IDALsPrecSolveFn is described in §4.6.8.

The function type IDALsPrecSetupFn is described in §4.6.9.

The previous routine IDASpilsSetPreconditioner is now a wrapper for this routine, and may still be used for backward-compatibility. However, this will be deprecated in future releases, so we recommend that users transition to the new routine name soon.

F2003 Name FIDASetPreconditioner

IDASetEpsLin

Call flag = IDASetEpsLin(ida_mem, eplifac);

Description The function IDASetEpsLin specifies the factor by which the Krylov linear solver’s convergence test constant is reduced from the nonlinear iteration test constant.

Arguments ida_mem (void *) pointer to the IDAS memory block.

eplifac (realtype) linear convergence safety factor (≥ 0.0).

Return value The return value flag (of type int) is one of

- IDALS_SUCCESS The optional value has been successfully set.
- IDALS_MEM_NULL The ida_mem pointer is NULL.
- IDALS_LMEM_NULL The IDALS linear solver has not been initialized.
- IDALS_ILL_INPUT The factor eplifac is negative.

Notes The default value is 0.05. This function must be called after the IDALS linear solver interface has been initialized through a call to IDASpilsSetPreconditioner. If eplifac= 0.0 is passed, the default value is used.

The previous routine IDASpilsSetEpsLin is now a wrapper for this routine, and may still be used for backward-compatibility. However, this will be deprecated in future releases, so we recommend that users transition to the new routine name soon.

F2003 Name FIDASetEpsLin

4.5.8.3 Initial condition calculation optional input functions

The following functions can be called just prior to calling IDACalcIC to set optional inputs controlling the initial condition calculation.

IDASetNonlinConvCoefIC

Call flag = IDASetNonlinConvCoefIC(ida_mem, epiccon);

Description The function IDASetNonlinConvCoefIC specifies the positive constant in the Newton iteration convergence test within the initial condition calculation.

Arguments ida_mem (void *) pointer to the IDAS memory block.

epiccon (realtype) coefficient in the Newton convergence test (> 0).

Return value The return value flag (of type int) is one of

- IDA_SUCCESS The optional value has been successfully set.
- IDA_MEM_NULL The ida_mem pointer is NULL.
- IDA_ILL_INPUT The epiccon factor is <= 0.0.
Notes

The default value is 0.01 · 0.33.

This test uses a weighted RMS norm (with weights defined by the tolerances). For
new initial value vectors \( y \) and \( \dot{y} \) to be accepted, the norm of \( J^{-1}F(t_0, y, \dot{y}) \) must be \( \leq \) epiccon, where \( J \) is the system Jacobian.

F2003 Name FIDASetNonlinConvCoefIC

**IDASetMaxNumStepsIC**

Call

\[
flag = \text{IDASetMaxNumStepsIC}(\text{ida_mem}, \text{maxnh});
\]

Description

The function IDASetMaxNumStepsIC specifies the maximum number of steps allowed
when \( \text{ic} = \text{IDA}_Y \text{A}_Y \text{DP}_\text{INIT} \) in IDACalcIC, where \( h \) appears in the system Jacobian,
\[
J = \frac{\partial F}{\partial y} + \frac{1}{h}\frac{\partial F}{\partial \dot{y}}.
\]

Arguments

- **ida_mem** (void *) pointer to the IDAS memory block.
- **maxnh** (int) maximum allowed number of values for \( h \).

Return value

The return value \( flag \) (of type int) is one of

- **IDA_SUCCESS** The optional value has been successfully set.
- **IDA_MEM_NULL** The ida_mem pointer is NULL.
- **IDA_Ill_INPUT** maxnh is non-positive.

Notes

The default value is 5.

F2003 Name FIDASetMaxNumStepsIC

**IDASetMaxNumJacsIC**

Call

\[
flag = \text{IDASetMaxNumJacsIC}(\text{ida_mem}, \text{maxnj});
\]

Description

The function IDASetMaxNumJacsIC specifies the maximum number of the approximate
Jacobian or preconditioner evaluations allowed when the Newton iteration appears to
be slowly converging.

Arguments

- **ida_mem** (void *) pointer to the IDAS memory block.
- **maxnj** (int) maximum allowed number of Jacobian or preconditioner evaluations.

Return value

The return value \( flag \) (of type int) is one of

- **IDA_SUCCESS** The optional value has been successfully set.
- **IDA_MEM_NULL** The ida_mem pointer is NULL.
- **IDA_Ill_INPUT** maxnj is non-positive.

Notes

The default value is 4.

F2003 Name FIDASetMaxNumJacsIC

**IDASetMaxNumItersIC**

Call

\[
flag = \text{IDASetMaxNumItersIC}(\text{ida_mem}, \text{maxnit});
\]

Description

The function IDASetMaxNumItersIC specifies the maximum number of Newton iter-
tations allowed in any one attempt to solve the initial conditions calculation problem.

Arguments

- **ida_mem** (void *) pointer to the IDAS memory block.
- **maxnit** (int) maximum number of Newton iterations.

Return value

The return value \( flag \) (of type int) is one of

- **IDA_SUCCESS** The optional value has been successfully set.
- **IDA_MEM_NULL** The ida_mem pointer is NULL.
- **IDA_Ill_INPUT** maxnit is non-positive.

Notes

The default value is 10.

F2003 Name FIDASetMaxNumItersIC
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**IDASetMaxBacksIC**

**Call**
```
flag = IDASetMaxBacksIC(ida_mem, maxbacks);
```

**Description**
The function `IDASetMaxBacksIC` specifies the maximum number of linesearch backtracks allowed in any Newton iteration, when solving the initial conditions calculation problem.

**Arguments**
- `ida_mem` (void *) pointer to the IDAS memory block.
- `maxbacks` (int) maximum number of linesearch backtracks per Newton step.

**Return value**
The return value `flag` (of type `int`) is one of
- `IDA_SUCCESS` The optional value has been successfully set.
- `IDA_MEM NULL` The `ida_mem` pointer is NULL.
- `IDA_Ill_INPUT maxbacks` is non-positive.

**Notes**
The default value is 100.

If `IDASetMaxBacksIC` is called in a Forward Sensitivity Analysis, the the limit `maxbacks` applies in the calculation of both the initial state values and the initial sensitivities.

F2003 Name FIDASetMaxBacksIC

**IDASetLineSearchOffIC**

**Call**
```
flag = IDASetLineSearchOffIC(ida_mem, lsoff);
```

**Description**
The function `IDASetLineSearchOffIC` specifies whether to turn on or off the linesearch algorithm.

**Arguments**
- `ida_mem` (void *) pointer to the IDAS memory block.
- `lsoff` (boolean type) a flag to turn off (`SUNTRUE`) or keep (`SUNFALSE`) the linesearch algorithm.

**Return value**
The return value `flag` (of type `int`) is one of
- `IDA_SUCCESS` The optional value has been successfully set.
- `IDA_MEM NULL` The `ida_mem` pointer is NULL.

**Notes**
The default value is `SUNFALSE`.

F2003 Name FIDASetLineSearchOffIC

**IDASetStepToleranceIC**

**Call**
```
flag = IDASetStepToleranceIC(ida_mem, steptol);
```

**Description**
The function `IDASetStepToleranceIC` specifies a positive lower bound on the Newton step.

**Arguments**
- `ida_mem` (void *) pointer to the IDAS memory block.
- `steptol` (int) Minimum allowed WRMS-norm of the Newton step (> 0.0).

**Return value**
The return value `flag` (of type `int`) is one of
- `IDA_SUCCESS` The optional value has been successfully set.
- `IDA_MEM NULL` The `ida_mem` pointer is NULL.
- `IDA_Ill_INPUT steptol` tolerance is <= 0.0.

**Notes**
The default value is `(unit roundoff)^2/3`.

F2003 Name FIDASetStepToleranceIC

### 4.5.8.4 Rootfinding optional input functions

The following functions can be called to set optional inputs to control the rootfinding algorithm.
Using IDAS for IVP Solution

### IDASSetRootDirection

**Call**

```c
flag = IDASSetRootDirection(ida_mem, rootdir);
```

**Description**
The function `IDASSetRootDirection` specifies the direction of zero-crossings to be located and returned to the user.

**Arguments**
- `ida_mem` (void *) pointer to the IDAS memory block.
- `rootdir` (int *) state array of length `nrtfn`, the number of root functions $g_i$, as specified in the call to the function `IDARootInit`. A value of 0 for `rootdir[i]` indicates that crossing in either direction should be reported for $g_i$. A value of +1 or −1 indicates that the solver should report only zero-crossings where $g_i$ is increasing or decreasing, respectively.

**Return value**
The return value `flag` (of type int) is one of
- `IDA_SUCCESS` The optional value has been successfully set.
- `IDA_MEM_NULL` The `ida_mem` pointer is NULL.
- `IDA_Ill_INPUT` rootfinding has not been activated through a call to `IDARootInit`.

**Notes**
The default behavior is to locate both zero-crossing directions.

F2003 Name FIDASetRootDirection

### IDASSetNoInactiveRootWarn

**Call**

```c
flag = IDASSetNoInactiveRootWarn(ida_mem);
```

**Description**
The function `IDASSetNoInactiveRootWarn` disables issuing a warning if some root function appears to be identically zero at the beginning of the integration.

**Arguments**
- `ida_mem` (void *) pointer to the IDAS memory block.

**Return value**
The return value `flag` (of type int) is one of
- `IDA_SUCCESS` The optional value has been successfully set.
- `IDA_MEM_NULL` The `ida_mem` pointer is NULL.

**Notes**

- IDAS will not report the initial conditions as a possible zero-crossing (assuming that one or more components $g_i$ are zero at the initial time). However, if it appears that some $g_i$ is identically zero at the initial time (i.e., $g_i$ is zero at the initial time and after the first step), IDAS will issue a warning which can be disabled with this optional input function.

F2003 Name FIDASetNoInactiveRootWarn

### 4.5.9 Interpolated output function

An optional function `IDAGetDky` is available to obtain additional output values. This function must be called after a successful return from `IDASolve` and provides interpolated values of $y$ or its derivatives of order up to the last internal order used for any value of $t$ in the last internal step taken by IDAS.

The call to the `IDAGetDky` function has the following form:

**Call**

```c
flag = IDAGetDky(ida_mem, t, k, dky);
```

**Description**
The function `IDAGetDky` computes the interpolated values of the $k^{th}$ derivative of $y$ for any value of $t$ in the last internal step taken by IDAS. The value of $k$ must be non-negative and smaller than the last internal order used. A value of 0 for $k$ means that the $y$ is interpolated. The value of $t$ must satisfy $t_n - h_u \leq t \leq t_n$, where $t_n$ denotes the current internal time reached, and $h_u$ is the last internal step size used successfully.

**Arguments**
- `ida_mem` (void *) pointer to the IDAS memory block.
- `t` (realtype) time at which to interpolate.
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k (int) integer specifying the order of the derivative of y wanted.
dky (N_Vector) vector containing the interpolated $k^{th}$ derivative of $y(t)$.

Return value

The return value flag (of type int) is one of

- IDA_SUCCESS: IDAGetDky succeeded.
- IDA_MEM_NULL: The ida_mem argument was NULL.
- IDA_BAD_T: $t$ is not in the interval $[t_n - h, t_n]$.
- IDA_BAD_K: $k$ is not one of $\{0, 1, \ldots, klast\}$.
- IDA_BAD_DKY: dky is NULL.

Notes

It is only legal to call the function IDAGetDky after a successful return from IDASolve. Functions IDAGetCurrentTime, IDAGetLastStep and IDAGetLastOrder (see §4.5.10.2) can be used to access $t_n$, $h_u$ and $klast$.

4.5.10 Optional output functions

IDAS provides an extensive list of functions that can be used to obtain solver performance information. Table 4.3 lists all optional output functions in IDAS, which are then described in detail in the remainder of this section.

Some of the optional outputs, especially the various counters, can be very useful in determining how successful the IDAS solver is in doing its job. For example, the counters nsteps and nrevals provide a rough measure of the overall cost of a given run, and can be compared among runs with differing input options to suggest which set of options is most efficient. The ratio nmiters/nsteps measures the performance of the nonlinear solver in solving the nonlinear systems at each time step; typical values for this range from 1.1 to 1.8. The ratio njevals/nmiters (in the case of a matrix-based linear solver), and the ratio npevals/nmiters (in the case of an iterative linear solver) measure the overall degree of nonlinearity in these systems, and also the quality of the approximate Jacobian or preconditioner being used. Thus, for example, njevals/nmiters can indicate if a user-supplied Jacobian is inaccurate, if this ratio is larger than for the case of the corresponding internal Jacobian. The ratio nliters/nmiters measures the performance of the Krylov iterative linear solver, and thus (indirectly) the quality of the preconditioner.

4.5.10.1 SUNDIALS version information

The following functions provide a way to get SUNDIALS version information at runtime.

```
SUNDIALSGetVersion
```

Call

```
flag = SUNDIALSGetVersion(version, len);
```

Description

The function SUNDIALSGetVersion fills a character array with SUNDIALS version information.

Arguments

- `version` (char *) character array to hold the SUNDIALS version information.
- `len` (int) allocated length of the `version` character array.

Return value

If successful, SUNDIALSGetVersion returns 0 and `version` contains the SUNDIALS version information. Otherwise, it returns -1 and `version` is not set (the input character array is too short).

Notes

A string of 25 characters should be sufficient to hold the version information. Any trailing characters in the `version` array are removed.
Table 4.3: Optional outputs from IDAS and IDALS

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<th>Optional output</th>
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<td>IDAGetWorkSpace</td>
</tr>
<tr>
<td>Cumulative number of internal steps</td>
<td>IDAGetNumSteps</td>
</tr>
<tr>
<td>No. of calls to residual function</td>
<td>IDAGetNumResEvals</td>
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</tr>
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</tr>
<tr>
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<td>IDAGetLastOrder</td>
</tr>
<tr>
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<td>IDAGetCurrentOrder</td>
</tr>
<tr>
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</tr>
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<tr>
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<tr>
<td>Step size to be attempted on the next step</td>
<td>IDAGetCurrentStep</td>
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<td>Current internal time reached by the solver</td>
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<td><strong>IDAS initial conditions calculation</strong></td>
<td></td>
</tr>
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<td><strong>IDALS linear solver interface</strong></td>
<td></td>
</tr>
<tr>
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</tr>
<tr>
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<td>IDAGetNumJacEvals</td>
</tr>
<tr>
<td>No. of residual calls for finite diff. Jacobian[-vector] evals.</td>
<td>IDAGetNumLinResEvals</td>
</tr>
<tr>
<td>No. of linear iterations</td>
<td>IDAGetNumLinIters</td>
</tr>
<tr>
<td>No. of linear convergence failures</td>
<td>IDAGetNumLinConvFails</td>
</tr>
<tr>
<td>No. of preconditioner evaluations</td>
<td>IDAGetNumPrecEvals</td>
</tr>
<tr>
<td>No. of preconditioner solves</td>
<td>IDAGetNumPrecSolves</td>
</tr>
<tr>
<td>No. of Jacobian-vector setup evaluations</td>
<td>IDAGetNumJTSetupEvals</td>
</tr>
<tr>
<td>No. of Jacobian-vector product evaluations</td>
<td>IDAGetNumJtimesEvals</td>
</tr>
<tr>
<td>Last return from a linear solver function</td>
<td>IDAGetLastLinFlag</td>
</tr>
<tr>
<td>Name of constant associated with a return flag</td>
<td>IDAGetLinReturnFlagName</td>
</tr>
</tbody>
</table>
SUNDIALSGetVersionNumber

Call: flag = SUNDIALSGetVersionNumber(&major, &minor, &patch, label, len);

Description: The function SUNDIALSGetVersionNumber sets integers for the SUNDIALS major, minor, and patch release numbers and fills a character array with the release label if applicable.

Arguments:
- major (int) SUNDIALS release major version number.
- minor (int) SUNDIALS release minor version number.
- patch (int) SUNDIALS release patch version number.
- label (char *) character array to hold the SUNDIALS release label.
- len (int) allocated length of the label character array.

Return value: If successful, SUNDIALSGetVersionNumber returns 0 and the major, minor, patch, and label values are set. Otherwise, it returns -1 and the values are not set (the input character array is too short).

Notes: A string of 10 characters should be sufficient to hold the label information. If a label is not used in the release version, no information is copied to label. Any trailing characters in the label array are removed.

4.5.10.2 Main solver optional output functions

IDAS provides several user-callable functions that can be used to obtain different quantities that may be of interest to the user, such as solver workspace requirements, solver performance statistics, as well as additional data from the IDAS memory block (a suggested tolerance scaling factor, the error weight vector, and the vector of estimated local errors). Also provided are functions to extract statistics related to the performance of the SUNNONLINSOL nonlinear solver being used. As a convenience, additional extraction functions provide the optional outputs in groups. These optional output functions are described next.

IDAGetWorkSpace

Call: flag = IDAGetWorkSpace(ida_mem, &lenrw, &leniw);

Description: The function IDAGetWorkSpace returns the IDAS real and integer workspace sizes.

Arguments:
- ida_mem (void *) pointer to the IDAS memory block.
- lenrw (long int) number of real values in the IDAS workspace.
- leniw (long int) number of integer values in the IDAS workspace.

Return value: The return value flag (of type int) is one of
- IDA_SUCCESS The optional output value has been successfully set.
- IDA_MEM_NULL The ida_mem pointer is NULL.

Notes: In terms of the problem size \( N \), the maximum method order \( \text{maxord} \), and the number \( \text{nrtfn} \) of root functions (see §4.5.6), the actual size of the real workspace, in \text{realttype} words, is given by the following:

- base value: \( \text{lenrw} = 55 + (m + 6) \cdot N_r + 3 \cdot \text{nrtfn} \);
- with IDASVtolerances: \( \text{lenrw} = \text{lenrw} + N_r \);
- with constraint checking (see IDASSetConstraints): \( \text{lenrw} = \text{lenrw} + N_r \);
- with id specified (see IDASSetId): \( \text{lenrw} = \text{lenrw} + N_r \);

where \( m = \text{max(} \text{maxord}, 3 \text{)} \), and \( N_r \) is the number of real words in one \text{N_Vector} (≈ \( N \)).

The size of the integer workspace (without distinction between int and long int words) is given by:

- base value: \( \text{leniw} = 38 + (m + 6) \cdot N_i + \text{nrtfn} \);
• with IDASVtolerances: leniw = leniw + N_i;
• with constraint checking: lenrw = lenrw + N_i;
• with id specified: lenrw = lenrw + N_i;

where \( N_i \) is the number of integer words in one \( \text{N}_\text{Vector} \) (= 1 for \( \text{N}_\text{VECTOR}_\text{SERIAL} \) and \( 2^\text{npes} \) for \( \text{N}_\text{VECTOR}_\text{PARALLEL} \) on \( \text{npes} \) processors).

For the default value of maxord, with no rootfinding, no id, no constraints, and with no call to IDASVtolerances, these lengths are given roughly by: lenrw = 55 + 11N, leniw = 49.

Note that additional memory is allocated if quadratures and/or forward sensitivity integration is enabled. See §4.7.1 and §5.2.1 for more details.

F2003 Name FIDAGetWorkSpace

```
IDAGetNumSteps
```

Call  \( \text{flag} = \text{IDAGetNumSteps}(\text{ida}_\text{mem}, \&\text{nsteps}); \)

Description The function IDAGetNumSteps returns the cumulative number of internal steps taken by the solver (total so far).

Arguments \( \text{ida}_\text{mem} \) (void *) pointer to the IDAS memory block.
\( \text{nsteps} \) (long int) number of steps taken by IDAS.

Return value The return value \( \text{flag} \) (of type int) is one of
\( \text{IDA}_{\text{SUCCESS}} \) The optional output value has been successfully set.
\( \text{IDA}_{\text{MEM}} \text{NULL} \) The \( \text{ida}_\text{mem} \) pointer is NULL.

F2003 Name FIDAGetNumSteps

```
IDAGetNumResEvals
```

Call  \( \text{flag} = \text{IDAGetNumResEvals}(\text{ida}_\text{mem}, \&\text{nrevals}); \)

Description The function IDAGetNumResEvals returns the number of calls to the user’s residual evaluation function.

Arguments \( \text{ida}_\text{mem} \) (void *) pointer to the IDAS memory block.
\( \text{nrevals} \) (long int) number of calls made to the user’s \( \text{res} \) function.

Return value The return value \( \text{flag} \) (of type int) is one of
\( \text{IDA}_{\text{SUCCESS}} \) The optional output value has been successfully set.
\( \text{IDA}_{\text{MEM}} \text{NULL} \) The \( \text{ida}_\text{mem} \) pointer is NULL.

Notes The \( \text{nrevals} \) value returned by IDAGetNumResEvals does not account for calls made to \( \text{res} \) from a linear solver or preconditioner module.

F2003 Name FIDAGetNumResEvals

```
IDAGetNumLinSolvSetups
```

Call  \( \text{flag} = \text{IDAGetNumLinSolvSetups}(\text{ida}_\text{mem}, \&\text{nlinsetups}); \)

Description The function IDAGetNumLinSolvSetups returns the cumulative number of calls made to the linear solver’s setup function (total so far).

Arguments \( \text{ida}_\text{mem} \) (void *) pointer to the IDAS memory block.
\( \text{nlinsetups} \) (long int) number of calls made to the linear solver setup function.

Return value The return value \( \text{flag} \) (of type int) is one of
\( \text{IDA}_{\text{SUCCESS}} \) The optional output value has been successfully set.
4.5 User-callable functions

IDA_MEM_NULL The ida_mem pointer is NULL.

F2003 Name FIDAGetNumLinSolvSetups

**FIDAGetNumErrTestFails**

Call `flag = FIDAGetNumErrTestFails(ida_mem, &netfails);`

Description The function `FIDAGetNumErrTestFails` returns the cumulative number of local error test failures that have occurred (total so far).

Arguments `ida_mem` (void *) pointer to the IDAS memory block.
`netfails` (long int) number of error test failures.

Return value The return value `flag` (of type `int`) is one of
- `IDA_SUCCESS` The optional output value has been successfully set.
- `IDA_MEM_NULL` The ida_mem pointer is NULL.

F2003 Name FIDAGetNumErrTestFails

**FIDAGetLastOrder**

Call `flag = FIDAGetLastOrder(ida_mem, &klast);`

Description The function `FIDAGetLastOrder` returns the integration method order used during the last internal step.

Arguments `ida_mem` (void *) pointer to the IDAS memory block.
`klast` (int) method order used on the last internal step.

Return value The return value `flag` (of type `int`) is one of
- `IDA_SUCCESS` The optional output value has been successfully set.
- `IDA_MEM_NULL` The ida_mem pointer is NULL.

F2003 Name FIDAGetLastOrder

**FIDAGetCurrentOrder**

Call `flag = FIDAGetCurrentOrder(ida_mem, &kcur);`

Description The function `FIDAGetCurrentOrder` returns the integration method order to be used on the next internal step.

Arguments `ida_mem` (void *) pointer to the IDAS memory block.
`kcur` (int) method order to be used on the next internal step.

Return value The return value `flag` (of type `int`) is one of
- `IDA_SUCCESS` The optional output value has been successfully set.
- `IDA_MEM_NULL` The ida_mem pointer is NULL.

F2003 Name FIDAGetCurrentOrder

**FIDAGetLastStep**

Call `flag = FIDAGetLastStep(ida_mem, &hlast);`

Description The function `FIDAGetLastStep` returns the integration step size taken on the last internal step (if from IDASolve), or the last value of the artificial step size $h$ (if from IDACalcIC).

Arguments `ida_mem` (void *) pointer to the IDAS memory block.
`hlast` (realtype) step size taken on the last internal step by IDAS, or last artificial step size used in IDACalcIC, whichever was called last.

Return value The return value `flag` (of type `int`) is one of
**IDAGetCurrentStep**

**Call**
```c
flag = IDAGetCurrentStep(ida_mem, &hcur);
```

**Description**
The function **IDAGetCurrentStep** returns the integration step size to be attempted on the next internal step.

**Arguments**
- *ida_mem* (void *) pointer to the IDAS memory block.
- *hcur* (realtype) step size to be attempted on the next internal step.

**Return value**
The return value *flag* (of type int) is one of
- **IDA_SUCCESS** The optional output value has been successfully set.
- **IDA_MEM_NULL** The *ida_mem* pointer is NULL.

F2003 Name **FIDAGetLastStep**

**IDAGetActualInitStep**

**Call**
```c
flag = IDAGetActualInitStep(ida_mem, &hinused);
```

**Description**
The function **IDAGetActualInitStep** returns the value of the integration step size used on the first step.

**Arguments**
- *ida_mem* (void *) pointer to the IDAS memory block.
- *hinused* (realtype) actual value of initial step size.

**Return value**
The return value *flag* (of type int) is one of
- **IDA_SUCCESS** The optional output value has been successfully set.
- **IDA_MEM_NULL** The *ida_mem* pointer is NULL.

**Notes**
Even if the value of the initial integration step size was specified by the user through a call to **IDASetInitStep**, this value might have been changed by IDAS to ensure that the step size is within the prescribed bounds ($h_{\text{min}} \leq h_0 \leq h_{\text{max}}$), or to meet the local error test.

F2003 Name **FIDAGetActualInitStep**

**IDAGetCurrentTime**

**Call**
```c
flag = IDAGetCurrentTime(ida_mem, &tcur);
```

**Description**
The function **IDAGetCurrentTime** returns the current internal time reached by the solver.

**Arguments**
- *ida_mem* (void *) pointer to the IDAS memory block.
- *tcur* (realtype) current internal time reached.

**Return value**
The return value *flag* (of type int) is one of
- **IDA_SUCCESS** The optional output value has been successfully set.
- **IDA_MEM_NULL** The *ida_mem* pointer is NULL.

F2003 Name **FIDAGetCurrentTime**
4.5 User-callable functions

**IDAGetTolScaleFactor**

**Call**

```c
flag = IDAGetTolScaleFactor(ida_mem, &tolsfac);
```

**Description**

The function `IDAGetTolScaleFactor` returns a suggested factor by which the user’s tolerances should be scaled when too much accuracy has been requested for some internal step.

**Arguments**

- `ida_mem` (void *) pointer to the IDAS memory block.
- `tolsfac` (realtype) suggested scaling factor for user tolerances.

**Return value**

The return value `flag` (of type int) is one of

- `IDA_SUCCESS` The optional output value has been successfully set.
- `IDA_MEM_NULL` The `ida_mem` pointer is NULL.

**F2003 Name** FIDAGetTolScaleFactor

**IDAGetErrWeights**

**Call**

```c
flag = IDAGetErrWeights(ida_mem, eweight);
```

**Description**

The function `IDAGetErrWeights` returns the solution error weights at the current time. These are the $W_i$ given by Eq. (2.7) (or by the user’s IDAEwtFn).

**Arguments**

- `ida_mem` (void *) pointer to the IDAS memory block.
- `eweight` (N_Vector) solution error weights at the current time.

**Return value**

The return value `flag` (of type int) is one of

- `IDA_SUCCESS` The optional output value has been successfully set.
- `IDA_MEM_NULL` The `ida_mem` pointer is NULL.

**Notes**

The user must allocate space for `eweight`.

**F2003 Name** FIDAGetErrWeights

**IDAGetEstLocalErrors**

**Call**

```c
flag = IDAGetEstLocalErrors(ida_mem, ele);
```

**Description**

The function `IDAGetEstLocalErrors` returns the estimated local errors.

**Arguments**

- `ida_mem` (void *) pointer to the IDAS memory block.
- `ele` (N_Vector) estimated local errors at the current time.

**Return value**

The return value `flag` (of type int) is one of

- `IDA_SUCCESS` The optional output value has been successfully set.
- `IDA_MEM_NULL` The `ida_mem` pointer is NULL.

**Notes**

The user must allocate space for `ele`.

The values returned in `ele` are only valid if `IDASolve` returned a non-negative value.

The `ele` vector, together with the `eweight` vector from `IDAGetErrWeights`, can be used to determine how the various components of the system contributed to the estimated local error test. Specifically, that error test uses the RMS norm of a vector whose components are the products of the components of these two vectors. Thus, for example, if there were recent error test failures, the components causing the failures are those with largest values for the products, denoted loosely as $eweight[i]*ele[i]$.

**F2003 Name** FIDAGetEstLocalErrors
IDAGetIntegratorStats

Call

```c
flag = IDAGetIntegratorStats(ida_mem, &nsteps, &nrevals, &nlinsetups, &netfails, &klast, &kcur, &hinused, &hlast, &hcur, &tcur);
```

Description The function **IDAGetIntegratorStats** returns the IDAS integrator statistics as a group.

Arguments

- `ida_mem` (void *) pointer to the IDAS memory block.
- `nsteps` (long int) cumulative number of steps taken by IDAS.
- `nrevals` (long int) cumulative number of calls to the user's `res` function.
- `nlinsetups` (long int) cumulative number of calls made to the linear solver setup function.
- `netfails` (long int) cumulative number of error test failures.
- `klast` (int) method order used on the last internal step.
- `kcur` (int) method order to be used on the next internal step.
- `hinused` (realtype) actual value of initial step size.
- `hlast` (realtype) step size taken on the last internal step.
- `hcur` (realtype) step size to be attempted on the next internal step.
- `tcur` (realtype) current internal time reached.

Return value The return value `flag` (of type int) is one of

- `IDA_SUCCESS` the optional output values have been successfully set.
- `IDA_MEM_NULL` the `ida_mem` pointer is `NULL`.

F2003 Name FIDAGetIntegratorStats

IDAGetNumNonlinSolvIters

Call

```c
flag = IDAGetNumNonlinSolvIters(ida_mem, &nniters);
```

Description The function **IDAGetNumNonlinSolvIters** returns the cumulative number of nonlinear iterations performed.

Arguments

- `ida_mem` (void *) pointer to the IDAS memory block.
- `nniters` (long int) number of nonlinear iterations performed.

Return value The return value `flag` (of type int) is one of

- `IDA_SUCCESS` The optional output value has been successfully set.
- `IDA_MEM_NULL` The `ida_mem` pointer is `NULL`.
- `IDA_MEM_FAIL` The `sunnonlinsol` module is `NULL`.

F2003 Name FIDAGetNumNonlinSolvIters

IDAGetNumNonlinSolvConvFails

Call

```c
flag = IDAGetNumNonlinSolvConvFails(ida_mem, &nncfails);
```

Description The function **IDAGetNumNonlinSolvConvFails** returns the cumulative number of nonlinear convergence failures that have occurred.

Arguments

- `ida_mem` (void *) pointer to the IDAS memory block.
- `nncfails` (long int) number of nonlinear convergence failures.

Return value The return value `flag` (of type int) is one of

- `IDA_SUCCESS` The optional output value has been successfully set.
- `IDA_MEM_NULL` The `ida_mem` pointer is `NULL`.

F2003 Name FIDAGetNumNonlinSolvConvFails
4.5 User-callable functions

**IDAGetNonlinSolvStats**

Call: `flag = IDAGetNonlinSolvStats(ida_mem, &nniters, &nncfails);`

Description: The function `IDAGetNonlinSolvStats` returns the IDAS nonlinear solver statistics as a group.

Arguments:
- `ida_mem` (void *) pointer to the IDAS memory block.
- `nniters` (long int) cumulative number of nonlinear iterations performed.
- `nncfails` (long int) cumulative number of nonlinear convergence failures.

Return value: The return value `flag` (of type `int`) is one of:
- `IDA_SUCCESS` The optional output value has been successfully set.
- `IDA_MEM_NULL` The `ida_mem` pointer is NULL.
- `IDA_MEM_FAIL` The SUNNONLINSOL module is NULL.

F2003 Name: `FIDAGetNonlinSolvStats`

**IDAGetReturnFlagName**

Call: `name = IDAGetReturnFlagName(flag);`

Description: The function `IDAGetReturnFlagName` returns the name of the IDAS constant corresponding to `flag`.

Arguments: The only argument, of type `int`, is a return flag from an IDAS function.

Return value: The return value is a string containing the name of the corresponding constant.

F2003 Name: `FIDAGetReturnFlagName`

### 4.5.10.3 Initial condition calculation optional output functions

**IDAGetNumBcktrackOps**

Call: `flag = IDAGetNumBacktrackOps(ida_mem, &nbacktr);`

Description: The function `IDAGetNumBacktrackOps` returns the number of backtrack operations done in the linesearch algorithm in `IDACalcIC`.

Arguments:
- `ida_mem` (void *) pointer to the IDAS memory block.
- `nbacktr` (long int) the cumulative number of backtrack operations.

Return value: The return value `flag` (of type `int`) is one of:
- `IDA_SUCCESS` The optional output value has been successfully set.
- `IDA_MEM_NULL` The `ida_mem` pointer is NULL.

F2003 Name: `FIDAGetNumBcktrackOps`

**IDAGetConsistentIC**

Call: `flag = IDAGetConsistentIC(ida_mem, yy0_mod, yp0_mod);`

Description: The function `IDAGetConsistentIC` returns the corrected initial conditions calculated by `IDACalcIC`.

Arguments:
- `ida_mem` (void *) pointer to the IDAS memory block.
- `yy0_mod` (N_Vector) consistent solution vector.
- `yp0_mod` (N_Vector) consistent derivative vector.

Return value: The return value `flag` (of type `int`) is one of:
- `IDA_SUCCESS` The optional output value has been successfully set.
Using IDAS for IVP Solution

IDA_ILL_INPUT  The function was not called before the first call to IDASolve.
IDA_MEM_NULL  The ida_mem pointer is NULL.

Notes  If the consistent solution vector or consistent derivative vector is not desired, pass NULL for the corresponding argument.

The user must allocate space for yy0_mod and yp0_mod (if not NULL).

F2003 Name  FIDAGetConsistentIC

4.5.10.4  Rootfinding optional output functions

There are two optional output functions associated with rootfinding.

IDAGetRootInfo

Call  flag = IDAGetRootInfo(ida_mem, rootsfound);

Description  The function IDAGetRootInfo returns an array showing which functions were found to have a root.

Arguments  ida_mem  (void *) pointer to the IDAS memory block.
rootsfound  (int *) array of length nrtfn with the indices of the user functions $g_i$ found to have a root. For $i = 0, \ldots, nrtfn - 1$, $rootsfound[i] \neq 0$ if $g_i$ has a root, and = 0 if not.

Return value  The return value flag (of type int) is one of

IDA_SUCCESS  The optional output values have been successfully set.
IDA_MEM_NULL  The ida_mem pointer is NULL.

Notes  Note that, for the components $g_i$ for which a root was found, the sign of $rootsfound[i]$ indicates the direction of zero-crossing. A value of +1 indicates that $g_i$ is increasing, while a value of −1 indicates a decreasing $g_i$.

The user must allocate memory for the vector rootsfound.

F2003 Name  FIDAGetRootInfo

IDAGetNumGEvals

Call  flag = IDAGetNumGEvals(ida_mem, &ngevals);

Description  The function IDAGetNumGEvals returns the cumulative number of calls to the user root function $g$.

Arguments  ida_mem  (void *) pointer to the IDAS memory block.
nergievals  (long int) number of calls to the user's function $g$ so far.

Return value  The return value flag (of type int) is one of

IDA_SUCCESS  The optional output value has been successfully set.
IDA_MEM_NULL  The ida_mem pointer is NULL.

F2003 Name  FIDAGetNumGEvals

4.5.10.5  IDALS linear solver interface optional output functions

The following optional outputs are available from the IDALS modules: workspace requirements, number of calls to the Jacobian routine, number of calls to the residual routine for finite-difference Jacobian or Jacobian-vector product approximation, number of linear iterations, number of linear convergence failures, number of calls to the preconditioner setup and solve routines, number of calls to the Jacobian-vector setup and product routines, and last return value from an IDALS function. Note that, where the name of an output would otherwise conflict with the name of an optional output from the main solver, a suffix LS (for Linear Solver) has been added (e.g., lenrwLS).
4.5 User-callable functions

**IDAGetLinWorkSpace**

**Call**

```c
flag = IDAGetLinWorkSpace(ida_mem, &lenrwLS, &leniwLS);
```

**Description**
The function IDAGetLinWorkSpace returns the sizes of the real and integer workspaces used by the IDALS linear solver interface.

**Arguments**

- `ida_mem` (void *) pointer to the IDAS memory block.
- `lenrwLS` (long int) the number of real values in the IDALS workspace.
- `leniwLS` (long int) the number of integer values in the IDALS workspace.

**Return value**
The return value `flag` (of type int) is one of:

- IDALS_SUCCESS The optional output value has been successfully set.
- IDALS_MEM_NULL The `ida_mem` pointer is NULL.
- IDALS_LMEM_NULL The IDALS linear solver has not been initialized.

**Notes**
The workspace requirements reported by this routine correspond only to memory allocated within this interface and to memory allocated by the SUNLINSOL object attached to it. The template Jacobian matrix allocated by the user outside of IDALS is not included in this report.

The previous routines IDADlsGetWorkspace and IDASpilsGetWorkspace are now wrappers for this routine, and may still be used for backward-compatibility. However, these will be deprecated in future releases, so we recommend that users transition to the new routine name soon.

F2003 Name FIDAGetLinWorkSpace

**IDAGetNumJacEvals**

**Call**

```c
flag = IDAGetNumJacEvals(ida_mem, &njevals);
```

**Description**
The function IDAGetNumJacEvals returns the cumulative number of calls to the IDALS Jacobian approximation function.

**Arguments**

- `ida_mem` (void *) pointer to the IDAS memory block.
- `njevals` (long int) the cumulative number of calls to the Jacobian function (total so far).

**Return value**
The return value `flag` (of type int) is one of:

- IDALS_SUCCESS The optional output value has been successfully set.
- IDALS_MEM_NULL The `ida_mem` pointer is NULL.
- IDALS_LMEM_NULL The IDALS linear solver has not been initialized.

**Notes**
The previous routine IDADlsGetNumJacEvals is now a wrapper for this routine, and may still be used for backward-compatibility. However, this will be deprecated in future releases, so we recommend that users transition to the new routine name soon.

F2003 Name FIDAGetNumJacEvals

**IDAGetNumLinResEvals**

**Call**

```c
flag = IDAGetNumLinResEvals(ida_mem, &nrevalsLS);
```

**Description**
The function IDAGetNumLinResEvals returns the cumulative number of calls to the user residual function due to the finite difference Jacobian approximation or finite difference Jacobian-vector product approximation.

**Arguments**

- `ida_mem` (void *) pointer to the IDAS memory block.
- `nrevalsLS` (long int) the cumulative number of calls to the user residual function.

**Return value**
The return value `flag` (of type int) is one of:

- IDALS_SUCCESS The optional output value has been successfully set.
- IDALS_MEM_NULL The `ida_mem` pointer is NULL.
- IDALS_LMEM_NULL The IDALS linear solver has not been initialized.

**Notes**
The previous routine IDADlsGetNumJacEvals is now a wrapper for this routine, and may still be used for backward-compatibility. However, this will be deprecated in future releases, so we recommend that users transition to the new routine name soon.

F2003 Name FIDAGetNumLinResEvals
IDALS_SUCCESS The optional output value has been successfully set.
IDALS_MEM_NULL The ida_mem pointer is NULL.
IDALS_LMEM_NULL The IDALS linear solver has not been initialized.

Notes The value nrevalsLS is incremented only if one of the default internal difference quotient functions is used.

The previous routines IDADlsGetNumRhsEvals and IDASpilsGetNumRhsEvals are now wrappers for this routine, and may still be used for backward-compatibility. However, these will be deprecated in future releases, so we recommend that users transition to the new routine name soon.

F2003 Name FIDAGetNumLinResEvals

**FIDAGetNumLinResEvals**

Call \( \text{flag} = \text{FIDAGetNumLinResEvals}(\text{ida_mem}, &\text{nrevals}) \);

Description The function FIDAGetNumLinResEvals returns the cumulative number of right-hand-side evaluations.

Arguments ida_mem (void *) pointer to the IDAS memory block.
nrevals (long int) the current number of right-hand-side evaluations.

Return value The return value flag (of type int) is one of
IDALS_SUCCESS The optional output value has been successfully set.
IDALS_MEM_NULL The ida_mem pointer is NULL.
IDALS_LMEM_NULL The IDALS linear solver has not been initialized.

Notes The previous routine IDASpilsGetNumLinResEvals is now a wrapper for this routine, and may still be used for backward-compatibility. However, this will be deprecated in future releases, so we recommend that users transition to the new routine name soon.

F2003 Name FIDAGetNumLinResEvals

**FIDAGetNumLinResEvals**

Call \( \text{flag} = \text{FIDAGetNumLinConvFails}(\text{ida_mem}, &\text{nlcfails}) \);

Description The function FIDAGetNumLinConvFails returns the cumulative number of linear convergence failures.

Arguments ida_mem (void *) pointer to the IDAS memory block.
nlcfails (long int) the current number of linear convergence failures.

Return value The return value flag (of type int) is one of
IDALS_SUCCESS The optional output value has been successfully set.
IDALS_MEM_NULL The ida_mem pointer is NULL.
IDALS_LMEM_NULL The IDALS linear solver has not been initialized.

Notes The previous routine IDASpilsGetNumLinConvFails is now a wrapper for this routine, and may still be used for backward-compatibility. However, this will be deprecated in future releases, so we recommend that users transition to the new routine name soon.

F2003 Name FIDAGetNumLinConvFails

**FIDAGetNumLinConvFails**

Call \( \text{flag} = \text{FIDAGetNumPrecEvals}(\text{ida_mem}, &\text{npevals}) \);

Description The function FIDAGetNumPrecEvals returns the cumulative number of preconditioner evaluations, i.e., the number of calls made to psetup.

Arguments ida_mem (void *) pointer to the IDAS memory block.
4.5 User-callable functions

**npevals** (long int) the cumulative number of calls to psetup.

Return value
The return value flag (of type int) is one of:
- IDALS_SUCCESS The optional output value has been successfully set.
- IDALS_MEM_NULL The ida_mem pointer is NULL.
- IDALS_LMEM_NULL The IDALS linear solver has not been initialized.

Notes
The previous routine IDASpilsGetNumPrecEvals is now a wrapper for this routine, and may still be used for backward-compatibility. However, this will be deprecated in future releases, so we recommend that users transition to the new routine name soon.

F2003 Name FIDAGetNumPrecEvals

**FIDAGetNumPrecSolves**

Call
flag = FIDAGetNumPrecSolves(ida_mem, &npsolves);

Description
The function FIDAGetNumPrecSolves returns the cumulative number of calls made to the preconditioner solve function, psolve.

Arguments
ida_mem (void *) pointer to the IDAS memory block.

Return value
The return value flag (of type int) is one of:
- IDALS_SUCCESS The optional output value has been successfully set.
- IDALS_MEM_NULL The ida_mem pointer is NULL.
- IDALS_LMEM_NULL The IDALS linear solver has not been initialized.

Notes
The previous routine IDASpilsGetNumPrecSolves is now a wrapper for this routine, and may still be used for backward-compatibility. However, this will be deprecated in future releases, so we recommend that users transition to the new routine name soon.

F2003 Name FIDAGetNumPrecSolves

**FIDAGetNumJTSetupEvals**

Call
flag = FIDAGetNumJTSetupEvals(ida_mem, &njtsetup);

Description
The function FIDAGetNumJTSetupEvals returns the cumulative number of calls made to the Jacobian-vector setup function jsetup.

Arguments
ida_mem (void *) pointer to the IDAS memory block.

Return value
The return value flag (of type int) is one of:
- IDA_SUCCESS The optional output value has been successfully set.
- IDA_MEM_NULL The ida_mem pointer is NULL.
- IDA_LMEM_NULL The IDA linear solver has not been initialized.

Notes
The previous routine IDASpilsGetNumJTSetupEvals is now a wrapper for this routine, and may still be used for backward-compatibility. However, this will be deprecated in future releases, so we recommend that users transition to the new routine name soon.

F2003 Name FIDAGetNumJTSetupEvals

**FIDAGetNumJtimesEvals**

Call
flag = FIDAGetNumJtimesEvals(ida_mem, &njvevals);

Description
The function FIDAGetNumJtimesEvals returns the cumulative number of calls made to the Jacobian-vector function, jtimes.

Arguments
ida_mem (void *) pointer to the IDAS memory block.
njvevals (long int) the cumulative number of calls to jtimes.

Return value The return value flag (of type int) is one of
  IDA_SUCCESS   The optional output value has been successfully set.
  IDA_MEM_NULL  The ida_mem pointer is NULL.
  IDA_LMEM_NULL The IDA linear solver has not been initialized.

Notes The previous routine IDASpilsGetNumJtimesEvals is now a wrapper for this routine, and may still be used for backward-compatibility. However, this will be deprecated in future releases, so we recommend that users transition to the new routine name soon.

F2003 Name FIDAGetNumJtimesEvals

---

IDAGetLastLinFlag

Call flag = IDAGetLastLinFlag(ida_mem, &lsflag);

Description The function IDAGetLastLinFlag returns the last return value from an IDALS routine.

Arguments ida_mem (void *) pointer to the IDAS memory block.
  lsflag (long int) the value of the last return flag from an IDALS function.

Return value The return value flag (of type int) is one of
  IDALS_SUCCESS   The optional output value has been successfully set.
  IDALS_MEM_NULL The ida_mem pointer is NULL.
  IDALS_LMEM_NULL The IDALS linear solver has not been initialized.

Notes If the IDALS setup function failed (i.e., IDASolve returned IDA_LSETUP_FAIL) when using the SUNLINSOL_DENSE or SUNLINSOL_BAND modules, then the value of lsflag is equal to the column index (numbered from one) at which a zero diagonal element was encountered during the LU factorization of the (dense or banded) Jacobian matrix.

If the IDALS setup function failed when using another SUNLINSOL module, then lsflag will be SUNLS_PSET_FAIL_UNREC, SUNLS_ASET_FAIL_UNREC, or SUNLS_PACKAGE_FAIL_UNREC.

If the IDALS solve function failed (IDASolve returned IDA_LSOLVE_FAIL), lsflag contains the error return flag from the SUNLINSOL object, which will be one of:
  SUNLS_MEM_NULL, indicating that the SUNLINSOL memory is NULL;
  SUNLS_ATIMES_FAIL_UNREC, indicating an unrecoverable failure in the J * v function;
  SUNLS_P SOLVE_FAIL_UNREC, indicating that the preconditioner solve function psolve failed unrecoverably;
  SUNLS_GS_FAIL, indicating a failure in the Gram-Schmidt procedure (generated only in SPGMR or SPFGMR); SUNLS_QRSOL_FAIL, indicating that the matrix R was found to be singular during the QR solve phase (SPGMR and SPFGMR only); or
  SUNLS_PACKAGE_FAIL_UNREC, indicating an unrecoverable failure in an external iterative linear solver package.

The previous routines IDAIsGetLastFlag and IDASpIsGetLastFlag are now wrappers for this routine, and may still be used for backward-compatibility. However, these will be deprecated in future releases, so we recommend that users transition to the new routine name soon.

F2003 Name FIDAGetLastLinFlag

---

IDAGetLinReturnFlagName

Call name = IDAGetLinReturnFlagName(lsflag);

Description The function IDAGetLinReturnFlagName returns the name of the IDA constant corresponding to lsflag.

Arguments The only argument, of type long int, is a return flag from an IDALS function.
4.5 User-callable functions

Return value  The return value is a string containing the name of the corresponding constant.

If $1 \leq \text{lsflag} \leq N$ (LU factorization failed), this function returns “NONE”.

Notes  The previous routines IDADlsGetReturnFlagName and IDASPilsGetReturnFlagName are now wrappers for this routine, and may still be used for backward-compatibility. However, these will be deprecated in future releases, so we recommend that users transition to the new routine name soon.

F2003 Name  FIDAGetLinReturnFlagName

4.5.11 IDAS reinitialization function

The function IDAReInit reinitializes the main IDAS solver for the solution of a new problem, where a prior call to IDAInit has been made. The new problem must have the same size as the previous one. IDAReInit performs the same input checking and initializations that IDAInit does, but does no memory allocation, as it assumes that the existing internal memory is sufficient for the new problem. A call to IDAReInit deletes the solution history that was stored internally during the previous integration. Following a successful call to IDAReInit, call IDASolve again for the solution of the new problem.

The use of IDAReInit requires that the maximum method order, maxord, is no larger for the new problem than for the problem specified in the last call to IDAInit. In addition, the same NVECTOR module set for the previous problem will be reused for the new problem.

If there are changes to the linear solver specifications, make the appropriate calls to either the linear solver objects themselves, or to the IDALS interface routines, as described in §4.5.3.

If there are changes to any optional inputs, make the appropriate IDASet*** calls, as described in §4.5.8. Otherwise, all solver inputs set previously remain in effect.

One important use of the IDAReInit function is in the treating of jump discontinuities in the residual function. Except in cases of fairly small jumps, it is usually more efficient to stop at each point of discontinuity and restart the integrator with a readjusted DAE model, using a call to IDAReInit. To stop when the location of the discontinuity is known, simply make that location a value of tout. To stop when the location of the discontinuity is determined by the solution, use the rootfinding feature. In either case, it is critical that the residual function not incorporate the discontinuity, but rather have a smooth extension over the discontinuity, so that the step across it (and subsequent rootfinding, if used) can be done efficiently. Then use a switch within the residual function (communicated through user_data) that can be flipped between the stopping of the integration and the restart, so that the restarted problem uses the new values (which have jumped). Similar comments apply if there is to be a jump in the dependent variable vector.

```
[IDAReInit]
.Call  flag = IDAReInit(ida_mem, t0, y0, yp0);
.Description  The function IDAReInit provides required problem specifications and reinitializes IDAS.
.Arguments  ida_mem (void *) pointer to the IDAS memory block.
            t0 (realtype) is the initial value of t.
            y0 (N_Vector) is the initial value of y.
            yp0 (N_Vector) is the initial value of \dot{y}.
.Return value  The return value flag (of type int) will be one of the following:
            IDA_SUCCESS  The call to IDAReInit was successful.
            IDA_MEM_NULL  The IDAS memory block was not initialized through a previous call to IDACreate.
            IDA_NO_MALLOC  Memory space for the IDAS memory block was not allocated through a previous call to IDAInit.
            IDA_Ill_INPUT  An input argument to IDAReInit has an illegal value.
```
Notes If an error occurred, IDAReInit also sends an error message to the error handler function.

F2003 Name FIDAReInit

4.6 User-supplied functions

The user-supplied functions consist of one function defining the DAE residual, (optionally) a function that handles error and warning messages, (optionally) a function that provides the error weight vector, (optionally) one or two functions that provide Jacobian-related information for the linear solver, and (optionally) one or two functions that define the preconditioner for use in any of the Krylov iteration algorithms.

4.6.1 Residual function

The user must provide a function of type IDAResFn defined as follows:

```c
typedef int (*IDAResFn)(realtype tt, N_Vector yy, N_Vector yp,
                        N_Vector rr, void *user_data);
```

**Definition**

This function computes the problem residual for given values of the independent variable \( t \), state vector \( y \), and derivative \( \dot{y} \).

**Purpose**

Arguments

- `tt` is the current value of the independent variable.
- `yy` is the current value of the dependent variable vector, \( y(t) \).
- `yp` is the current value of \( \dot{y}(t) \).
- `rr` is the output residual vector \( F(t, y, \dot{y}) \).
- `user_data` is a pointer to user data, the same as the `user_data` parameter passed to IDASetUserData.

**Return value**

An IDAResFn function type should return a value of 0 if successful, a positive value if a recoverable error occurred (e.g., `yy` has an illegal value), or a negative value if a nonrecoverable error occurred. In the last case, the integrator halts. If a recoverable error occurred, the integrator will attempt to correct and retry.

**Notes**

A recoverable failure error return from the IDAResFn is typically used to flag a value of the dependent variable `y` that is “illegal” in some way (e.g., negative where only a non-negative value is physically meaningful). If such a return is made, IDAS will attempt to recover (possibly repeating the nonlinear solve, or reducing the step size) in order to avoid this recoverable error return.

For efficiency reasons, the DAE residual function is not evaluated at the converged solution of the nonlinear solver. Therefore, in general, a recoverable error in that converged value cannot be corrected. (It may be detected when the right-hand side function is called the first time during the following integration step, but a successful step cannot be undone.) However, if the user program also includes quadrature integration, the state variables can be checked for legality in the call to IDAQuadRhsFn, which is called at the converged solution of the nonlinear system, and therefore IDAS can be flagged to attempt to recover from such a situation. Also, if sensitivity analysis is performed with the staggered method, the DAE residual function is called at the converged solution of the nonlinear system, and a recoverable error at that point can be flagged, and IDAS will then try to correct it.

Allocation of memory for `yp` is handled within IDAS.
4.6 User-supplied functions

4.6.2 Error message handler function

As an alternative to the default behavior of directing error and warning messages to the file pointed to by \texttt{errfp} (see \texttt{IDASetErrFile}), the user may provide a function of type \texttt{IDAErrHandlerFn} to process any such messages. The function type \texttt{IDAErrHandlerFn} is defined as follows:

\begin{verbatim}
typedef void (*IDAErrHandlerFn)(int error_code, const char *module, const char *function, char *msg, void *eh_data);
\end{verbatim}

\textbf{Definition} This function processes error and warning messages from IDAS and its sub-modules.

\textbf{Arguments}
- \texttt{error\_code} is the error code.
- \texttt{module} is the name of the IDAS module reporting the error.
- \texttt{function} is the name of the function in which the error occurred.
- \texttt{msg} is the error message.
- \texttt{eh\_data} is a pointer to user data, the same as the \texttt{eh\_data} parameter passed to \texttt{IDASetErrHandlerFn}.

\textbf{Return value} A \texttt{IDAErrHandlerFn} function has no return value.

\textbf{Notes} \texttt{error\_code} is negative for errors and positive (\texttt{IDA\_WARNING}) for warnings. If a function that returns a pointer to memory encounters an error, it sets \texttt{error\_code} to 0.

4.6.3 Error weight function

As an alternative to providing the relative and absolute tolerances, the user may provide a function of type \texttt{IDAEwtFn} to compute a vector \( e\texttt{wt} \) containing the multiplicative weights \( W_i \) used in the WRMS norm \( \| v \|_{\text{WRMS}} = \sqrt{(1/N) \sum_{i=1}^{N} (W_i \cdot v_i)^2} \). These weights will used in place of those defined by Eq. (2.7). The function type \texttt{IDAEwtFn} is defined as follows:

\begin{verbatim}
typedef int (*IDAEwtFn)(NVector y, NVector ewt, void *user_data);
\end{verbatim}

\textbf{Definition} This function computes the WRMS error weights for the vector \( y \).

\textbf{Arguments}
- \texttt{y} is the value of the dependent variable vector at which the weight vector is to be computed.
- \texttt{ewt} is the output vector containing the error weights.
- \texttt{user\_data} is a pointer to user data, the same as the \texttt{user\_data} parameter passed to \texttt{IDASetUserData}.

\textbf{Return value} An \texttt{IDAEwtFn} function type must return 0 if it successfully set the error weights and \(-1\) otherwise.

\textbf{Notes} Allocation of memory for \texttt{ewt} is handled within IDAS.

The error weight vector must have all components positive. It is the user’s responsibility to perform this test and return \(-1\) if it is not satisfied.

4.6.4 Rootfinding function

If a rootfinding problem is to be solved during the integration of the DAE system, the user must supply a C function of type \texttt{IDARootFn}, defined as follows:
IDARootFn

**Definition**
typedef int (*IDARootFn)(realtype t, N_Vector y, N_Vector yp,
realtype *gout, void *user_data);

**Purpose**
This function computes a vector-valued function \( g(t, y, \dot{y}) \) such that the roots of the \( nrtfn \) components \( g_i(t, y, \dot{y}) \) are to be found during the integration.

**Arguments**
t is the current value of the independent variable.
y is the current value of the dependent variable vector, \( y(t) \).
yp is the current value of \( \dot{y}(t) \), the \( t \)-derivative of \( y \).
gout is the output array, of length \( nrtfn \), with components \( g_i(t, y, \dot{y}) \).
user_data is a pointer to user data, the same as the user_data parameter passed to IDASolve.

**Return value**
An IDARootFn should return 0 if successful or a non-zero value if an error occurred (in which case the integration is halted and IDASolve returns IDA_RTFUNC_FAIL).

**Notes**
Allocation of memory for gout is handled within IDAS.

### 4.6.5 Jacobian construction (matrix-based linear solvers)

If a matrix-based linear solver module is used (i.e. a non-NULL SUNMATRIX object was supplied to IDASSetLinearSolver), the user may provide a function of type IDALSJacFn defined as follows:

IDALSJacFn

**Definition**
typedef int (*IDALSJacFn)(realtype tt, realtype cj,
N_Vector yy, N_Vector yp, N_Vector rr,
SUNMatrix Jac, void *user_data,
N_Vector tmp1, N_Vector tmp2, N_Vector tmp3);

**Purpose**
This function computes the Jacobian matrix \( J \) of the DAE system (or an approximation to it), defined by Eq. (2.6).

**Arguments**
tt is the current value of the independent variable \( t \).
cj is the scalar in the system Jacobian, proportional to the inverse of the step size (\( \alpha \) in Eq. (2.6)).
yy is the current value of the dependent variable vector, \( y(t) \).
yp is the current value of \( \dot{y}(t) \).
rr is the current value of the residual vector \( F(t, y, \dot{y}) \).
Jac is the output (approximate) Jacobian matrix (of type SUNMatrix), \( J = \partial F/\partial y + c_j \partial F/\partial \dot{y} \).
user_data is a pointer to user data, the same as the user_data parameter passed to IDASSetUserData.
tmp1
tmp2
tmp3 are pointers to memory allocated for variables of type N_Vector which can be used by IDALSJacFn function as temporary storage or work space.

**Return value**
An IDALSJacFn should return 0 if successful, a positive value if a recoverable error occurred, or a negative value if a nonrecoverable error occurred.

In the case of a recoverable error return, the integrator will attempt to recover by reducing the stepsize, and hence changing \( \alpha \) in (2.6).

**Notes**
Information regarding the structure of the specific SUNMATRIX structure (e.g., number of rows, upper/lower bandwidth, sparsity type) may be obtained through using the implementation-specific SUNMATRIX interface functions (see Chapter 9 for details).
4.6 User-supplied functions

With direct linear solvers (i.e., linear solvers with type `SUNDIALS_SOLVER_DIRECT`), the Jacobian matrix \( J(t, y) \) is zeroed out prior to calling the user-supplied Jacobian function so only nonzero elements need to be loaded into \( \text{Jac} \).

If the user’s `IDALSJacFn` function uses difference quotient approximations, it may need to access quantities not in the call list. These quantities may include the current stepsizes, the error weights, etc. To obtain these, the user will need to add a pointer to `ida_mem` to `user_data` and then use the `IDAGet*` functions described in §4.5.10.2. The unit roundoff can be accessed as `UNIT_ROUNDOFF` defined in `sundials_types.h`.

dense:
A user-supplied dense Jacobian function must load the \( \text{Neq} \times \text{Neq} \) dense matrix \( \text{Jac} \) with an approximation to the Jacobian matrix \( J(t, y, \dot{y}) \) at the point \((tt, yy, yp)\). The accessor macros `SM_ELEMENT_D` and `SM_COLUMN_D` allow the user to read and write dense matrix elements without making explicit references to the underlying representation of the `sundials` type. `SM_ELEMENT_D(J, i, j)` references the \((i, j)\)-th element of the dense matrix \( \text{Jac} \) (with \( i, j = 0 \ldots N - 1 \)). This macro is meant for small problems for which efficiency of access is not a major concern. Thus, in terms of the indices \( m \) and \( n \) ranging from 1 to \( N \), the Jacobian element \( J_{m,n} \) can be set using the statement `SM_ELEMENT_D(J, m-1, n-1) = J_{m,n}`. Alternatively, `SM_COLUMN_D(J, j)` returns a pointer to the first element of the \( j \)-th column of \( \text{Jac} \) (with \( j = 0 \ldots N - 1 \)), and the elements of the \( j \)-th column can then be accessed using ordinary array indexing. Consequently, \( J_{m,n} \) can be loaded using the statements `col_n = SM_COLUMN_D(J, n-1); col_n[m-1] = J_{m,n}`. For large problems, it is more efficient to use `SM_COLUMN_D` than to use `SM_ELEMENT_D`. Note that both of these macros number rows and columns starting from 0. The `sundials` type and accessor macros are documented in §9.3.

banded:
A user-supplied banded Jacobian function must load the \( \text{Neq} \times \text{Neq} \) banded matrix \( \text{Jac} \) with an approximation to the Jacobian matrix \( J(t, y, \dot{y}) \) at the point \((tt, yy, yp)\). The accessor macros `SM_ELEMENT_B`, `SM_COLUMN_B`, and `SM_COLUMN_ELEMENT_B` allow the user to read and write banded matrix elements without making specific references to the underlying representation of the `sundials` type. `SM_ELEMENT_B(J, i, j)` references the \((i, j)\)-th element of the banded matrix \( \text{Jac} \), counting from 0. This macro is meant for use in small problems for which efficiency of access is not a major concern. Thus, in terms of the indices \( m \) and \( n \) ranging from 1 to \( N \) with \((m, n)\) within the band defined by `mupper` and `mlower`, the Jacobian element \( J_{m,n} \) can be loaded using the statement `SM_ELEMENT_B(J, m-1, n-1) = J_{m,n}`. The elements within the band are those with \(-\text{mupper} \leq m-n \leq \text{mlower}\). Alternatively, `SM_COLUMN_B(J, j)` returns a pointer to the diagonal element of the \( j \)-th column of \( \text{Jac} \), and if we assign this address to `realtype *col_j`, then the \( i \)-th element of the \( j \)-th column is given by `SM_COLUMN_ELEMENT_B(col_j, i, j)`, counting from 0. Thus, for \((m, n)\) within the band, \( J_{m,n} \) can be loaded by setting `col_n = SM_COLUMN_B(J, n-1);` and `SM_COLUMN_ELEMENT_B(col_n, m-1, n-1) = J_{m,n}`. The elements of the \( j \)-th column can also be accessed via ordinary array indexing, but this approach requires knowledge of the underlying storage for a banded matrix of type `sundials`. The array `col_n` can be indexed from `mupper` to `mlower`. For large problems, it is more efficient to use `SM_COLUMN_B` and `SM_COLUMN_ELEMENT_B` than to use the `SM_ELEMENT_B` macro. As in the dense case, these macros all number rows and columns starting from 0. The `sundials` type and accessor macros are documented in §9.4.

sparse:
A user-supplied sparse Jacobian function must load the \( \text{Neq} \times \text{Neq} \) compressed-sparse-column or compressed-sparse-row matrix \( \text{Jac} \) with an approximation to the Jacobian matrix \( J(t, y, \dot{y}) \) at the point \((tt, yy, yp)\). Storage for \( \text{Jac} \) already exists on entry to this function, although the user should ensure that sufficient space is allocated in \( \text{Jac} \) to hold the nonzero values to be set; if the existing space is insufficient the user may
reallocate the data and index arrays as needed. The amount of allocated space in a
SUNMATRIX\_SPARSE object may be accessed using the macro \texttt{SM\_NNZ\_S} or the routine
\texttt{SUNSparseMatrix\_NNZ}. The SUNMATRIX\_SPARSE type and accessor macros are docu-
mented in §9.5.

The previous function type \texttt{IDADlsJacFn} is identical to \texttt{IDALsJacFn}, and may still be
used for backward-compatibility. However, this will be deprecated in future releases, so
we recommend that users transition to the new function type name soon.

4.6.6 Jacobian-vector product (matrix-free linear solvers)

If a matrix-free linear solver is to be used (i.e., a NULL-valued SUNMATRIX was supplied to
\texttt{IDASetLinearSolver}), the user may provide a function of type \texttt{IDALsJacTimesVecFn} in the following
form, to compute matrix-vector products $Jv$. If such a function is not supplied, the default is a
difference quotient approximation to these products.

\begin{verbatim}
typedef int (*IDALsJacTimesVecFn)(realtype tt, N_Vector yy,
                                  N_Vector yp, N_Vector rr,
                                  N_Vector v, N_Vector Jv,
                                  realtype cj, void *user_data,
                                  N_Vector tmp1, N_Vector tmp2);
\end{verbatim}

\textbf{Definition} This function computes the product $Jv$ of the DAE system Jacobian $J$ (or an approxi-
mation to it) and a given vector $v$, where $J$ is defined by Eq. (2.6).

\begin{itemize}
  \item \textbf{Arguments} \texttt{tt} is the current value of the independent variable.
  \item \texttt{yy} is the current value of the dependent variable vector, $y(t)$.
  \item \texttt{yp} is the current value of $\dot{y}(t)$.
  \item \texttt{rr} is the current value of the residual vector $F(t,y,\dot{y})$.
  \item \texttt{v} is the vector by which the Jacobian must be multiplied to the right.
  \item \texttt{Jv} is the computed output vector.
  \item \texttt{cj} is the scalar in the system Jacobian, proportional to the inverse of the step
        size ($\alpha$ in Eq. (2.6)).
  \item \texttt{user_data} is a pointer to user data, the same as the \texttt{user_data} parameter passed to
        \texttt{IDASetUserData}.
  \item \texttt{tmp1} and \texttt{tmp2} are pointers to memory allocated for variables of type \texttt{N\_Vector}
        which can be used by \texttt{IDALsJacTimesVecFn} as temporary storage or work space.
\end{itemize}

\textbf{Return value} The value returned by the Jacobian-times-vector function should be 0 if successful. A
nonzero value indicates that a nonrecoverable error occurred.

\begin{itemize}
  \item \textbf{Notes} This function must return a value of $J \cdot v$ that uses the \textit{current} value of $J$, i.e. as
evaluated at the current $(t,y,\dot{y})$.
  \item If the user’s \texttt{IDALsJacTimesVecFn} function uses difference quotient approximations, it
        may need to access quantities not in the call list. These include the current stepsize, the
        error weights, etc. To obtain these, the user will need to add a pointer to \texttt{ida\_mem} to
        \texttt{user_data} and then use the \texttt{IDAGet\_*} functions described in §4.5.10.2. The unit roundoff
        can be accessed as \texttt{UNIT\_ROUNDOFF} defined in \texttt{sundials\_types.h}.
\end{itemize}

The previous function type \texttt{IDASpilsJacTimesVecFn} is identical to
\texttt{IDALsJacTimesVecFn}, and may still be used for backward-compatibility. However, this
will be deprecated in future releases, so we recommend that users transition to the new
function type name soon.
4.6.7 Jacobian-vector product setup (matrix-free linear solvers)

If the user’s Jacobian-times-vector requires that any Jacobian-related data be preprocessed or evaluated, then this needs to be done in a user-supplied function of type IDALSJacTimesSetupFn, defined as follows:

**IDAJacTimesSetupFn**

**Definition**

typedef int (*IDAJacTimesSetupFn)(realtype tt, N_Vector yy, N_Vector yp, N_Vector rr, realtype cj, void *user_data);

**Purpose**

This function preprocesses and/or evaluates Jacobian data needed by the Jacobian-times-vector routine.

**Arguments**

- `tt` is the current value of the independent variable.
- `yy` is the current value of the dependent variable vector, \( y(t) \).
- `yp` is the current value of \( \dot{y}(t) \).
- `rr` is the current value of the residual vector \( F(t, y, \dot{y}) \).
- `cj` is the scalar in the system Jacobian, proportional to the inverse of the step size (\( \alpha \) in Eq. (2.6)).
- `user_data` is a pointer to user data, the same as the `user_data` parameter passed to IDASetUserData.

**Return value**

The value returned by the Jacobian-vector setup function should be 0 if successful, positive for a recoverable error (in which case the step will be retried), or negative for an unrecoverable error (in which case the integration is halted).

**Notes**

Each call to the Jacobian-vector setup function is preceded by a call to the IDAResFn user function with the same \((t, y, yp)\) arguments. Thus, the setup function can use any auxiliary data that is computed and saved during the evaluation of the DAE residual.

If the user’s IDALSJacTimesVecFn function uses difference quotient approximations, it may need to access quantities not in the call list. These include the current stepsize, the error weights, etc. To obtain these, the user will need to add a pointer to `ida_mem` to `user_data` and then use the IDAGet* functions described in §4.5.10.2. The unit roundoff can be accessed as `UNIT_ROUNDOFF` defined in sundials_types.h.

The previous function type IDASpilsJacTimesSetupFn is identical to IDALSJacTimesSetupFn, and may still be used for backward-compatibility. However, this will be deprecated in future releases, so we recommend that users transition to the new function type name soon.

4.6.8 Preconditioner solve (iterative linear solvers)

If a user-supplied preconditioner is to be used with a SUNLINSOL solver module, then the user must provide a function to solve the linear system \( Pz = r \) where \( P \) is a left preconditioner matrix which approximates (at least crudely) the Jacobian matrix \( J = \partial F/\partial y + cj \partial F/\partial \dot{y} \). This function must be of type IDALSPrecSolveFn, defined as follows:

**IDALSPrecSolveFn**

**Definition**

typedef int (*IDALSPrecSolveFn)(realtype tt, N_Vector yy, N_Vector yp, N_Vector rr, N_Vector rvec, N_Vector zvec, realtype cj, realtype delta, void *user_data);

**Purpose**

This function solves the preconditioning system \( Pz = r \).
Arguments

- **tt** is the current value of the independent variable.
- **yy** is the current value of the dependent variable vector, \(y(t)\).
- **yp** is the current value of \(\dot{y}(t)\).
- **rr** is the current value of the residual vector \(F(t, y, \dot{y})\).
- **rvec** is the right-hand side vector \(r\) of the linear system to be solved.
- **zvec** is the computed output vector.
- **cj** is the scalar in the system Jacobian, proportional to the inverse of the step size (\(\alpha\) in Eq. (2.6)).
- **delta** is an input tolerance to be used if an iterative method is employed in the solution. In that case, the residual vector \(\text{Res} = r - Pz\) of the system should be made less than \(\delta\) in weighted \(\ell_2\) norm, i.e., \(\sqrt{\sum_i (\text{Res}_i \cdot \text{ewt}_i)^2} < \delta\). To obtain the \(\text{N}\_\text{Vector ewt}\), call \(\text{IDAGetErrWeights}\) (see §4.5.10.2).
- **user_data** is a pointer to user data, the same as the user_data parameter passed to the function \(\text{IDASetUserData}\).

Return value

The value to be returned by the preconditioner solve function is a flag indicating whether it was successful. This value should be 0 if successful, positive for a recoverable error (in which case the step will be retried), negative for an unrecoverable error (in which case the integration is halted).

Notes

- The previous function type \(\text{IDASpilsPrecSolveFn}\) is identical to \(\text{IDALsPrecSolveFn}\), and may still be used for backward-compatibility. However, this will be deprecated in future releases, so we recommend that users transition to the new function type name soon.

### 4.6.9 Preconditioner setup (iterative linear solvers)

If the user’s preconditioner requires that any Jacobian-related data be evaluated or preprocessed, then this needs to be done in a user-supplied function of type \(\text{IDALsPrecSetupFn}\), defined as follows:

```c
typedef int (*IDALsPrecSetupFn)(realtype tt, N_Vector yy, N_Vector yp, N_Vector rr, realtype cj, void *user_data);
```

**Definition**

- **tt** is the current value of the independent variable.
- **yy** is the current value of the dependent variable vector, \(y(t)\).
- **yp** is the current value of \(\dot{y}(t)\).
- **rr** is the current value of the residual vector \(F(t, y, \dot{y})\).
- **cj** is the scalar in the system Jacobian, proportional to the inverse of the step size (\(\alpha\) in Eq. (2.6)).
- **user_data** is a pointer to user data, the same as the user_data parameter passed to the function \(\text{IDASetUserData}\).

**Return value**

The value returned by the preconditioner setup function is a flag indicating whether it was successful. This value should be 0 if successful, positive for a recoverable error (in which case the step will be retried), negative for an unrecoverable error (in which case the integration is halted).

**Notes**

- The operations performed by this function might include forming a crude approximate Jacobian, and performing an LU factorization on the resulting approximation.
- Each call to the preconditioner setup function is preceded by a call to the \(\text{IDAResFn}\) user function with the same (tt, yy, yp) arguments. Thus the preconditioner setup
4.7 Integration of pure quadrature equations

IDAS allows the DAE system to include pure quadratures. In this case, it is more efficient to treat the quadratures separately by excluding them from the nonlinear solution stage. To do this, begin by excluding the quadrature variables from the vectors y and yp and the quadrature equations from within res. Thus a separate vector yQ of quadrature variables is to satisfy \( (d/dt)y_Q = f_Q(t, y, \dot{y}) \). The following is an overview of the sequence of calls in a user’s main program in this situation. Steps that are unchanged from the skeleton program presented in §4.4 are grayed out.

1. Initialize parallel or multi-threaded environment, if appropriate
2. Set problem dimensions, etc.
   This generally includes \( N \), the problem size \( N \) (excluding quadrature variables), \( N_q \), the number of quadrature variables, and may include the local vector length \( N_{\text{local}} \) (excluding quadrature variables), and local number of quadrature variables \( N_{q\text{local}} \).
3. Set vectors of initial values
4. Create IDAS object
5. Initialize IDAS solver
6. Specify integration tolerances
7. Create matrix object
8. Create linear solver object
9. Set linear solver optional inputs
10. Attach linear solver module
11. Set optional inputs
12. Create nonlinear solver object
13. Attach nonlinear solver module
14. Set nonlinear solver optional inputs
15. Correct initial values
16. **Set vector of initial values for quadrature variables**
   Typically, the quadrature variables should be initialized to 0.

17. **Initialize quadrature integration**
   Call `IDAQuadInit` to specify the quadrature equation right-hand side function and to allocate internal memory related to quadrature integration. See §4.7.1 for details.

18. **Set optional inputs for quadrature integration**
   Call `IDASetQuadErrCon` to indicate whether or not quadrature variables should be used in the step size control mechanism. If so, one of the `IDAQuad*tolerances` functions must be called to specify the integration tolerances for quadrature variables. See §4.7.4 for details.

19. **Advance solution in time**

20. **Extract quadrature variables**
   Call `IDAGetQuad` or `IDAGetQuadDky` to obtain the values of the quadrature variables or their derivatives at the current time. See §4.7.3 for details.

21. **Get optional outputs**

22. **Get quadrature optional outputs**
   Call `IDAGetQuad*` functions to obtain optional output related to the integration of quadratures. See §4.7.5 for details.

23. **Deallocation memory for solution vectors and for the vector of quadrature variables**

24. **Free solver memory**

25. **Free nonlinear solver memory**

26. **Free linear solver and matrix memory**

27. **Finalize MPI, if used**
   `IDAQuadInit` can be called and quadrature-related optional inputs (step 18 above) can be set, anywhere between steps 4 and 19.

### 4.7.1 Quadrature initialization and deallocation functions

The function `IDAQuadInit` activates integration of quadrature equations and allocates internal memory related to these calculations. The form of the call to this function is as follows:

```c
Call flag = IDAQuadInit(ida_mem, rhsQ, yQ0);
```

**Description**
The function `IDAQuadInit` provides required problem specifications, allocates internal memory, and initializes quadrature integration.

**Arguments**
- `ida_mem` (void *) pointer to the IDAS memory block returned by `IDACreate`.
- `rhsQ` (IDAQuadRhsFn) is the C function which computes \( f_Q \), the right-hand side of the quadrature equations. This function has the form \( f_Q(t, yy, yp, rhsQ, user_data) \) (for full details see §4.7.6).
- `yQ0` (N_Vector) is the initial value of \( y_Q \).

**Return value**
The return value `flag` (of type `int`) will be one of the following:
- `IDA_SUCCESS` The call to `IDAQuadInit` was successful.
4.7 Integration of pure quadrature equations

IDA_MEM_NULL The IDAS memory was not initialized by a prior call to IDACreate.
IDA_MEM_FAIL A memory allocation request failed.

Notes If an error occurred, IDAQuadInit also sends an error message to the error handler function.

F2003 Name FIDAQuadInit

In terms of the number of quadrature variables \( N_q \) and maximum method order \( \text{maxord} \), the size of the real workspace is increased as follows:

- Base value: \( \text{lenrw} = \text{lenrw} + (\text{maxord}+5)N_q \)
- If IDAQuadSVtolerances is called: \( \text{lenrw} = \text{lenrw} + N_q \)

and the size of the integer workspace is increased as follows:

- Base value: \( \text{leniw} = \text{leniw} + (\text{maxord}+5)N_q \)
- If IDAQuadSVtolerances is called: \( \text{leniw} = \text{leniw} + N_q \)

The function IDAQuadReInit, useful during the solution of a sequence of problems of same size, reinitializes the quadrature-related internal memory and must follow a call to IDAQuadInit (and maybe a call to IDAReInit). The number \( N_q \) of quadratures is assumed to be unchanged from the prior call to IDAQuadInit. The call to the IDAQuadReInit function has the following form:

```
IDAQuadReInit
Call flag = IDAQuadReInit(ida_mem, yQ0);
```

Description The function IDAQuadReInit provides required problem specifications and reinitializes the quadrature integration.

Arguments
- \( \text{idamem} \) (void *) pointer to the IDAS memory block.
- \( \text{yQ0} \) (N_Vector) is the initial value of \( y_Q \).

Return value The return value \( \text{flag} \) (of type \text{int}) will be one of the following:

- IDA_SUCCESS The call to IDAReInit was successful.
- IDA_MEM_NULL The IDAS memory was not initialized by a prior call to IDACreate.
- IDA_NO_QUAD Memory space for the quadrature integration was not allocated by a prior call to IDAQuadInit.

Notes If an error occurred, IDAQuadReInit also sends an error message to the error handler function.

F2003 Name FIDAQuadReInit

```
IDAQuadFree
Call IDAQuadFree(ida_mem);
```

Description The function IDAQuadFree frees the memory allocated for quadrature integration.

Arguments The argument is the pointer to the IDAS memory block (of type void *).

Return value The function IDAQuadFree has no return value.

Notes In general, IDAQuadFree need not be called by the user as it is invoked automatically by IDAFree.

F2003 Name FIDAQuadFree
4.7.2 IDAS solver function

Even if quadrature integration was enabled, the call to the main solver function `IDASolve` is exactly the same as in §4.5.7. However, in this case the return value `flag` can also be one of the following:

- `IDA_QRHS_FAIL` The quadrature right-hand side function failed in an unrecoverable manner.
- `IDA_FIRST_QRHS_ERR` The quadrature right-hand side function failed at the first call.
- `IDA_REP_QRHS_ERR` Convergence test failures occurred too many times due to repeated recoverable errors in the quadrature right-hand side function. This value will also be returned if the quadrature right-hand side function had repeated recoverable errors during the estimation of an initial step size (assuming the quadrature variables are included in the error tests).

4.7.3 Quadrature extraction functions

If quadrature integration has been initialized by a call to `IDAQuadInit`, or reinitialized by a call to `IDAQuadReInit`, then IDAS computes both a solution and quadratures at time \( t \). However, `IDASolve` will still return only the solution \( y \) in \( y \). Solution quadratures can be obtained using the following function:

```c
flag = IDAGetQuad(ida_mem, &tret, yQ);
```

**Description**
The function `IDAGetQuad` returns the quadrature solution vector after a successful return from `IDASolve`.

**Arguments**
- `ida_mem` (`void *`) pointer to the memory previously allocated by `IDAInit`.
- `tret` (`realtype`) the time reached by the solver (output).
- `yQ` (`N_Vector`) the computed quadrature vector.

**Return value**
The return value `flag` of `IDAGetQuad` is one of:
- `IDA_SUCCES` `IDAGetQuad` was successful.
- `IDA_MEM_NULL` `ida_mem` was NULL.
- `IDA_NO_QUAD` Quadrature integration was not initialized.
- `IDA_BAD_DKY` \( yQ \) is NULL.

**F2003 Name** `FIDAGetQuad`

The function `IDAGetQuadDky` computes the \( k \)-th derivatives of the interpolating polynomials for the quadrature variables at time \( t \). This function is called by `IDAGetQuad` with \( k = 0 \) and with the current time at which `IDASolve` has returned, but may also be called directly by the user.

```c
flag = IDAGetQuadDky(ida_mem, t, k, dkyQ);
```

**Description**
The function `IDAGetQuadDky` returns derivatives of the quadrature solution vector after a successful return from `IDASolve`.

**Arguments**
- `ida_mem` (`void *`) pointer to the memory previously allocated by `IDAInit`.
- `t` (`realtype`) the time at which quadrature information is requested. The time \( t \) must fall within the interval defined by the last successful step taken by IDAS.
- `k` (`int`) order of the requested derivative. This must be \( \leq k_{last} \).
- `dkyQ` (`N_Vector`) the vector containing the derivative. This vector must be allocated by the user.

**Return value**
The return value `flag` of `IDAGetQuadDky` is one of:
- `IDA_SUCCESS` `IDAGetQuadDky` succeeded.
4.7.4 Optional inputs for quadrature integration

**F2003 Name** FIDASetQuadErrCon

**IDA** provides the following optional input functions to control the integration of quadrature equations.

**IDASetQuadErrCon**

**Call**

\[ \text{flag} = \text{IDASetQuadErrCon}(\text{idamem}, \text{errconQ}); \]

**Description**

The function **IDASetQuadErrCon** specifies whether or not the quadrature variables are to be used in the step size control mechanism within **IDA**. If they are, the user must call either **IDAQuadSS tolerances** or **IDAQuadSV tolerances** to specify the integration tolerances for the quadrature variables.

**Arguments**

- **ida_mem** (void *) pointer to the **IDA** memory block.
- **errconQ** (boolean type) specifies whether quadrature variables are included (**SUNTRUE**) or not (**SUNFALSE**) in the error control mechanism.

**Return value**

The return value **flag** (of type **int**) is one of:

- **IDA_SUCCESS** The optional value has been successfully set.
- **IDA_MEM_NULL** The **ida_mem** pointer is **NULL**.
- **IDA_NO QUAD** Quadrature integration was not initialized.

**Notes**

By default, **errconQ** is set to **SUNFALSE**. It is illegal to call **IDASetQuadErrCon** before a call to **IDAQuadInit**.

**F2003 Name** FIDASetQuadErrCon

If the quadrature variables are part of the step size control mechanism, one of the following functions must be called to specify the integration tolerances for quadrature variables.

**IDAQuadSS tolerances**

**Call**

\[ \text{flag} = \text{IDAQuadSV tolerances}(\text{idamem}, \text{reltolQ}, \text{abstolQ}); \]

**Description**

The function **IDAQuadSS tolerances** specifies scalar relative and absolute tolerances.

**Arguments**

- **ida_mem** (void *) pointer to the **IDA** memory block.
- **reltolQ** (real type) is the scalar relative error tolerance.
- **abstolQ** (real type) is the scalar absolute error tolerance.

**Return value**

The return value **flag** (of type **int**) is one of:

- **IDA_SUCCESS** The optional value has been successfully set.
- **IDA_NO QUAD** Quadrature integration was not initialized.
- **IDA_MEM NULL** The **ida_mem** pointer is **NULL**.
- **IDA_Ill_INPUT** One of the input tolerances was negative.

**F2003 Name** FIDAQuadSS tolerances
IDAQuadSVtolerances

Call
flag = IDAQuadSVtolerances(ida_mem, reltolQ, abstolQ);

Description
The function IDAQuadSVtolerances specifies scalar relative and vector absolute tolerances.

Arguments
ida_mem (void*) pointer to the IDAS memory block.
reltolQ (realtype) is the scalar relative error tolerance.
abstolQ (N_Vector) is the vector absolute error tolerance.

Return value
The return value flag (of type int) is one of:
IDA_SUCCESS The optional value has been successfully set.
IDA_NO_QUAD Quadrature integration was not initialized.
IDA_MEM_NULL The ida_mem pointer is NULL.
IDA_ILL_INPUT One of the input tolerances was negative.

F2003 Name FIDAQuadSVtolerances

4.7.5 Optional outputs for quadrature integration

IDAS provides the following functions that can be used to obtain solver performance information related to quadrature integration.

IDAGetQuadNumRhsEvals

Call
flag = IDAGetQuadNumRhsEvals(ida_mem, &nrhsQevals);

Description
The function IDAGetQuadNumRhsEvals returns the number of calls made to the user’s quadrature right-hand side function.

Arguments
ida_mem (void*) pointer to the IDAS memory block.
nrhsQevals (long int) number of calls made to the user’s rhsQ function.

Return value
The return value flag (of type int) is one of:
IDA_SUCCESS The optional output value has been successfully set.
IDA_MEM_NULL The ida_mem pointer is NULL.
IDA_NO_QUAD Quadrature integration has not been initialized.

F2003 Name FIDAGetQuadNumRhsEvals

IDAGetQuadNumErrTestFails

Call
flag = IDAGetQuadNumErrTestFails(ida_mem, &nQetfails);

Description
The function IDAGetQuadNumErrTestFails returns the number of local error test failures due to quadrature variables.

Arguments
ida_mem (void*) pointer to the IDAS memory block.
nQetfails (long int) number of error test failures due to quadrature variables.

Return value
The return value flag (of type int) is one of:
IDA_SUCCESS The optional output value has been successfully set.
IDA_MEM_NULL The ida_mem pointer is NULL.
IDA_NO_QUAD Quadrature integration has not been initialized.

F2003 Name FIDAGetQuadNumErrTestFails
4.7 Integration of pure quadrature equations

**IDAGetQuadErrWeights**

**Call**

```c
flag = IDAGetQuadErrWeights(ida_mem, eQweight);
```

**Description**
The function `IDAGetQuadErrWeights` returns the quadrature error weights at the current time.

**Arguments**
- `ida_mem` (void *) pointer to the IDAS memory block.
- `eQweight` (N_Vector) quadrature error weights at the current time.

**Return value**
The return value `flag` (of type int) is one of:
- `IDA_SUCCESS` The optional output value has been successfully set.
- `IDA_MEM_NULL` The `ida_mem` pointer is `NULL`.
- `IDA_NO_QUAD` Quadrature integration has not been initialized.

**Notes**
The user must allocate memory for `eQweight`.

If quadratures were not included in the error control mechanism (through a call to `IDASetQuadErrCon` with `errconQ = SUNTRUE`), `IDAGetQuadErrWeights` does not set the `eQweight` vector.

**F2003 Name**
FIDAGetQuadErrWeights

**IDAGetQuadStats**

**Call**

```c
flag = IDAGetQuadStats(ida_mem, &nrhsQevals, &nQetfails);
```

**Description**
The function `IDAGetQuadStats` returns the IDAS integrator statistics as a group.

**Arguments**
- `ida_mem` (void *) pointer to the IDAS memory block.
- `nrhsQevals` (long int) number of calls to the user's `rhsQ` function.
- `nQetfails` (long int) number of error test failures due to quadrature variables.

**Return value**
The return value `flag` (of type int) is one of
- `IDA_SUCCESS` the optional output values have been successfully set.
- `IDA_MEM_NULL` the `ida_mem` pointer is `NULL`.
- `IDA_NO_QUAD` Quadrature integration has not been initialized.

**F2003 Name**
FIDAGetQuadStats

### 4.7.6 User-supplied function for quadrature integration

For integration of quadrature equations, the user must provide a function that defines the right-hand side of the quadrature equations (in other words, the integrand function of the integral that must be evaluated). This function must be of type `IDAQuadRhsFn` defined as follows:

**Definition**

```c
typedef int (*IDAQuadRhsFn)(realtype t, N_Vector yy, N_Vector yp,
                          N_Vector rhsQ, void *user_data);
```

**Purpose**
This function computes the quadrature equation right-hand side for a given value of the independent variable \( t \) and state vectors \( y \) and \( \dot{y} \).

**Arguments**
- `t` is the current value of the independent variable.
- `yy` is the current value of the dependent variable vector, \( y(t) \).
- `yp` is the current value of the dependent variable derivative vector, \( \dot{y}(t) \).
- `rhsQ` is the output vector \( f_Q(t, y, \dot{y}) \).
- `user_data` is the `user_data` pointer passed to `IDASetUserData`.

**F2003 Name**
FIDAQuadRhsFn
Return value A IDAQuadRhsFn should return 0 if successful, a positive value if a recoverable error occurred (in which case IDAS will attempt to correct), or a negative value if it failed unrecoverably (in which case the integration is halted and IDA_QRHS_FAIL is returned).

Notes Allocation of memory for rhsQ is automatically handled within IDAS.

Both y and rhsQ are of type N_Vector, but they typically have different internal representations. It is the user’s responsibility to access the vector data consistently (including the use of the correct accessor macros from each NVECTOR implementation). For the sake of computational efficiency, the vector functions in the two NVECTOR implementations provided with IDAS do not perform any consistency checks with respect to their N_Vector arguments (see §8.3 and §8.4).

There is one situation in which recovery is not possible even if IDAQuadRhsFn function returns a recoverable error flag. This is when this occurs at the very first call to the IDAQuadRhsFn (in which case IDAS returns IDA_FIRST_QRHS_ERR).

4.8 A parallel band-block-diagonal preconditioner module

A principal reason for using a parallel DAE solver such as IDAS lies in the solution of partial differential equations (PDEs). Moreover, the use of a Krylov iterative method for the solution of many such problems is motivated by the nature of the underlying linear system of equations (2.5) that must be solved at each time step. The linear algebraic system is large, sparse, and structured. However, if a Krylov iterative method is to be effective in this setting, then a nontrivial preconditioner needs to be used. Otherwise, the rate of convergence of the Krylov iterative method is usually unacceptably slow. Unfortunately, an effective preconditioner tends to be problem-specific.

However, we have developed one type of preconditioner that treats a rather broad class of PDE-based problems. It has been successfully used for several realistic, large-scale problems [35] and is included in a software module within the IDAS package. This module works with the parallel vector module NVECTOR_PARALLEL and generates a preconditioner that is a block-diagonal matrix with each block being a band matrix. The blocks need not have the same number of super- and sub-diagonals, and these numbers may vary from block to block. This Band-Block-Diagonal Preconditioner module is called IDABBDDPRE.

One way to envision these preconditioners is to think of the domain of the computational PDE problem as being subdivided into M non-overlapping sub-domains. Each of these sub-domains is then assigned to one of the M processors to be used to solve the DAE system. The basic idea is to isolate the preconditioning so that it is local to each processor, and also to use a (possibly cheaper) approximate residual function. This requires the definition of a new function \( G(t, y, \dot{y}) \) which approximates the function \( F(t, y, \dot{y}) \) in the definition of the DAE system (2.1). However, the user may set \( G = F \). Corresponding to the domain decomposition, there is a decomposition of the solution vectors \( y \) and \( \dot{y} \) into M disjoint blocks \( y_m \) and \( \dot{y}_m \), and a decomposition of \( G \) into blocks \( G_m \). The block \( G_m \) depends on \( y_m \) and \( \dot{y}_m \), and also on components of \( y_{m'} \) and \( \dot{y}_{m'} \) associated with neighboring sub-domains (so-called ghost-cell data). Let \( \bar{y}_m \) and \( \bar{\dot{y}}_m \) denote \( y_m \) and \( \dot{y}_m \) (respectively) augmented with those other components on which \( G_m \) depends. Then we have

\[
G(t, y, \dot{y}) = [G_1(t, \bar{y}_1, \bar{\dot{y}}_1), G_2(t, \bar{y}_2, \bar{\dot{y}}_2), \ldots, G_M(t, \bar{y}_M, \bar{\dot{y}}_M)]^T, \quad (4.1)
\]

and each of the blocks \( G_m(t, \bar{y}_m, \bar{\dot{y}}_m) \) is uncoupled from the others.

The preconditioner associated with this decomposition has the form

\[
P = diag[P_1, P_2, \ldots, P_M] \quad (4.2)
\]

where

\[
P_m \approx \partial G_m / \partial y_m + \alpha \partial G_m / \partial \dot{y}_m \quad (4.3)
\]

This matrix is taken to be banded, with upper and lower half-bandwidths mbdq and mldq defined as the number of non-zero diagonals above and below the main diagonal, respectively. The difference
quotient approximation is computed using \( m_u + m_l + 2 \) evaluations of \( G_m \), but only a matrix of bandwidth \( m_k + 1 \) is retained.

Neither pair of parameters need be the true half-bandwidths of the Jacobians of the local block of \( G \), if smaller values provide a more efficient preconditioner. Such an efficiency gain may occur if the couplings in the DAE system outside a certain bandwidth are considerably weaker than those within the band. Reducing \( m_k \) and \( mlkeep \) while keeping \( m_u \) and \( mlq \) at their true values, discards the elements outside the narrower band. Reducing both pairs has the additional effect of lumping the outer Jacobian elements into the computed elements within the band, and requires more caution and experimentation.

The solution of the complete linear system

\[
Px = b
\]

reduces to solving each of the equations

\[
P_m x_m = b_m
\]

and this is done by banded LU factorization of \( P_m \) followed by a banded backsolve.

Similar block-diagonal preconditioners could be considered with different treatment of the blocks \( P_m \). For example, incomplete LU factorization or an iterative method could be used instead of banded LU factorization.

The \texttt{idabbdpre} module calls two user-provided functions to construct \( P \): a required function \texttt{Gres} (of type \texttt{IDABBDLocalFn}) which approximates the residual function \( G(t, y, \dot{y}) \approx F(t, y, \dot{y}) \) and which is computed locally, and an optional function \texttt{Gcomm} (of type \texttt{IDABBDCommFn}) which performs all inter-process communication necessary to evaluate the approximate residual \( G \). These are in addition to the user-supplied residual function \texttt{res}. Both functions take as input the same pointer \texttt{user_data} as passed by the user to \texttt{IDASetUserData} and passed to the user’s function \texttt{res}. The user is responsible for providing space (presumably within \texttt{user_data}) for components of \( yy \) and \( yp \) that are communicated by \texttt{Gcomm} from the other processors, and that are then used by \texttt{Gres}, which should not do any communication.

### IDABBDLocalFn

**Definition**

\[
\text{typedef int (*IDABBDLocalFn)(sunindextype Nlocal, realtype tt, N_Vector yy, N_Vector yp, N_Vector gval, void *user_data);}
\]

**Purpose**

This \texttt{Gres} function computes \( G(t, y, \dot{y}) \). It loads the vector \texttt{gval} as a function of \texttt{tt}, \texttt{yy}, and \texttt{yp}.

**Arguments**

- \texttt{Nlocal} is the local vector length.
- \texttt{tt} is the value of the independent variable.
- \texttt{yy} is the dependent variable.
- \texttt{yp} is the derivative of the dependent variable.
- \texttt{gval} is the output vector.
- \texttt{user_data} is a pointer to user data, the same as the \texttt{user_data} parameter passed to \texttt{IDASetUserData}.

**Return value**

An \texttt{IDABBDLocalFn} function type should return 0 to indicate success, 1 for a recoverable error, or -1 for a non-recoverable error.

**Notes**

This function must assume that all inter-processor communication of data needed to calculate \texttt{gval} has already been done, and this data is accessible within \texttt{user_data}.

The case where \( G \) is mathematically identical to \( F \) is allowed.
**IDABBDCommFn**

**Definition**

typedef int (*IDABBDCommFn)(sunindextype Nlocal, realtype tt, N_Vector yy, N_Vector yp, void *user_data);

**Purpose**

This Gcomm function performs all inter-processor communications necessary for the execution of the Gres function above, using the input vectors yy and yp.

**Arguments**

- **Nlocal** is the local vector length.
- **tt** is the value of the independent variable.
- **yy** is the dependent variable.
- **yp** is the derivative of the dependent variable.
- **user_data** is a pointer to user data, the same as the user_data parameter passed to IDASetUserData.

**Return value**

An IDABBDCommFn function type should return 0 to indicate success, 1 for a recoverable error, or -1 for a non-recoverable error.

**Notes**

The Gcomm function is expected to save communicated data in space defined within the structure user_data.

Each call to the Gcomm function is preceded by a call to the residual function res with the same (tt, yy, yp) arguments. Thus Gcomm can omit any communications done by res if relevant to the evaluation of Gres. If all necessary communication was done in res, then Gcomm = NULL can be passed in the call to IDABBDPreInit (see below).

Besides the header files required for the integration of the DAE problem (see §4.3), to use the IDABBDPre module, the main program must include the header file idas_bbdpre.h which declares the needed function prototypes.

The following is a summary of the usage of this module and describes the sequence of calls in the user main program. Steps that are unchanged from the user main program presented in §4.4 are grayed-out.

1. Initialize MPI
2. Set problem dimensions etc.
3. Set vectors of initial values
4. Create IDAS object
5. Initialize IDAS solver
6. Specify integration tolerances
7. Create linear solver object
   - When creating the iterative linear solver object, specify the use of left preconditioning (PREC_LEFT) as IDAS only supports left preconditioning.
8. Set linear solver optional inputs
9. Attach linear solver module
10. Set optional inputs
    - Note that the user should not overwrite the preconditioner setup function or solve function through calls to idIDASetPreconditioner optional input function.
11. Initialize the IDABBDPre preconditioner module
    - Specify the upper and lower bandwidths mudq, mldq and mukeep, mlkeep and call
flag = IDABBDPrecInit(ida_mem, Nlocal, mudq, mldq,
mukeep, mlkeep, dq_rel_yy, Gres, Gcomm);

to allocate memory and initialize the internal preconditioner data. The last two arguments of
IDABBDPrecInit are the two user-supplied functions described above.

12. Create nonlinear solver object
13. Attach nonlinear solver module
14. Set nonlinear solver optional inputs
15. Correct initial values
16. Specify rootfinding problem
17. Advance solution in time
18. Get optional outputs
Additional optional outputs associated with IDABBDPRE are available by way of two routines
described below, IDABBDPrecGetWorkSpace and IDABBDPrecGetNumGfnEvals.

19. Deallocate memory for solution vectors
20. Free solver memory
21. Free nonlinear solver memory
22. Free linear solver memory
23. Finalize MPI

The user-callable functions that initialize (step 11 above) or re-initialize the IDABBDPRE preconditioner
module are described next.

IDABBDPrecInit[ Call flag = IDABBDPrecInit(ida_mem, Nlocal, mudq, mldq,
mukeep, mlkeep, dq_rel_yy, Gres, Gcomm);

Description The function IDABBDPrecInit initializes and allocates (internal) memory for the ID-
ABBDPRE preconditioner.

Arguments ida_mem (void *) pointer to the IDAS memory block.
Nlocal (sunindextype) local vector dimension.
mudq (sunindextype) upper half-bandwidth to be used in the difference-quotient
Jacobian approximation.
mldq (sunindextype) lower half-bandwidth to be used in the difference-quotient
Jacobian approximation.
mukeep (sunindextype) upper half-bandwidth of the retained banded approximate
Jacobian block.
mlkeep (sunindextype) lower half-bandwidth of the retained banded approximate
Jacobian block.
dq_rel_yy (realtype) the relative increment in components of y used in the difference
quotient approximations. The default is dq_rel_yy = \sqrt{\text{unit roundoff}}, which
can be specified by passing dq_rel_yy = 0.0.
Gres (IDABBDLocalFn) the C function which computes the local residual approx-
imation \(G(t, y, \dot{y})\).
Using IDAS for IVP Solution

Gcomm (IDABBDCommFn) the optional C function which performs all inter-process communication required for the computation of $G(t, y, \dot{y})$.

Return value The return value flag (of type int) is one of

- IDALS_SUCCESS: The call to IDABBDPrecInit was successful.
- IDALS_MEM_NULL: The ida_mem pointer was NULL.
- IDALS_MEM_FAIL: A memory allocation request has failed.
- IDALS_LMEM_NULL: An IDALS linear solver memory was not attached.
- IDALS_ILL_INPUT: The supplied vector implementation was not compatible with the block band preconditioner.

Notes If one of the half-bandwidths mudq or mldq to be used in the difference-quotient calculation of the approximate Jacobian is negative or exceeds the value $N_{local}-1$, it is replaced by 0 or $N_{local}-1$ accordingly.

Also, the half-bandwidths mukeep and mlkeep of the retained banded approximate Jacobian block may be even smaller, to reduce storage and computation costs further.

For all four half-bandwidths, the values need not be the same on every processor.

F2003 Name FIDABBDPrecInit

The IDABBDPre module also provides a reinitialization function to allow for a sequence of problems of the same size, with the same linear solver choice, provided there is no change in local_N, mukeep, or mlkeep. After solving one problem, and after calling IDAREInit to re-initialize IDAS for a subsequent problem, a call to IDABBDPrecReInit can be made to change any of the following: the half-bandwidths mudq and mldq used in the difference-quotient Jacobian approximations, the relative increment dq_rel_yy, or one of the user-supplied functions Gres and Gcomm. If there is a change in any of the linear solver inputs, an additional call to the “Set” routines provided by the SUNLINSOL module, and/or one or more of the corresponding IDASSet*** functions, must also be made (in the proper order).

F2003 Name FIDABBDPrecReInit

The following two optional output functions are available for use with the IDABBDPre module:

IDABBDPrecReInit

Call flag = IDABBDPrecReInit(ida_mem, mudq, mldq, dq_rel_yy);

Description The function IDABBDPrecReInit reinitializes the IDABBDPre preconditioner.

Arguments ida_mem (void *) pointer to the IDAS memory block.

mudq (sunindextype) upper half-bandwidth to be used in the difference-quotient Jacobian approximation.

mldq (sunindextype) lower half-bandwidth to be used in the difference-quotient Jacobian approximation.

dq_rel_yy (realtype) the relative increment in components of y used in the difference quotient approximations. The default is $dq_{rel\_yy} = \sqrt{\text{unit roundoff}}$, which can be specified by passing $dq_{rel\_yy} = 0.0$.

Return value The return value flag (of type int) is one of

- IDALS_SUCCESS: The call to IDABBDPrecReInit was successful.
- IDALS_MEM_NULL: The ida_mem pointer was NULL.
- IDALS_LMEM_NULL: An IDALS linear solver memory was not attached.
- IDALS_PMEM_NULL: The function IDABBDPrecInit was not previously called.

Notes If one of the half-bandwidths mudq or mldq is negative or exceeds the value $N_{local}-1$, it is replaced by 0 or $N_{local}-1$, accordingly.

F2003 Name FIDABBDPrecReInit
4.8 A parallel band-block-diagonal preconditioner module

**IDABBDPrecGetWorkSpace**

<table>
<thead>
<tr>
<th>Call</th>
<th>flag = IDABBDPrecGetWorkSpace(ida_mem, &amp;lenrwBBDP, &amp;leniwBBDP);</th>
</tr>
</thead>
<tbody>
<tr>
<td>Description</td>
<td>The function IDABBDPrecGetWorkSpace returns the local sizes of the IDABBDPRE real and integer workspaces.</td>
</tr>
<tr>
<td>Arguments</td>
<td>ida_mem (void *) pointer to the IDAS memory block.</td>
</tr>
<tr>
<td></td>
<td>lenrwBBDP (long int) local number of real values in the IDABBDPRE workspace.</td>
</tr>
<tr>
<td></td>
<td>leniwBBDP (long int) local number of integer values in the IDABBDPRE workspace.</td>
</tr>
<tr>
<td>Return value</td>
<td>The return value flag (of type int) is one of</td>
</tr>
<tr>
<td></td>
<td>IDALS_SUCCESS The optional output value has been successfully set.</td>
</tr>
<tr>
<td></td>
<td>IDALS_MEM_NULL The ida_mem pointer was NULL.</td>
</tr>
<tr>
<td></td>
<td>IDALS_PMEM_NULL The IDABBDPRE preconditioner has not been initialized.</td>
</tr>
<tr>
<td>Notes</td>
<td>The workspace requirements reported by this routine correspond only to memory allocated within the IDABBDPRE module (the banded matrix approximation, banded SUNLINSOL object, temporary vectors). These values are local to each process.</td>
</tr>
<tr>
<td></td>
<td>The workspaces referred to here exist in addition to those given by the corresponding function IDAGetLinWorkSpace.</td>
</tr>
<tr>
<td>F2003 Name</td>
<td>FIDABBDPrecGetWorkSpace</td>
</tr>
</tbody>
</table>

**IDABBDPrecGetNumGfnEvals**

<table>
<thead>
<tr>
<th>Call</th>
<th>flag = IDABBDPrecGetNumGfnEvals(ida_mem, &amp;ngevalsBBDP);</th>
</tr>
</thead>
<tbody>
<tr>
<td>Description</td>
<td>The function IDABBDPrecGetNumGfnEvals returns the cumulative number of calls to the user Gres function due to the finite difference approximation of the Jacobian blocks used within IDABBDPRE's preconditioner setup function.</td>
</tr>
<tr>
<td>Arguments</td>
<td>ida_mem (void *) pointer to the IDAS memory block.</td>
</tr>
<tr>
<td></td>
<td>ngevalsBBDP (long int) the cumulative number of calls to the user Gres function.</td>
</tr>
<tr>
<td>Return value</td>
<td>The return value flag (of type int) is one of</td>
</tr>
<tr>
<td></td>
<td>IDALS_SUCCESS The optional output value has been successfully set.</td>
</tr>
<tr>
<td></td>
<td>IDALS_MEM_NULL The ida_mem pointer was NULL.</td>
</tr>
<tr>
<td></td>
<td>IDALS_PMEM_NULL The IDABBDPRE preconditioner has not been initialized.</td>
</tr>
<tr>
<td>F2003 Name</td>
<td>FIDABBDPrecGetNumGfnEvals</td>
</tr>
</tbody>
</table>

In addition to the ngevalsBBDP Gres evaluations, the costs associated with IDABBDPRE also include nlinsetups LU factorizations, nlinsetups calls to Gcomm, npsolves banded backsolve calls, and nrevalsLS residual function evaluations, where nlinsetups is an optional IDAS output (see §4.5.10.2), and npsolves and nrevalsLS are linear solver optional outputs (see §4.5.10.5).
Chapter 5

Using IDAS for Forward Sensitivity Analysis

This chapter describes the use of IDAS to compute solution sensitivities using forward sensitivity analysis. One of our main guiding principles was to design the IDAS user interface for forward sensitivity analysis as an extension of that for IVP integration. Assuming a user main program and user-defined support routines for IVP integration have already been defined, in order to perform forward sensitivity analysis the user only has to insert a few more calls into the main program and (optionally) define an additional routine which computes the residuals for sensitivity systems (2.12). The only departure from this philosophy is due to the IDAResFn type definition (§4.6.1). Without changing the definition of this type, the only way to pass values of the problem parameters to the DAE residual function is to require the user data structure user data to contain a pointer to the array of real parameters p.

IDAS uses various constants for both input and output. These are defined as needed in this chapter, but for convenience are also listed separately in Appendix B.

We begin with a brief overview, in the form of a skeleton user program. Following that are detailed descriptions of the interface to the various user-callable routines and of the user-supplied routines that were not already described in Chapter 4.

5.1 A skeleton of the user’s main program

The following is a skeleton of the user’s main program (or calling program) as an application of IDAS. The user program is to have these steps in the order indicated, unless otherwise noted. For the sake of brevity, we defer many of the details to the later sections. As in §4.4, most steps are independent of the nvector, sunmatrix, sunlinsol, and sunnonlinsol implementations used. For the steps that are not, refer to Chapter 8, 9, 10, and 11 for the specific name of the function to be called or macro to be referenced.

Differences between the user main program in §4.4 and the one below start only at step (16). Steps that are unchanged from the skeleton program presented in §4.4 are grayed out.

First, note that no additional header files need be included for forward sensitivity analysis beyond those for IVP solution (§4.4).

1. Initialize parallel or multi-threaded environment, if appropriate

2. Set problem dimensions etc.

3. Set vectors of initial values

4. Create IDAS object

5. Initialize IDAS solver
6. Specify integration tolerances
7. Create matrix object
8. Create linear solver object
9. Set linear solver optional inputs
10. Attach linear solver module
11. Set optional inputs
12. Create nonlinear solver object
13. Attach nonlinear solver module
14. Set nonlinear solver optional inputs
15. Initialize quadrature problem, if not sensitivity-dependent

16. Define the sensitivity problem

- Number of sensitivities (required)
  Set \( N_s = N_s \), the number of parameters with respect to which sensitivities are to be computed.

- Problem parameters (optional)
  If IDAS is to evaluate the residuals of the sensitivity systems, set \( \mathbf{p} \), an array of \( N_p \) real parameters upon which the IVP depends. Only parameters with respect to which sensitivities are (potentially) desired need to be included. Attach \( \mathbf{p} \) to the user data structure \texttt{user.data}.
  For example, \texttt{user.data->p = p;}
  If the user provides a function to evaluate the sensitivity residuals, \( \mathbf{p} \) need not be specified.

- Parameter list (optional)
  If IDAS is to evaluate the sensitivity residuals, set \( \mathbf{plist} \), an array of \( N_s \) integers to specify the parameters \( \mathbf{p} \) with respect to which solution sensitivities are to be computed. If sensitivities with respect to the \( j \)-th parameter \( \mathbf{p}[j] \) \((0 \leq j < N_p)\) are desired, set \( \mathbf{plist}_i = j \), for some \( i = 0, \ldots, N_s - 1 \).
  If \( \mathbf{plist} \) is not specified, IDAS will compute sensitivities with respect to the first \( N_s \) parameters; i.e., \( \mathbf{plist}_i = i \ (i = 0, \ldots, N_s - 1) \).
  If the user provides a function to evaluate the sensitivity residuals, \( \mathbf{plist} \) need not be specified.

- Parameter scaling factors (optional)
  If IDAS is to estimate tolerances for the sensitivity solution vectors (based on tolerances for the state solution vector) or if IDAS is to evaluate the residuals of the sensitivity systems using the internal difference-quotient function, the results will be more accurate if order of magnitude information is provided.
  Set \( \mathbf{pbar} \), an array of \( N_s \) positive scaling factors. Typically, if \( p_i \neq 0 \), the value \( \bar{p}_i = |p_{\mathbf{plist}_i}| \) can be used.
  If \( \mathbf{pbar} \) is not specified, IDAS will use \( \bar{p}_i = 1.0 \).
  If the user provides a function to evaluate the sensitivity residual and specifies tolerances for the sensitivity variables, \( \mathbf{pbar} \) need not be specified.

Note that the names for \( \mathbf{p}, \mathbf{pbar}, \mathbf{plist} \), as well as the field \( p \) of \texttt{user.data} are arbitrary, but they must agree with the arguments passed to \texttt{IDASetsensParams} below.
17. Set sensitivity initial conditions

Set the \( N_s \) vectors \( y_S[0] \) and \( y_pS[0] \) of initial values for sensitivities (for \( i = 0, \ldots, N_s - 1 \)), using the appropriate functions defined by the particular NVECTOR implementation chosen.

First, create an array of \( N_s \) vectors by making the appropriate call

\[
y_S[0] = N_{VCloneVectorArray}**(Ns, y0);
\]

or

\[
y_S[0] = N_{VCloneVectorArrayEmpty}**(Ns, y0);
\]

Here the argument \( y_0 \) serves only to provide the N_Vector type for cloning.

Then, for each \( i = 0, \ldots, N_s - 1 \), load initial values for the \( i \)-th sensitivity vector \( y_S[i] \).

Set the initial conditions for the \( N_s \) sensitivity derivative vectors \( y_pS[0] \) of \( \dot{y} \) similarly.

18. Activate sensitivity calculations

Call \( \text{flag} = \text{IDASensInit}(\ldots) \); to activate forward sensitivity computations and allocate internal memory for IDAS related to sensitivity calculations (see §5.2.1).

19. Set sensitivity tolerances

Call IDASensSSStolerances, IDASensSVtolerances, or IDASensEEtolerances. See §5.2.2.

20. Set sensitivity analysis optional inputs

Call IDASensSet\* routines to change from their default values any optional inputs that control the behavior of IDAS in computing forward sensitivities. See §5.2.7.

21. Create sensitivity nonlinear solver object (optional)

If using a non-default nonlinear solver (see §5.2.3), then create the desired nonlinear solver object by calling the appropriate constructor function defined by the particular SUNNONLINSOL implementation e.g.,

\[
\text{NLSSens} = \text{SUNNonlinSol}_***Sens(\ldots);
\]

where *** is the name of the nonlinear solver and ... are constructor specific arguments (see Chapter 11 for details).

22. Attach the sensitivity nonlinear solver module (optional)

If using a non-default nonlinear solver, then initialize the nonlinear solver interface by attaching the nonlinear solver object by calling

\[
\text{ier} = \text{IDASSetNonlinearSolverSensSim(ida_mem, NLSSens)};
\]

when using the IDA_SIMULTANEOUS corrector method or

\[
\text{ier} = \text{IDASSetNonlinearSolverSensStg(ida_mem, NLSSens)};
\]

when using the IDA_STAGGERED corrector method (see §5.2.3 for details).

23. Set sensitivity nonlinear solver optional inputs (optional)

Call the appropriate set functions for the selected nonlinear solver module to change optional inputs specific to that nonlinear solver. These must be called after IDASensInit if using the default nonlinear solver or after attaching a new nonlinear solver to IDAS, otherwise the optional inputs will be overridden by IDAS defaults. See Chapter 11 for more information on optional inputs.
24. Correct initial values
25. Specify rootfinding problem
26. Advance solution in time
27. Extract sensitivity solution
   After each successful return from IDASolve, the solution of the original IVP is available in the y argument of IDASolve, while the sensitivity solution can be extracted into yS and ypS (which can be the same as yS0 and ypS0, respectively) by calling one of the following routines: IDAGetSens, IDAGetSens1, IDAGetSensDky or IDAGetSensDky1 (see §5.2.6).
28. Get optional outputs
29. Deallocate memory for solution vector
30. Deallocate memory for sensitivity vectors
   Upon completion of the integration, deallocate memory for the vectors contained in yS0 and ypS0:
   N_VDestroyVectorArray_***(yS0, Ns);
   If yS was created from realtype arrays yS_i, it is the user’s responsibility to also free the space for the arrays yS_i, and likewise for ypS.
31. Free user data structure
32. Free solver memory
33. Free nonlinear solver memory
34. Free vector specification memory
35. Free linear solver and matrix memory
36. Finalize MPI, if used

5.2 User-callable routines for forward sensitivity analysis

This section describes the IDAS functions, in addition to those presented in §4.5, that are called by the user to set up and solve a forward sensitivity problem.

5.2.1 Forward sensitivity initialization and deallocation functions

Activation of forward sensitivity computation is done by calling IDASensInit. The form of the call to this routine is as follows:

```
IDASensInit
```

Call

```
flag = IDASensInit(ida_mem, Ns, ism, resS, yS0, ypS0);
```

Description

The routine IDASensInit activates forward sensitivity computations and allocates internal memory related to sensitivity calculations.

Arguments

- `ida_mem` (void *) pointer to the IDAS memory block returned by IDACreate.
- `Ns` (int) the number of sensitivities to be computed.
- `ism` (int) a flag used to select the sensitivity solution method. Its value can be either `IDA_SIMULTANEOUS` or `IDA_STAGGERED`. 

• In the IDA_SIMULTANEOUS approach, the state and sensitivity variables are corrected at the same time. If the default Newton nonlinear solver is used, this amounts to performing a modified Newton iteration on the combined nonlinear system;

• In the IDA_STAGGERED approach, the correction step for the sensitivity variables takes place at the same time for all sensitivity equations, but only after the correction of the state variables has converged and the state variables have passed the local error test;

ress (IDASensResFn) is the C function which computes the residual of the sensitivity DAE. For full details see §5.3.

ys0 (N_Vector *) a pointer to an array of Ns vectors containing the initial values of the sensitivities of y.

ypS0 (N_Vector *) a pointer to an array of Ns vectors containing the initial values of the sensitivities of ˙y.

Return value The return value flag (of type int) will be one of the following:

IDA_SUCCESS The call to IDASensInit was successful.

IDA_MEM_NULL The IDAS memory block was not initialized through a previous call to IDACreate.

IDA_MEM_FAIL A memory allocation request has failed.

IDA_ILL_INPUT An input argument to IDASensInit has an illegal value.

Notes Passing resS=NULL indicates using the default internal difference quotient sensitivity residual routine.

If an error occurred, IDASensInit also prints an error message to the file specified by the optional input errfp.

F2003 Name FIDASensInit

In terms of the problem size N, number of sensitivity vectors Ns, and maximum method order maxord, the size of the real workspace is increased as follows:

• Base value: lenrw = lenrw + (maxord+5)N_s,N

• With IDASensSVtolerances: lenrw = lenrw +N_s,N

the size of the integer workspace is increased as follows:

• Base value: leniw = leniw + (maxord+5)N_s,N_i

• With IDASensSVtolerances: leniw = leniw +N_s,N_i

where N_i is the number of integer words in one N_Vector.

The routine IDASensReInit, useful during the solution of a sequence of problems of same size, reinitializes the sensitivity-related internal memory and must follow a call to IDASensInit (and maybe a call to IDAReInit). The number Ns of sensitivities is assumed to be unchanged since the call to IDASensInit. The call to the IDASensReInit function has the form:

```
IDASensReInit
```

Call flag = IDASensReInit(ida_mem, ism, yS0, ypS0);

Description The routine IDASensReInit reinitializes forward sensitivity computations.

Arguments ida_mem (void *) pointer to the IDAS memory block returned by IDACreate.

ism (int) a flag used to select the sensitivity solution method. Its value can be either IDA_SIMULTANEOUS or IDA_STAGGERED.

ys0 (N_Vector *) a pointer to an array of Ns variables of type N_Vector containing the initial values of the sensitivities of y.
ypS0 \((\text{N\_Vector } *)\) a pointer to an array of \(N\) variables of type \text{N\_Vector} containing the initial values of the sensitivities of \(\dot{y}\).

Return value The return value \texttt{flag} (of type \texttt{int}) will be one of the following:

- \texttt{IDA\_SUCCESS} The call to \texttt{IDAReInit} was successful.
- \texttt{IDA\_MEM\_NULL} The IDAS memory block was not initialized through a previous call to \texttt{IDACreate}.
- \texttt{IDA\_NO\_SENS} Memory space for sensitivity integration was not allocated through a previous call to \texttt{IDASensInit}.
- \texttt{IDA\_ILL\_INPUT} An input argument to \texttt{IDASensReInit} has an illegal value.
- \texttt{IDA\_MEM\_FAIL} A memory allocation request has failed.

Notes All arguments of \texttt{IDASensReInit} are the same as those of \texttt{IDASensInit}.

If an error occurred, \texttt{IDASensReInit} also prints an error message to the file specified by the optional input \texttt{errfp}.

F2003 Name \texttt{FIDASensReInit}

To deallocate all forward sensitivity-related memory (allocated in a prior call to \texttt{IDASensInit}), the user must call

\begin{verbatim}
FIDASensReInit
\end{verbatim}

5.2.2 Forward sensitivity tolerance specification functions

One of the following three functions must be called to specify the integration tolerances for sensitivities. Note that this call must be made after the call to \texttt{IDASensInit}.
5.2 User-callable routines for forward sensitivity analysis

**IDASensSSStolerances**

*Call*  
`flag = IDASensSSStolerances(ida_mem, reltolS, abstolS);`

*Description*  
The function `IDASensSSStolerances` specifies scalar relative and absolute tolerances.

*Arguments*  
- `ida_mem` (void *) pointer to the IDAS memory block returned by `IDACreate`.
- `reltolS` (realtype) is the scalar relative error tolerance.
- `abstolS` (realtype*) is a pointer to an array of length `Ns` containing the scalar absolute error tolerances.

*Return value*  
The return flag `flag` (of type `int`) will be one of the following:
- `IDA_SUCCESS` The call to `IDASStolerances` was successful.
- `IDA_MEM_NULL` The IDAS memory block was not initialized through a previous call to `IDACreate`.
- `IDA_NO_SENS` The sensitivity allocation function `IDASensInit` has not been called.
- `IDA_Ill_INPUT` One of the input tolerances was negative.

F2003 Name `FIDASensSSStolerances`

**IDASensSVtolerances**

*Call*  
`flag = IDASensSVtolerances(ida_mem, reltolS, abstolS);`

*Description*  
The function `IDASensSVtolerances` specifies scalar relative tolerance and vector absolute tolerances.

*Arguments*  
- `ida_mem` (void *) pointer to the IDAS memory block returned by `IDACreate`.
- `reltolS` (realtype) is the scalar relative error tolerance.
- `abstolS` (N_Vector*) is an array of `Ns` variables of type `N_Vector`. The `N_Vector` from `abstolS[iS]` specifies the vector tolerances for `iS-th` sensitivity.

*Return value*  
The return flag `flag` (of type `int`) will be one of the following:
- `IDA_SUCCESS` The call to `IDASVtolerances` was successful.
- `IDA_MEM_NULL` The IDAS memory block was not initialized through a previous call to `IDACreate`.
- `IDA_NO_SENS` The sensitivity allocation function `IDASensInit` has not been called.
- `IDA_Ill_INPUT` The relative error tolerance was negative or one of the absolute tolerance vectors had a negative component.

*Notes*  
This choice of tolerances is important when the absolute error tolerance needs to be different for each component of any vector `yS[i]`.

F2003 Name `FIDASensSVtolerances`

**IDASensEEtolerances**

*Call*  
`flag = IDASensEEtolerances(ida_mem);`

*Description*  
When `IDASensEEtolerances` is called, IDAS will estimate tolerances for sensitivity variables based on the tolerances supplied for states variables and the scaling factors \( \bar{p} \).

*Arguments*  
- `ida_mem` (void *) pointer to the IDAS memory block returned by `IDACreate`.

*Return value*  
The return flag `flag` (of type `int`) will be one of the following:
- `IDA_SUCCESS` The call to `IDASensEEtolerances` was successful.
- `IDA_MEM_NULL` The IDAS memory block was not initialized through a previous call to `IDACreate`.
- `IDA_NO_SENS` The sensitivity allocation function `IDASensInit` has not been called.

F2003 Name `FIDASensEEtolerances`
5.2.3 Forward sensitivity nonlinear solver interface functions

As in the pure DAE case, when computing solution sensitivities using forward sensitivity analysis IDAS uses the SUNNONLINSOL implementation of Newton’s method defined by the SUNNONLINSOL_Newton module (see §11.3) by default. To specify a different nonlinear solver in IDAS, the user’s program must create a SUNNONLINSOL object by calling the appropriate constructor routine. The user must then attach the SUNNONLINSOL object to IDAS by calling either IDASetNonlinearSolverSensSim when using the IDA_SIMULTANEOUS corrector option, or IDASetNonlinearSolver (see §4.5.4) and IDASetNonlinearSolverSensStg when using the IDA_STAGGERED corrector option, as documented below.

When changing the nonlinear solver in IDAS, IDASetNonlinearSolver must be called after IDAInit; similarly IDASetNonlinearSolverSensSim and IDASetNonlinearSolverSensStg must be called after IDASensInit. If any calls to IDASolve have been made, then IDAS will need to be reinitialized by calling IDAREinit to ensure that the nonlinear solver is initialized correctly before any subsequent calls to IDASolve.

The first argument passed to the routines IDASetNonlinearSolverSensSim and IDASetNonlinearSolverSensStg is the IDAS memory pointer returned by IDACreate and the second argument is the SUNNONLINSOL object to use for solving the nonlinear system 2.4. A call to this function attaches the nonlinear solver to the main IDAS integrator. We note that at present, the SUNNONLINSOL object must be of type SUNNONLINEARSOLVER_ROOTFIND.

```c
int IDASetNonlinearSolverSensSim(void *ida_mem, SUNNonlinearSolver NLS);
```

**Call**

flag = IDASetNonlinearSolverSensSim(ida_mem, NLS);

**Description**

The function IDASetNonlinearSolverSensSim attaches a SUNNONLINSOL object (NLS) to IDAS when using the IDA_SIMULTANEOUS approach to correct the state and sensitivity variables at the same time.

**Arguments**

- `ida_mem` (void *) pointer to the IDAS memory block.
- `NLS` (SUNNonlinearSolver) SUNNONLINSOL object to use for solving nonlinear systems.

**Return value**

The return value `flag` (of type int) is one of:

- `IDA_SUCCESS` The nonlinear solver was successfully attached.
- `IDA_MEM_NULL` The `ida_mem` pointer is NULL.
- `IDA_IILL_INPUT` The SUNNONLINSOL object is NULL, does not implement the required nonlinear solver operations, is not of the correct type, or the residual function, convergence test function, or maximum number of nonlinear iterations could not be set.

F2003 Name  FIDASetNonlinearSolverSensSim

```c
int IDASetNonlinearSolverSensStg(void *ida_mem, SUNNonlinearSolver NLS);
```

**Call**

flag = IDASetNonlinearSolverSensStg(ida_mem, NLS);

**Description**

The function IDASetNonlinearSolverSensStg attaches a SUNNONLINSOL object (NLS) to IDAS when using the IDA_STAGGERED approach to correct the sensitivity variables after the correction of the state variables.

**Arguments**

- `ida_mem` (void *) pointer to the IDAS memory block.
- `NLS` (SUNNonlinearSolver) SUNNONLINSOL object to use for solving nonlinear systems.

**Return value**

The return value `flag` (of type int) is one of:

- `IDA_SUCCESS` The nonlinear solver was successfully attached.
- `IDA_MEM_NULL` The `ida_mem` pointer is NULL.
5.2 User-callable routines for forward sensitivity analysis

IDA_Ill_INPUT The SUNNONLINSOL object is NULL, does not implement the required nonlinear solver operations, is not of the correct type, or the residual function, convergence test function, or maximum number of nonlinear iterations could not be set.

Notes This function only attaches the SUNNONLINSOL object for correcting the sensitivity variables. To attach a SUNNONLINSOL object for the state variable correction use IDASetNonlinearSolver (see §4.5.4).

F2003 Name FIDASetNonlinearSolverSensStg

5.2.4 Forward sensitivity initial condition calculation function

IDACalcIC also calculates corrected initial conditions for sensitivity variables of a DAE system. When used for initial conditions calculation of the forward sensitivities, IDACalcIC must be preceded by successful calls to IDASensInit (or IDASensReInit) and should precede the call(s) to IDASolve. For restrictions that apply for initial conditions calculation of the state variables, see §4.5.5.

Calling IDACalcIC is optional. It is only necessary when the initial conditions do not satisfy the sensitivity systems. Even if forward sensitivity analysis was enabled, the call to the initial conditions calculation function IDACalcIC is exactly the same as for state variables.

flag = IDACalcIC(ida_mem, icopt, tout1);

See §4.5.5 for a list of possible return values.

5.2.5 IDAS solver function

Even if forward sensitivity analysis was enabled, the call to the main solver function IDASolve is exactly the same as in §4.5.7. However, in this case the return value flag can also be one of the following:
IDA_SRRES_FAIL The sensitivity residual function failed in an unrecoverable manner.
IDA REP_SRRES_ERR The user’s residual function repeatedly returned a recoverable error flag, but the solver was unable to recover.

5.2.6 Forward sensitivity extraction functions

If forward sensitivity computations have been initialized by a call to IDASensInit, or reinitialized by a call to IDASensReInit, then IDAS computes both a solution and sensitivities at time t. However, IDASolve will still return only the solutions y and ẏ in yret and ypret, respectively. Solution sensitivities can be obtained through one of the following functions:

flag = IDAGetSens(ida_mem, &tret, yS);

The function IDAGetSens returns the sensitivity solution vectors after a successful return from IDASolve.

Arguments ida_mem (void *) pointer to the memory previously allocated by IDAInit.
tret (realtype) the time reached by the solver (output).
yS (N_Vector *) the array of Ns computed forward sensitivity vectors.

Return value The return value flag of IDAGetSens is one of:
IDA_SUCCESS IDAGetSens was successful.
IDA_MEM_NULL ida_mem was NULL.
IDA_NO_SENS Forward sensitivity analysis was not initialized.
IDA_BAD_DKY yS is NULL.
Using IDAS for Forward Sensitivity Analysis

Notes

Note that the argument `tret` is an output for this function. Its value will be the same as that returned at the last `IDASolve` call.

F2003 Name: FIDAGetSens

The function `IDAGetSensDky` computes the $k$-th derivatives of the interpolating polynomials for the sensitivity variables at time $t$. This function is called by `IDAGetSens` with $k = 0$, but may also be called directly by the user.

```
IDAGetSensDky
```

Call

```c
flag = IDAGetSensDky(ida_mem, t, k, dkyS);
```

Description

The function `IDAGetSensDky` returns derivatives of the sensitivity solution vectors after a successful return from `IDASolve`.

Arguments

- `ida_mem` (void *) pointer to the memory previously allocated by `IDAInit`.
- `t` (realtype) specifies the time at which sensitivity information is requested. The time $t$ must fall within the interval defined by the last successful step taken by IDAS.
- `k` (int) order of derivatives. $k$ must be in the range $0, 1, ..., klast$ where $klast$ is the method order of the last successful step.
- `dkyS` (N_Vector *) array of $N_s$ vectors containing the derivatives on output. The space for `dkyS` must be allocated by the user.

Return value

The return value `flag` of `IDAGetSensDky` is one of:

- `IDA_SUCCESS` `IDAGetSensDky` succeeded.
- `IDA_MEM_NULL` `ida_mem` was NULL.
- `IDA_NO_SENS` Forward sensitivity analysis was not initialized.
- `IDA_BAD_DKY` `dkyS` or one of the vectors $dkyS[i]$ is NULL.
- `IDA_BAD_K` $k$ is not in the range $0, 1, ..., klast$.
- `IDA_BAD_T` The time $t$ is not in the allowed range.

F2003 Name: FIDAGetSensDky

Forward sensitivity solution vectors can also be extracted separately for each parameter in turn through the functions `IDAGetSens1` and `IDAGetSensDky1`, defined as follows:

```
IDAGetSens1
```

Call

```c
flag = IDAGetSens1(ida_mem, &tret, is, yS);
```

Description

The function `IDAGetSens1` returns the $is$-th sensitivity solution vector after a successful return from `IDASolve`.

Arguments

- `ida_mem` (void *) pointer to the memory previously allocated by `IDAInit`.
- `tret` (realtype *) the time reached by the solver (output).
- `is` (int) specifies which sensitivity vector is to be returned ($0 \leq is < N_s$).
- `yS` (N_Vector) the computed forward sensitivity vector.

Return value

The return value `flag` of `IDAGetSens1` is one of:

- `IDA_SUCCESS` `IDAGetSens1` was successful.
- `IDA_MEM_NULL` `ida_mem` was NULL.
- `IDA_NO_SENS` Forward sensitivity analysis was not initialized.
- `IDA_BAD_IS` The index `is` is not in the allowed range.
- `IDA_BAD_DKY` `yS` is NULL.
- `IDA_BAD_T` The time $t$ is not in the allowed range.

Notes

Note that the argument `tret` is an output for this function. Its value will be the same as that returned at the last `IDASolve` call.

F2003 Name: FIDAGetSens1
5.2 User-callable routines for forward sensitivity analysis

**IDAGetSensDky1**

Call

```c
flag = IDAGetSensDky1(ida_mem, t, k, is, dkyS);
```

Description

The function `IDAGetSensDky1` returns the \( k \)-th derivative of the \( is \)-th sensitivity solution vector after a successful return from `IDASolve`.

Arguments

- `ida_mem` (void *) pointer to the memory previously allocated by `IDAInit`.
- \( t \) (realtype) specifies the time at which sensitivity information is requested. The time \( t \) must fall within the interval defined by the last successful step taken by `IDAS`.
- \( k \) (int) order of derivative. \( k \) must be in the range 0, 1, ..., \( klast \) where \( klast \) is the method order of the last successful step.
- \( is \) (int) specifies the sensitivity derivative vector to be returned (0 \( \leq \) \( is < Ns \)).
- `dkyS` (N_Vector) the vector containing the derivative on output. The space for `dkyS` must be allocated by the user.

Return value

The return value `flag` of `IDAGetSensDky1` is one of:

- `IDA_SUCCESS` `IDAGetQuadDky1` succeeded.
- `IDA_MEM_NULL` `ida_mem` was NULL.
- `IDA_NO_SENS` Forward sensitivity analysis was not initialized.
- `IDA_BAD_DKY` `dkyS` is NULL.
- `IDA_BAD_IS` The index `is` is not in the allowed range.
- `IDA_BAD_K` \( k \) is not in the range 0, 1, ..., \( klast \).
- `IDA_BAD_T` The time \( t \) is not in the allowed range.

5.2.7 Optional inputs for forward sensitivity analysis

Optional input variables that control the computation of sensitivities can be changed from their default values through calls to `IDASetSens*` functions. Table 5.1 lists all forward sensitivity optional input functions in `IDAS` which are described in detail in the remainder of this section.

**IDASetSensParams**

Call

```c
flag = IDASetSensParams(ida_mem, p, pbar, plist);
```

Description

The function `IDASetSensParams` specifies problem parameter information for sensitivity calculations.

Arguments

- `ida_mem` (void *) pointer to the `IDAS` memory block.
- `p` (realtype *) a pointer to the array of real problem parameters used to evaluate \( F(t, y, \dot{y}, p) \). If non-NULL, `p` must point to a field in the user’s data structure `user_data` passed to the user’s residual function. (See §5.1).
- `pbar` (realtype *) an array of \( Ns \) positive scaling factors. If non-NULL, `pbar` must have all its components > 0.0. (See §5.1).

<table>
<thead>
<tr>
<th>Optional input</th>
<th>Routine name</th>
<th>Default</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sensitivity scaling factors</td>
<td><code>IDASetSensParams</code></td>
<td><code>NULL</code></td>
</tr>
<tr>
<td>DQ approximation method</td>
<td><code>IDASetSensDQMethod</code></td>
<td><code>centered,0.0</code></td>
</tr>
<tr>
<td>Error control strategy</td>
<td><code>IDASetSensErrCon</code></td>
<td><code>SUNFALSE</code></td>
</tr>
<tr>
<td>Maximum no. of nonlinear iterations</td>
<td><code>IDASetSensMaxNonlinIters</code></td>
<td><code>4</code></td>
</tr>
</tbody>
</table>
Using IDAS for Forward Sensitivity Analysis

`plist (int *)` an array of Ns non-negative indices to specify which components of p to use in estimating the sensitivity equations. If non-NULL, `plist` must have all components ≥ 0. (See §5.1).

Return value The return value `flag` (of type `int`) is one of:

- **IDA_SUCCESS**: The optional value has been successfully set.
- **IDA_MEM_NULL**: The `ida_mem` pointer is NULL.
- **IDA_NO_SENS**: Forward sensitivity analysis was not initialized.
- **IDA_ILL_INPUT**: An argument has an illegal value.

Notes This function must be preceded by a call to `IDASensInit`.

F2003 Name `FIDASetSensParams`

**IDASSetSensDQMethod**

Call

```c
flag = IDASSetSensDQMethod(ida_mem, DQtype, DQrhomax);
```

Description The function `IDASSetSensDQMethod` specifies the difference quotient strategy in the case in which the residual of the sensitivity equations are to be computed by IDAS.

Arguments `ida_mem (void *)` pointer to the IDAS memory block.

- **DQtype (int)** specifies the difference quotient type and can be either `IDA_CENTERED` or `IDA_FORWARD`.

- **DQrhomax (realtype)** positive value of the selection parameter used in deciding switching between a simultaneous or separate approximation of the two terms in the sensitivity residual.

Return value The return value `flag` (of type `int`) is one of:

- **IDA_SUCCESS**: The optional value has been successfully set.
- **IDA_MEM_NULL**: The `ida_mem` pointer is NULL.
- **IDA_ILL_INPUT**: An argument has an illegal value.

Notes

- If `DQrhomax` = 0.0, then no switching is performed. The approximation is done simultaneously using either centered or forward finite differences, depending on the value of `DQtype`. For values of `DQrhomax` ≥ 1.0, the simultaneous approximation is used whenever the estimated finite difference perturbations for states and parameters are within a factor of `DQrhomax`, and the separate approximation is used otherwise. Note that a value `DQrhomax` < 1.0 will effectively disable switching. See §2.5 for more details.

- The default value are `DQtype` = `IDA_CENTERED` and `DQrhomax` = 0.0.

F2003 Name `FIDASetSensDQMethod`

**IDASSetSensErrCon**

Call

```c
flag = IDASSetSensErrCon(ida_mem, errconS);
```

Description The function `IDASSetSensErrCon` specifies the error control strategy for sensitivity variables.

Arguments `ida_mem (void *)` pointer to the IDAS memory block.

- **errconS (booleantype)** specifies whether sensitivity variables are included (`SUNTRUE`) or not (`SUNFALSE`) in the error control mechanism.

Return value The return value `flag` (of type `int`) is one of:

- **IDA_SUCCESS**: The optional value has been successfully set.
- **IDA_MEM_NULL**: The `ida_mem` pointer is NULL.
5.2 User-callable routines for forward sensitivity analysis

Notes
By default, errconS is set to SUNFALSE. If errconS=SUNTRUE then both state variables and sensitivity variables are included in the error tests. If errconS=SUNFALSE then the sensitivity variables are excluded from the error tests. Note that, in any event, all variables are considered in the convergence tests.

F2003 Name FIDASetsensErrCon

**IDASetsensMaxNonlinIters**

Call
flag = IDASetsensMaxNonlinIters(ida_mem, maxcorS);

Description
The function IDASetsensMaxNonlinIters specifies the maximum number of nonlinear solver iterations for sensitivity variables per step.

Arguments
ida_mem (void *) pointer to the IDAS memory block.
maxcorS (int) maximum number of nonlinear solver iterations allowed per step (> 0).

Return value
The return value flag (of type int) is one of:
- IDA_SUCCESS The optional value has been successfully set.
- IDA_MEM_NULL The ida_mem pointer is NULL.
- IDA_MEM_FAIL The ida_mem SUNNONLINSOL module is NULL.

Notes
The default value is 4.

F2003 Name FIDASetsensMaxNonlinIters

5.2.8 Optional outputs for forward sensitivity analysis

5.2.8.1 Main solver optional output functions

Optional output functions that return statistics and solver performance information related to forward sensitivity computations are listed in Table 5.2 and described in detail in the remainder of this section.

**IDAGetsensNumResEvals**

Call
flag = IDAGetsensNumResEvals(ida_mem, &nfSevals);

Description
The function IDAGetsensNumResEvals returns the number of calls to the sensitivity residual function.

Arguments
ida_mem (void *) pointer to the IDAS memory block.
nfSevals (long int) number of calls to the sensitivity residual function.

Return value
The return value flag (of type int) is one of:
- IDA_SUCCESS The optional output value has been successfully set.

Table 5.2: Forward sensitivity optional outputs

<table>
<thead>
<tr>
<th>Optional output</th>
<th>Routine name</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of calls to sensitivity residual function</td>
<td>IDAGetsensNumResEvals</td>
</tr>
<tr>
<td>No. of calls to residual function for sensitivity</td>
<td>IDAGetsensNumResEvalsSens</td>
</tr>
<tr>
<td>No. of sensitivity local error test failures</td>
<td>IDAGetsensNumErrTestFails</td>
</tr>
<tr>
<td>No. of calls to lin. solv. setup routine for sens.</td>
<td>IDAGetsensNumLinSolvSetups</td>
</tr>
<tr>
<td>Sensitivity-related statistics as a group</td>
<td>IDAGetsensStats</td>
</tr>
<tr>
<td>Error weight vector for sensitivity variables</td>
<td>IDAGetsensErrWeights</td>
</tr>
<tr>
<td>No. of sens. nonlinear solver iterations</td>
<td>IDAGetsensNumNonlinSolvIters</td>
</tr>
<tr>
<td>No. of sens. convergence failures</td>
<td>IDAGetsensNumNonlinSolvConvFails</td>
</tr>
<tr>
<td>Sens. nonlinear solver statistics as a group</td>
<td>IDAGetsensNonlinSolvStats</td>
</tr>
</tbody>
</table>
IDAGetNumResEvalsSens

Call
flag = IDAGetNumResEvalsSens(ida_mem, &nfevalsS);

Description
The function IDAGetNumResEvalsSens returns the number of calls to the user's residual function due to the internal finite difference approximation of the sensitivity residuals.

Arguments
ida_mem (void *) pointer to the IDAS memory block.
nfevalsS (long int) number of calls to the user residual function for sensitivity residuals.

Return value
The return value flag (of type int) is one of:
IDA_SUCCESS The optional output value has been successfully set.
IDA_MEM_NULL The ida_mem pointer is NULL.
IDA_NO_SENS Forward sensitivity analysis was not initialized.

Notes
This counter is incremented only if the internal finite difference approximation routines are used for the evaluation of the sensitivity residuals.

F2003 Name FIDAGetNumResEvalsSens

IDAGetSensNumErrTestFails

Call
flag = IDAGetSensNumErrTestFails(ida_mem, &nSetfails);

Description
The function IDAGetSensNumErrTestFails returns the number of local error test failures for the sensitivity variables that have occurred.

Arguments
ida_mem (void *) pointer to the IDAS memory block.
nSetfails (long int) number of error test failures.

Return value
The return value flag (of type int) is one of:
IDA_SUCCESS The optional output value has been successfully set.
IDA_MEM_NULL The ida_mem pointer is NULL.
IDA_NO_SENS Forward sensitivity analysis was not initialized.

Notes
This counter is incremented only if the sensitivity variables have been included in the error test (see IDASetSensErrCon in §5.2.7). Even in that case, this counter is not incremented if the ism=IDA_SIMULTANEOUS sensitivity solution method has been used.

F2003 Name FIDAGetSensNumErrTestFails

IDAGetSensNumLinSolvSetups

Call
flag = IDAGetSensNumLinSolvSetups(ida_mem, &nlinsetupsS);

Description
The function IDAGetSensNumLinSolvSetups returns the number of calls to the linear solver setup function due to forward sensitivity calculations.

Arguments
ida_mem (void *) pointer to the IDAS memory block.
nlinsetupsS (long int) number of calls to the linear solver setup function.

Return value
The return value flag (of type int) is one of:
IDA_SUCCESS The optional output value has been successfully set.
IDA_MEM_NULL The ida_mem pointer is NULL.
IDA_NO_SENS Forward sensitivity analysis was not initialized.
5.2 User-callable routines for forward sensitivity analysis

Notes This counter is incremented only if a nonlinear solver requiring linear solves has been used and staggered sensitivity solution method ($ism=IDA_{STAGGERED}$) was specified in the call to IDASensInit (see §5.2.1).

F2003 Name FIDAGetSensNumLinSolvSetups

IDAGetSensStats

Call flag = IDAGetSensStats(ida_mem, &nfSevals, &nfevalsS, &nSetfails, &nlinsetupsS);

Description The function IDAGetSensStats returns all of the above sensitivity-related solver statistics as a group.

Arguments ida_mem (void *) pointer to the IDAS memory block.
    nfSevals (long int) number of calls to the sensitivity residual function.
    nfevalsS (long int) number of calls to the user-supplied residual function.
    nSetfails (long int) number of error test failures.
    nlinsetupsS (long int) number of calls to the linear solver setup function.

Return value The return value flag (of type int) is one of:
    IDA_SUCCESS The optional output values have been successfully set.
    IDA_MEM_NULL The ida_mem pointer is NULL.
    IDA_NO_SENS Forward sensitivity analysis was not initialized.

F2003 Name FIDAGetSensStats

IDAGetSensErrWeights

Call flag = IDAGetSensErrWeights(ida_mem, eSweight);

Description The function IDAGetSensErrWeights returns the sensitivity error weight vectors at the current time. These are the reciprocals of the $W_i$ of (2.7) for the sensitivity variables.

Arguments ida_mem (void *) pointer to the IDAS memory block.
    eSweight (N_Vector_S) pointer to the array of error weight vectors.

Return value The return value flag (of type int) is one of:
    IDA_SUCCESS The optional output value has been successfully set.
    IDA_MEM_NULL The ida_mem pointer is NULL.
    IDA_NO_SENS Forward sensitivity analysis was not initialized.

Notes The user must allocate memory for eweightS.

F2003 Name FIDAGetSensErrWeights

IDAGetSensNumNonlinSolvIters

Call flag = IDAGetSensNumNonlinSolvIters(ida_mem, &nSniters);

Description The function IDAGetSensNumNonlinSolvIters returns the number of nonlinear iterations performed for sensitivity calculations.

Arguments ida_mem (void *) pointer to the IDAS memory block.
    nSniters (long int) number of nonlinear iterations performed.

Return value The return value flag (of type int) is one of:
    IDA_SUCCESS The optional output value has been successfully set.
    IDA_MEM_NULL The ida_mem pointer is NULL.
    IDA_NO_SENS Forward sensitivity analysis was not initialized.
IDA_MEM_FAIL The SUNNOLINSOL module is NULL.

Notes This counter is incremented only if ism was IDA_STAGGERED in the call to IDASensInit (see §5.2.1).

F2003 Name FIDAGetSensNumNonlinSolvIters

IDAGetSensNumNonlinSolvConvFails

Call flag = IDAGetSensNumNonlinSolvConvFails(ida_mem, &nSncfails);

Description The function IDAGetSensNumNonlinSolvConvFails returns the number of nonlinear convergence failures that have occurred for sensitivity calculations.

Arguments ida_mem (void *) pointer to the IDAS memory block.

nSncfails (long int) number of nonlinear convergence failures.

Return value The return value flag (of type int) is one of:

IDA_SUCCESS The optional output value has been successfully set.
IDA_MEM_NULL The ida_mem pointer is NULL.
IDA_NO_SENS Forward sensitivity analysis was not initialized.

Notes This counter is incremented only if ism was IDA_STAGGERED in the call to IDASensInit (see §5.2.1).

F2003 Name FIDAGetSensNumNonlinSolvConvFails

IDAGetSensNonlinSolvStats

Call flag = IDAGetSensNonlinSolvStats(ida_mem, &nSniters, &nSncfails);

Description The function IDAGetSensNonlinSolvStats returns the sensitivity-related nonlinear solver statistics as a group.

Arguments ida_mem (void *) pointer to the IDAS memory block.

nSniters (long int) number of nonlinear iterations performed.

nSncfails (long int) number of nonlinear convergence failures.

Return value The return value flag (of type int) is one of:

IDA_SUCCESS The optional output values have been successfully set.
IDA_MEM_NULL The ida_mem pointer is NULL.
IDA_NO_SENS Forward sensitivity analysis was not initialized.
IDA_MEM_FAIL The SUNNOLINSOL module is NULL.

F2003 Name FIDAGetSensNonlinSolvStats

5.2.8.2 Initial condition calculation optional output functions

The sensitivity consistent initial conditions found by IDAS (after a successful call to IDACalcIC) can be obtained by calling the following function:

IDAGetSensConsistentIC

Call flag = IDAGetSensConsistentIC(ida_mem, yyS0_mod, ypS0_mod);

Description The function IDAGetSensConsistentIC returns the corrected initial conditions calculated by IDACalcIC for sensitivities variables.

Arguments ida_mem (void *) pointer to the IDAS memory block.

yyS0_mod (N_Vector *) a pointer to an array of Ns vectors containing consistent sensitivity vectors.
5.3 User-supplied routines for forward sensitivity analysis

In addition to the required and optional user-supplied routines described in §4.6, when using IDAS for forward sensitivity analysis, the user has the option of providing a routine that calculates the residual of the sensitivity equations (2.12).

By default, IDAS uses difference quotient approximation routines for the residual of the sensitivity equations. However, IDAS allows the option for user-defined sensitivity residual routines (which also provides a mechanism for interfacing IDAS to routines generated by automatic differentiation).

The user may provide the residuals of the sensitivity equations (2.12), for all sensitivity parameters at once, through a function of type IDASensResFn defined by:

```c
typedef int (*IDASensResFn)(int Ns, realtype t, N_Vector yy, N_Vector yp, N_Vector resval, N_Vector *yS, N_Vector *ypS, N_Vector *resvalS, void *user_data, N_Vector tmp1, N_Vector tmp2, N_Vector tmp3);
```

**Definition** This function computes the sensitivity residual for all sensitivity equations. It must compute the vectors $(\partial F/\partial y)_{si}(t) + (\partial F/\partial \dot{y})_{si}(t) + (\partial F/\partial p_i)$ and store them in resvalS[i].

**Arguments**
- `Ns` is the number of sensitivities.
- `t` is the current value of the independent variable.
- `yy` is the current value of the state vector, $y(t)$.
- `yp` is the current value of $\dot{y}(t)$.
- `resval` contains the current value $F$ of the original DAE residual.
- `yS` contains the current values of the sensitivities $s_i$.
- `ypS` contains the current values of the sensitivity derivatives $\dot{s}_i$.
- `resvalS` contains the output sensitivity residual vectors. Memory allocation for resvalS is handled within IDAS.
- `user_data` is a pointer to user data.
- `tmp1` `tmp2` `tmp3` are `N_Vectors` of length $N$ which can be used as temporary storage.

**Return value** An IDASensResFn should return 0 if successful, a positive value if a recoverable error occurred (in which case IDAS will attempt to correct), or a negative value if it failed unrecoverably (in which case the integration is halted and IDA_SRES_FAIL is returned).

**Notes** There is one situation in which recovery is not possible even if IDASensResFn function returns a recoverable error flag. That is when this occurs at the very first call to the IDASensResFn, in which case IDAS returns IDA_FIRST_RES_FAIL.
5.4 Integration of quadrature equations depending on forward sensitivities

IDAS provides support for integration of quadrature equations that depends not only on the state variables but also on forward sensitivities.

The following is an overview of the sequence of calls in a user’s main program in this situation. Steps that are unchanged from the skeleton program presented in §5.1 are grayed out. See also §4.7.

1. Initialize parallel or multi-threaded environment
2. Set problem dimensions, etc.
3. Set vectors of initial values
4. Create IDAS object
5. Initialize IDAS solver
6. Specify integration tolerances
7. Create matrix object
8. Create linear solver object
9. Set linear solver optional inputs
10. Attach linear solver module
11. Set optional inputs
12. Create nonlinear solver object
13. Attach nonlinear solver module
14. Set nonlinear solver optional inputs
15. Initialize sensitivity-independent quadrature problem
16. Define the sensitivity problem
17. Set sensitivity initial conditions
18. Activate sensitivity calculations
19. Set sensitivity tolerances
20. Set sensitivity analysis optional inputs
21. Create sensitivity nonlinear solver object
22. Attach the sensitivity nonlinear solver module
23. Set sensitivity nonlinear solver optional inputs
24. Set vector of initial values for quadrature variables
   Typically, the quadrature variables should be initialized to 0.
25. Initialize sensitivity-dependent quadrature integration
   Call IDAQuadSensInit to specify the quadrature equation right-hand side function and to allocate internal memory related to quadrature integration. See §5.4.1 for details.
26. Set optional inputs for sensitivity-dependent quadrature integration
   Call IDASetQuadSensErrCon to indicate whether or not quadrature variables should be used in
   the step size control mechanism. If so, one of the IDAQuadSens* tolerances functions must be
   called to specify the integration tolerances for quadrature variables. See §5.4.4 for details.

27. Advance solution in time

28. Extract sensitivity-dependent quadrature variables
   Call IDAGetQuadSens, IDAGetQuadSens1, IDAGetQuadSensDky or IDAGetQuadSensDky1 to obtain
   the values of the quadrature variables or their derivatives at the current time. See §5.4.3 for details.

29. Get optional outputs

30. Extract sensitivity solution

31. Get sensitivity-dependent quadrature optional outputs
   Call IDAQuadSens* functions to obtain optional output related to the integration of sensitivity-
   dependent quadratures. See §5.4.5 for details.

32. Deallocate memory for solutions vector

33. Deallocate memory for sensitivity vectors

34. Deallocate memory for sensitivity-dependent quadrature variables

35. Free solver memory

36. Free nonlinear solver memory

37. Free vector specification memory

38. Free linear solver and matrix memory

39. Finalize MPI, if used

Note: IDAQuadSensInit (step 25 above) can be called and quadrature-related optional inputs (step
26 above) can be set, anywhere between steps 16 and 27.

5.4.1 Sensitivity-dependent quadrature initialization and deallocation

The function IDAQuadSensInit activates integration of quadrature equations depending on sensitivities
and allocates internal memory related to these calculations. If rhsQS is input as NULL, then
IDAS uses an internal function that computes difference quotient approximations to the functions
\( \bar{q}_i = (\partial q/\partial y) s_i + (\partial q/\partial \dot{y}) \dot{s}_i + \partial q/\partial p_i \), in the notation of (2.10). The form of the call to this function
is as follows:

\[
\text{flag } = \text{IDAQuadSensInit}(\text{ida_mem}, \text{rhsQS}, \text{yQS0});
\]

Call

Description The function IDAQuadSensInit provides required problem specifications, allocates in-
            internal memory, and initializes quadrature integration.

Arguments

\text{ida_mem} (void *) pointer to the IDAS memory block returned by IDACreate.

\text{rhsQS} (IDAQuadSensRhsFn) is the C function which computes \( f_{\text{QS}} \), the right-hand
side of the sensitivity-dependent quadrature equations (for full details see §5.4.6).

\text{yQS0} (N_Vector *) contains the initial values of sensitivity-dependent quadratures.
Return value The return value flag (of type int) will be one of the following:

- **IDA_SUCCESS** The call to IDAQuadSensInit was successful.
- **IDA_MEM_NULL** The IDAS memory was not initialized by a prior call to IDACreate.
- **IDA_MEM_FAIL** A memory allocation request failed.
- **IDA_NOSENS** The sensitivities were not initialized by a prior call to IDASensInit.
- **IDA_IILL_INPUT** The parameter yQS0 is NULL.

Notes Before calling IDAQuadSensInit, the user must enable the sensitivities by calling IDASensInit.

If an error occurred, IDAQuadSensInit also sends an error message to the error handler function.

F2003 Name FIDAQuadSensInit

In terms of the number of quadrature variables \( N_q \) and maximum method order \( \text{maxord} \), the size of the real workspace is increased as follows:

- **Base value:** \( \text{lenrw} = \text{lenrw} + (\text{maxord}+5)N_q \)

- **If IDAQuadSensSVtolerances is called:** \( \text{lenrw} = \text{lenrw} + N_q N_s \)

and the size of the integer workspace is increased as follows:

- **Base value:** \( \text{leniw} = \text{leniw} + (\text{maxord}+5)N_q \)

- **If IDAQuadSensSVtolerances is called:** \( \text{leniw} = \text{leniw} + N_q N_s \)

The function IDAQuadSensReInit, useful during the solution of a sequence of problems of same size, reinitializes the quadrature related internal memory and must follow a call to IDAQuadSensInit. The number \( N_q \) of quadratures as well as the number \( N_s \) of sensitivities are assumed to be unchanged from the prior call to IDAQuadSensInit. The call to the IDAQuadSensReInit function has the form:

```c
IDAQuadSensReInit
```

Call

```c
flag = IDAQuadSensReInit(ida_mem, yQS0);
```

Description The function IDAQuadSensReInit provides required problem specifications and reinitializes the sensitivity-dependent quadrature integration.

Arguments

- **ida_mem** (void *) pointer to the IDAS memory block.
- **yQS0** (N_Vector *) contains the initial values of sensitivity-dependent quadratures.

Return value The return value flag (of type int) will be one of the following:

- **IDA_SUCCESS** The call to IDAQuadSensReInit was successful.
- **IDA_MEM_NULL** The IDAS memory was not initialized by a prior call to IDACreate.
- **IDA_NOSENS** Memory space for the sensitivity calculation was not allocated by a prior call to IDASensInit.
- **IDA_NOQUADSENS** Memory space for the sensitivity quadratures integration was not allocated by a prior call to IDAQuadSensInit.
- **IDA_IILL_INPUT** The parameter yQS0 is NULL.

Notes If an error occurred, IDAQuadSensReInit also sends an error message to the error handler function.

F2003 Name FIDAQuadSensReInit
5.4 Integration of quadrature equations depending on forward sensitivities

**IDAQuadSensFree**

Call: \( \text{IDAQuadSensFree}(\text{ida_mem}); \)

Description: The function \( \text{IDAQuadSensFree} \) frees the memory allocated for sensitivity quadrature integration.

Arguments: The argument is the pointer to the IDAS memory block (of type void *).

Return value: The function \( \text{IDAQuadSensFree} \) has no return value.

Notes: In general, \( \text{IDAQuadSensFree} \) need not be called by the user as it is called automatically by IDAFree.

F2003 Name: FIDAQuadSensFree

5.4.2 IDAS solver function

Even if quadrature integration was enabled, the call to the main solver function \( \text{IDASolve} \) is exactly the same as in §4.5.7. However, in this case the return value \( \text{flag} \) can also be one of the following:

- \( \text{IDA\_QSRHS\_FAIL} \): The sensitivity quadrature right-hand side function failed in an unrecoverable manner.
- \( \text{IDA\_FIRST\_QSRHS\_ERR} \): The sensitivity quadrature right-hand side function failed at the first call.
- \( \text{IDA\_REP\_QSRHS\_ERR} \): Convergence test failures occurred too many times due to repeated recoverable errors in the quadrature right-hand side function. The \( \text{IDA\_REP\_RES\_ERR} \) will also be returned if the quadrature right-hand side function had repeated recoverable errors during the estimation of an initial step size (assuming the sensitivity quadrature variables are included in the error tests).

5.4.3 Sensitivity-dependent quadrature extraction functions

If sensitivity-dependent quadratures have been initialized by a call to \( \text{IDAQuadSensInit} \), or reinitialized by a call to \( \text{IDAQuadSensReInit} \), then IDAS computes a solution, sensitivities, and quadratures depending on sensitivities at time \( t \). However, \( \text{IDASolve} \) will still return only the solutions \( y \) and \( \dot{y} \). Sensitivity-dependent quadratures can be obtained using one of the following functions:

**IDAGetQuadSens**

Call: \( \text{flag} = \text{IDAGetQuadSens}(\text{ida_mem, \&tret, yQS}); \)

Description: The function \( \text{IDAGetQuadSens} \) returns the quadrature sensitivity solution vectors after a successful return from \( \text{IDASolve} \).

Arguments: \( \text{ida_mem} \) (void *) pointer to the memory previously allocated by \( \text{IDAInit} \).
- \( \text{tret} \) (realtype) the time reached by the solver (output).
- \( yQS \) (N\_Vector *) array of \( Ns \) computed sensitivity-dependent quadrature vectors. This array of vectors must be allocated by the user.

Return value: The return value \( \text{flag} \) of \( \text{IDAGetQuadSens} \) is one of:
- \( \text{IDA\_SUCCESS} \): \( \text{IDAGetQuadSens} \) was successful.
- \( \text{IDA\_MEM\_NULL} \): \( \text{ida_mem} \) was NULL.
- \( \text{IDA\_NO\_SENS} \): Sensitivities were not activated.
- \( \text{IDA\_NO\_QUADSENS} \): Quadratures depending on the sensitivities were not activated.
- \( \text{BAD\_DKY} \): \( yQS \) or one of the \( yQS[i] \) is NULL.

F2003 Name: FIDAGetQuadSens

The function \( \text{IDAGetQuadSensDky} \) computes the \( k \)-th derivatives of the interpolating polynomials for the sensitivity-dependent quadrature variables at time \( t \). This function is called by \( \text{IDAGetQuadSens} \) with \( k = 0 \), but may also be called directly by the user.
`IDAGetQuadSensDky`

Call

```c
flag = IDAGetQuadSensDky(ida_mem, t, k, dkyQS);
```

Description The function `IDAGetQuadSensDky` returns derivatives of the quadrature sensitivities solution vectors after a successful return from `IDASolve`.

Arguments

- `ida_mem` (void *) pointer to the memory previously allocated by `IDAInit`.
- `t` (realtype) the time at which information is requested. The time `t` must fall within the interval defined by the last successful step taken by `IDAS`.  
- `k` (int) order of the requested derivative. `k` must be in the range 0, 1, ..., `klast` where `klast` is the method order of the last successful step.
- `dkyQS` (N_Vector *) array of `Ns` vectors containing the derivatives. This vector array must be allocated by the user.

Return value The return value `flag` of `IDAGetQuadSensDky` is one of:

- `IDA_SUCCESS` `IDAGetQuadSensDky` succeeded.
- `IDA_MEM_NULL` `ida_mem` was `NULL`.
- `IDA_NO_SENS` Sensitivities were not activated.
- `IDA_NO_QUADSENS` Quadratures depending on the sensitivities were not activated.
- `IDA_BAD_DKY` `dkyQS` or one of the vectors `dkyQS[i]` is `NULL`.
- `IDA_BAD_K` `k` is not in the range 0, 1, ..., `klast`.
- `IDA_BAD_T` The time `t` is not in the allowed range.

F2003 Name `FIDAGetQuadSensDky`

Quadrature sensitivity solution vectors can also be extracted separately for each parameter in turn through the functions `IDAGetQuadSens1` and `IDAGetQuadSensDky1`, defined as follows:

`IDAGetQuadSens1`

Call

```c
flag = IDAGetQuadSens1(ida_mem, &tret, is, yQS);
```

Description The function `IDAGetQuadSens1` returns the `is`-th sensitivity of quadratures after a successful return from `IDASolve`.

Arguments

- `ida_mem` (void *) pointer to the memory previously allocated by `IDAInit`.
- `tret` (realtype) the time reached by the solver (output).
- `is` (int) specifies which sensitivity vector is to be returned (0 ≤ `is` < `Ns`).
- `yQS` (N_Vector) the computed sensitivity-dependent quadrature vector. This vector must be allocated by the user.

Return value The return value `flag` of `IDAGetQuadSens1` is one of:

- `IDA_SUCCESS` `IDAGetQuadSens1` was successful.
- `IDA_MEM_NULL` `ida_mem` was `NULL`.
- `IDA_NO_SENS` Forward sensitivity analysis was not initialized.
- `IDA_NO_QUADSENS` Quadratures depending on the sensitivities were not activated.
- `IDA_BAD_IS` The index `is` is not in the allowed range.
- `IDA_BAD_DKY` `yQS` is `NULL`.

F2003 Name `FIDAGetQuadSens1`

`IDAGetQuadSensDky1`

Call

```c
flag = IDAGetQuadSensDky1(ida_mem, t, k, is, dkyQS);
```

Description The function `IDAGetQuadSensDky1` returns the `k`-th derivative of the `is`-th sensitivity solution vector after a successful return from `IDASolve`. 

- `Is`-th sensitivity of quadratures after a successful return from `IDASolve`.

Arguments

- `ida_mem` (void *) pointer to the memory previously allocated by `IDAInit`.
- `t` (realtype) the time at which information is requested. The time `t` must fall within the interval defined by the last successful step taken by `IDAS`.  
- `k` (int) order of the requested derivative. `k` must be in the range 0, 1, ..., `klast` where `klast` is the method order of the last successful step.
- `is` (int) specifies which sensitivity vector is to be returned (0 ≤ `is` < `Ns`).
- `yQS` (N_Vector *) array of `Ns` vectors containing the derivatives. This vector array must be allocated by the user.

Return value The return value `flag` of `IDAGetQuadSensDky1` is one of:

- `IDA_SUCCESS` `IDAGetQuadSensDky1` returned successfully.
- `IDA_MEM_NULL` `ida_mem` was `NULL`.
- `IDA_NO_SENS` Sensitivities were not activated.
- `IDA_NO_QUADSENS` Quadratures depending on the sensitivities were not activated.
- `IDA_BAD_DKY` `dkyQS` or one of the vectors `dkyQS[i]` is `NULL`.
- `IDA_BAD_K` `k` is not in the range 0, 1, ..., `klast`.
- `IDA_BAD_T` The time `t` is not in the allowed range.

F2003 Name `FIDAGetQuadSensDky1`
Arguments

- \( \text{idamem} \) (void *) pointer to the memory previously allocated by IDAInit.
- \( t \) (realtype) specifies the time at which sensitivity information is requested. The time \( t \) must fall within the interval defined by the last successful step taken by IDAS.
- \( k \) (int) order of derivative. \( k \) must be in the range \( 0, 1, ..., klast \) where \( klast \) is the method order of the last successful step.
- \( is \) (int) specifies the sensitivity derivative vector to be returned (\( 0 \leq is < N_s \)).
- \( dkyQS \) (N_Vector) the vector containing the derivative. The space for \( dkyQS \) must be allocated by the user.

Return value

The return value \( \text{flag} \) of IDAGetQuadSensDky1 is one of:

- IDA_SUCCESS IDAGetQuadDky1 succeeded.
- IDA_MEM_NULL \( \text{idamem} \) was NULL.
- IDA_NO_SENS Forward sensitivity analysis was not initialized.
- IDA_NO_QUADSENS Quadratures depending on the sensitivities were not activated.
- IDA_BAD_DKY \( dkyQS \) is NULL.
- IDA_BAD_IS The index \( is \) is not in the allowed range.
- IDA_BAD_K \( k \) is not in the range \( 0, 1, ..., klast \).
- IDA_BAD_T The time \( t \) is not in the allowed range.

5.4.4 Optional inputs for sensitivity-dependent quadrature integration

IDAS provides the following optional input functions to control the integration of sensitivity-dependent quadrature equations.

**IDASSetQuadSensErrCon**

Call

\[
\text{flag} = \text{IDASSetQuadSensErrCon(idamem, errconQS)}
\]

Description

The function IDASSetQuadSensErrCon specifies whether or not the quadrature variables are to be used in the local error control mechanism. If they are, the user must specify the error tolerances for the quadrature variables by calling IDAQuadSensSSStolerances, IDAQuadSensSVtolerances, or IDAQuadSensEEtolerances.

Arguments

- \( idamem \) (void *) pointer to the IDAS memory block.
- \( errconQS \) (booleantype) specifies whether sensitivity quadrature variables are included (SUNTRUE) or not (SUNFALSE) in the error control mechanism.

Return value

The return value \( \text{flag} \) (of type int) is one of:

- IDA_SUCCESS The optional value has been successfully set.
- IDA_MEM_NULL The \( idamem \) pointer is NULL.
- IDA_NO_SENS Sensitivities were not activated.
- IDA_NO_QUADSENS Quadratures depending on the sensitivities were not activated.

Notes

By default, \( errconQS \) is set to SUNFALSE. It is illegal to call IDASSetQuadSensErrCon before a call to IDAQuadSensInit.

F2003 Name FIDASSetQuadSensErrCon

If the quadrature variables are part of the step size control mechanism, one of the following functions must be called to specify the integration tolerances for quadrature variables.
### IDAQuadSensSSStolerances

**Call**
```
flag = IDAQuadSensSSStolerances(ida_mem, reltolQS, abstolQS);
```

**Description**
The function **IDAQuadSensSSStolerances** specifies scalar relative and absolute tolerances.

**Arguments**
- `ida_mem` *(void *)* pointer to the IDAS memory block.
- `reltolQS` *(realtype)* is the scalar relative error tolerance.
- `abstolQS` *(realtype*) is a pointer to an array containing the `Ns` scalar absolute error tolerances.

**Return value**
The return value `flag` (of type `int`) is one of:
- **IDA_SUCCESS** The optional value has been successfully set.
- **IDA_MEM_NULL** The `ida_mem` pointer is NULL.
- **IDA_NO_SENS** Sensitivities were not activated.
- **IDA_NO_QUADSENS** Quadratures depending on the sensitivities were not activated.
- **IDA_Ill_INPUT** One of the input tolerances was negative.

F2003 Name: FIDAQuadSensSSStolerances

### IDAQuadSensSVtolerances

**Call**
```
flag = IDAQuadSensSVtolerances(ida_mem, reltolQS, abstolQS);
```

**Description**
The function **IDAQuadSensSVtolerances** specifies scalar relative and vector absolute tolerances.

**Arguments**
- `ida_mem` *(void *)* pointer to the IDAS memory block.
- `reltolQS` *(realtype)* is the scalar relative error tolerance.
- `abstolQS` *(N_Vector*) is an array of `Ns` variables of type `N_Vector`. The `N_Vector` from `abstolS[is]` specifies the vector tolerances for `is`-th quadrature sensitivity.

**Return value**
The return value `flag` (of type `int`) is one of:
- **IDA_SUCCESS** The optional value has been successfully set.
- **IDA_NO_QUAD** Quadrature integration was not initialized.
- **IDA_MEM_NULL** The `ida_mem` pointer is NULL.
- **IDA_NO_SENS** Sensitivities were not activated.
- **IDA_NO_QUADSENS** Quadratures depending on the sensitivities were not activated.
- **IDA_Ill_INPUT** One of the input tolerances was negative.

F2003 Name: FIDAQuadSensSVtolerances

### IDAQuadSensEEtolerances

**Call**
```
flag = IDAQuadSensEEtolerances(ida_mem);
```

**Description**
The function **IDAQuadSensEEtolerances** specifies that the tolerances for the sensitivity-dependent quadratures should be estimated from those provided for the pure quadrature variables.

**Arguments**
- `ida_mem` *(void *)* pointer to the IDAS memory block.

**Return value**
The return value `flag` (of type `int`) is one of:
- **IDA_SUCCESS** The optional value has been successfully set.
- **IDA_MEM_NULL** The `ida_mem` pointer is NULL.
- **IDA_NO_SENS** Sensitivities were not activated.
- **IDA_NO_QUADSENS** Quadratures depending on the sensitivities were not activated.
5.4 Integration of quadrature equations depending on forward sensitivities

Notes When IDAQuadsSensEEtolerances is used, before calling IDASolve, integration of pure quadratures must be initialized (see 4.7.1) and tolerances for pure quadratures must be also specified (see 4.7.4).

F2003 Name FIDAQuadsSensEEtolerances

5.4.5 Optional outputs for sensitivity-dependent quadrature integration

IDAS provides the following functions that can be used to obtain solver performance information related to quadrature integration.

[IDAGetQuadSensNumRhsEvals]

Call flag = IDAGetQuadSensNumRhsEvals(ida_mem, &nrhsQSevals);

Description The function IDAGetQuadSensNumRhsEvals returns the number of calls made to the user’s quadrature right-hand side function.

Arguments ida_mem (void *) pointer to the IDAS memory block.
nrhsQSevals (long int) number of calls made to the user’s rhsQS function.

Return value The return value flag (of type int) is one of:
- IDA_SUCCESS The optional output value has been successfully set.
- IDA_MEM_NULL The ida_mem pointer is NULL.
- IDA_NO_QUADSENS Sensitivity-dependent quadrature integration has not been initialized.

F2003 Name FIDAGetQuadSensNumRhsEvals

[IDAGetQuadSensNumErrTestFails]

Call flag = IDAGetQuadSensNumErrTestFails(ida_mem, &nQSetfails);

Description The function IDAGetQuadSensNumErrTestFails returns the number of local error test failures due to quadrature variables.

Arguments ida_mem (void *) pointer to the IDAS memory block.
nQSetfails (long int) number of error test failures due to quadrature variables.

Return value The return value flag (of type int) is one of:
- IDA_SUCCESS The optional output value has been successfully set.
- IDA_MEM_NULL The ida_mem pointer is NULL.
- IDA_NO_QUADSENS Sensitivity-dependent quadrature integration has not been initialized.

F2003 Name FIDAGetQuadSensNumErrTestFails

[IDAGetQuadSensErrWeights]

Call flag = IDAGetQuadSensErrWeights(ida_mem, eQSweight);

Description The function IDAGetQuadSensErrWeights returns the quadrature error weights at the current time.

Arguments ida_mem (void *) pointer to the IDAS memory block.
eQSweight (N_Vector *) array of quadrature error weight vectors at the current time.

Return value The return value flag (of type int) is one of:
- IDA_SUCCESS The optional output value has been successfully set.
- IDA_MEM_NULL The ida_mem pointer is NULL.
- IDA_NO_QUADSENS Sensitivity-dependent quadrature integration has not been initialized.
Notes

The user must allocate memory for eQSweight.

If quadratures were not included in the error control mechanism (through a call to IDASetQuadSensErrCon with errconQS=SUNTRUE), IDAGetQuadSensErrWeights does not set the eQSweight vector.

F2003 Name FIDAGetQuadSensErrWeights

Call

flag = IDAGetQuadSensErrWeights(ida_mem, &nrhsQSevals, &nQSetfails);

Description

The function IDAGetQuadSensErrWeights returns the IDAS integrator statistics as a group.

Arguments

ida_mem (void *) pointer to the IDAS memory block.

nrhsQSevals (long int) number of calls to the user’s rhsQS function.

nQSetfails (long int) number of error test failures due to quadrature variables.

Return value

The return value flag (of type int) is one of

IDA_SUCCESS the optional output values have been successfully set.

IDA_MEM_NULL the ida_mem pointer is NULL.

IDA_NO_QUADSENS Sensitivity-dependent quadrature integration has not been initialized.

F2003 Name FIDAGetQuadSensStats

5.4.6 User-supplied function for sensitivity-dependent quadrature integration

For the integration of sensitivity-dependent quadrature equations, the user must provide a function that defines the right-hand side of the sensitivity quadrature equations. For sensitivities of quadratures (2.10) with integrands $q$, the appropriate right-hand side functions are given by $\bar{q}_i = (\partial q / \partial y)s_i + (\partial q / \partial \dot{y})\dot{s}_i + \partial q / \partial p_i$. This user function must be of type IDAQuadSensRhsFn, defined as follows:

typedef int (*IDAQuadSensRhsFn)(int Ns, realtype t, N_Vector yy, N_Vector yp, N_Vector *yyS, N_Vector *ypS, N_Vector rrQ, N_Vector *rhsvalQS, void *user_data, N_Vector tmp1, N_Vector tmp2, N_Vector tmp3)

Purpose

This function computes the sensitivity quadrature equation right-hand side for a given value of the independent variable $t$ and state vector $y$.

Arguments

Ns is the number of sensitivity vectors.

t is the current value of the independent variable.

yy is the current value of the dependent variable vector, $y(t)$.

yp is the current value of the dependent variable vector, $\dot{y}(t)$.

yyS is an array of $Ns$ variables of type N_Vector containing the dependent sensitivity vectors $s_i$.

ypS is an array of $Ns$ variables of type N_Vector containing the dependent sensitivity derivatives $\dot{s}_i$.

rrQ is the current value of the quadrature right-hand side $q$.

rhsvalQS contains the $Ns$ output vectors.

user_data is the user_data pointer passed to IDASetUserData.

tmp1
tmp2
5.5 Note on using partial error control

... are \texttt{N\_Vector}s which can be used as temporary storage.

Return value An \texttt{IDAQuadSensRhsFn} should return 0 if successful, a positive value if a recoverable error occurred (in which case \texttt{IDAS} will attempt to correct), or a negative value if it failed unrecoverably (in which case the integration is halted and \texttt{IDA\_QRHS\_FAIL} is returned).

Notes Allocation of memory for \texttt{rhsvalQS} is automatically handled within \texttt{IDAS}.

Both \texttt{yy} and \texttt{yp} are of type \texttt{N\_Vector} and both \texttt{yyS} and \texttt{ypS} are pointers to an array containing \texttt{Ns} vectors of type \texttt{N\_Vector}. It is the user’s responsibility to access the vector data consistently (including the use of the correct accessor macros from each \texttt{NVECTOR} implementation). For the sake of computational efficiency, the vector functions in the two \texttt{NVECTOR} implementations provided with \texttt{IDAS} do not perform any consistency checks with respect to their \texttt{N\_Vector} arguments (see \S8.3 and \S8.4).

There is one situation in which recovery is not possible even if \texttt{IDAQuadSensRhsFn} function returns a recoverable error flag. That is when this occurs at the very first call to the \texttt{IDAQuadSensRhsFn}, in which case \texttt{IDAS} returns \texttt{IDA\_FIRST\_QSRHS\_ERR}.

5.5 Note on using partial error control

For some problems, when sensitivities are excluded from the error control test, the behavior of \texttt{IDAS} may appear at first glance to be erroneous. One would expect that, in such cases, the sensitivity variables would not influence in any way the step size selection.

The short explanation of this behavior is that the step size selection implemented by the error control mechanism in \texttt{IDAS} is based on the magnitude of the correction calculated by the nonlinear solver. As mentioned in \S5.2.1, even with partial error control selected in the call to \texttt{IDASensInit}, the sensitivity variables are included in the convergence tests of the nonlinear solver.

When using the simultaneous corrector method (\S2.5), the nonlinear system that is solved at each step involves both the state and sensitivity equations. In this case, it is easy to see how the sensitivity variables may affect the convergence rate of the nonlinear solver and therefore the step size selection. The case of the staggered corrector approach is more subtle. The sensitivity variables at a given step are computed only once the solver for the nonlinear state equations has converged. However, if the nonlinear system corresponding to the sensitivity equations has convergence problems, \texttt{IDAS} will attempt to improve the initial guess by reducing the step size in order to provide a better prediction of the sensitivity variables. Moreover, even if there are no convergence failures in the solution of the sensitivity system, \texttt{IDAS} may trigger a call to the linear solver’s setup routine which typically involves reevaluation of Jacobian information (Jacobian approximation in the case of \texttt{IDADENSE} and \texttt{IDABAND}, or preconditioner data in the case of the Krylov solvers). The new Jacobian information will be used by subsequent calls to the nonlinear solver for the state equations and, in this way, potentially affect the step size selection.

When using the simultaneous corrector method it is not possible to decide whether nonlinear solver convergence failures or calls to the linear solver setup routine have been triggered by convergence problems due to the state or the sensitivity equations. When using one of the staggered corrector methods, however, these situations can be identified by carefully monitoring the diagnostic information provided through optional outputs. If there are no convergence failures in the sensitivity nonlinear solver, and none of the calls to the linear solver setup routine were made by the sensitivity nonlinear solver, then the step size selection is not affected by the sensitivity variables.

Finally, the user must be warned that the effect of appending sensitivity equations to a given system of DAEs on the step size selection (through the mechanisms described above) is problem-dependent and can therefore lead to either an increase or decrease of the total number of steps that \texttt{IDAS} takes to complete the simulation. At first glance, one would expect that the impact of the sensitivity variables, if any, would be in the direction of increasing the step size and therefore reducing the total number of steps. The argument for this is that the presence of the sensitivity variables in the convergence test of the nonlinear solver can only lead to additional iterations (and therefore a smaller iteration error), or to additional calls to the linear solver setup routine (and therefore more up-to-date Jacobian
information), both of which will lead to larger steps being taken by IDAS. However, this is true only locally. Overall, a larger integration step taken at a given time may lead to step size reductions at later times, due to either nonlinear solver convergence failures or error test failures.
Chapter 6

Using IDAS for Adjoint Sensitivity Analysis

This chapter describes the use of IDAS to compute sensitivities of derived functions using adjoint sensitivity analysis. As mentioned before, the adjoint sensitivity module of IDAS provides the infrastructure for integrating backward in time any system of DAEs that depends on the solution of the original IVP, by providing various interfaces to the main IDAS integrator, as well as several supporting user-callable functions. For this reason, in the following sections we refer to the **backward problem** and not to the **adjoint problem** when discussing details relevant to the DAEs that are integrated backward in time. The backward problem can be the adjoint problem (2.20) or (2.25), and can be augmented with some quadrature differential equations.

IDAS uses various constants for both input and output. These are defined as needed in this chapter, but for convenience are also listed separately in Appendix B.

We begin with a brief overview, in the form of a skeleton user program. Following that are detailed descriptions of the interface to the various user-callable functions and of the user-supplied functions that were not already described in Chapter 4.

### 6.1 A skeleton of the user’s main program

The following is a skeleton of the user’s main program as an application of IDAS. The user program is to have these steps in the order indicated, unless otherwise noted. For the sake of brevity, we defer many of the details to the later sections. As in §4.4, most steps are independent of the NVECTOR, SUNMATRIX, SUNLINSOL, and SUNNONLINSOL implementations used. For the steps that are not, refer to Chapters 8, 9, 10, and 11 for the specific name of the function to be called or macro to be referenced.

Steps that are unchanged from the skeleton programs presented in §4.4, §5.1, and §5.4, are grayed out.

1. Include necessary header files
   
   The idas.h header file also defines additional types, constants, and function prototypes for the adjoint sensitivity module user-callable functions. In addition, the main program should include an NVECTOR implementation header file (for the particular implementation used) and, if a nonlinear solver requiring a linear solver (e.g., the default Newton iteration) will be used, the header file of the desired linear solver module.

2. Initialize parallel or multi-threaded environment

   **Forward problem**

3. Set problem dimensions etc. for the forward problem
4. Set initial conditions for the forward problem
5. Create IDAS object for the forward problem
6. Initialize IDAS solver for the forward problem
7. Specify integration tolerances for forward problem
8. Set optional inputs for the forward problem
9. Create matrix object for the forward problem
10. Create linear solver object for the forward problem
11. Set linear solver optional inputs for the forward problem
12. Attach linear solver module for the forward problem
13. Create nonlinear solver module for the forward problem
14. Attach nonlinear solver module for the forward problem
15. Set nonlinear solver optional inputs for the forward problem
16. Initialize quadrature problem or problems for forward problems, using IDAQuadInit and/or IDAQuadSensInit.
17. Initialize forward sensitivity problem
18. Specify rootfinding
19. Allocate space for the adjoint computation
   Call IDAAdjInit() to allocate memory for the combined forward-backward problem (see §6.2.1 for details). This call requires Nd, the number of steps between two consecutive checkpoints. IDAAdjInit also specifies the type of interpolation used (see §2.6.3).
20. Integrate forward problem
   Call IDASolveF, a wrapper for the IDAS main integration function IDASolve, either in IDA_NORMAL mode to the time tout or in IDA_ONE_STEP mode inside a loop (if intermediate solutions of the forward problem are desired (see §6.2.3)). The final value of tret is then the maximum allowable value for the endpoint T of the backward problem.

   **Backward problem(s)**

21. Set problem dimensions etc. for the backward problem
   This generally includes NB, the number of variables in the backward problem and possibly the local vector length NBlocal.
22. Set initial values for the backward problem
   Set the endpoint time tB0 = T, and set the corresponding vectors yB0 and ypB0 at which the backward problem starts.
23. Create the backward problem
   Call IDACreateB, a wrapper for IDACreate, to create the IDAS memory block for the new backward problem. Unlike IDACreate, the function IDACreateB does not return a pointer to the newly created memory block (see §6.2.4). Instead, this pointer is attached to the internal adjoint memory block (created by IDAAdjInit) and returns an identifier called which that the user must later specify in any actions on the newly created backward problem.
24. **Allocate memory for the backward problem**

   Call IDAInitB (or IDAInitBS, when the backward problem depends on the forward sensitivities). The two functions are actually wrappers for IDAInit and allocate internal memory, specify problem data, and initialize IDAS at $t_B$ for the backward problem (see §6.2.4).

25. **Specify integration tolerances for backward problem**

   Call IDASSTolerancesB(...) or IDASVTolerancesB(...) to specify a scalar relative tolerance and scalar absolute tolerance, or a scalar relative tolerance and a vector of absolute tolerances, respectively. The functions are wrappers for IDASSTolerances(...) and IDASVTolerances(...) but they require an extra argument which, the identifier of the backward problem returned by IDACreateB. See §6.2.5 for more information.

26. **Set optional inputs for the backward problem**

   Call IDASet* functions to change from their default values any optional inputs that control the behavior of IDAS. Unlike their counterparts for the forward problem, these functions take an extra argument which, the identifier of the backward problem returned by IDACreateB (see §6.2.9).

27. **Create matrix object for the backward problem**

   If a nonlinear solver requiring a linear solve will be used (e.g., the default Newton iteration) and the linear solver will be a direct linear solver, then a template Jacobian matrix must be created by calling the appropriate constructor function defined by the particular SUNMATRIX implementation. NOTE: The dense, banded, and sparse matrix objects are usable only in a serial or threaded environment.

   Note also that it is not required to use the same matrix type for both the forward and the backward problems.

28. **Create linear solver object for the backward problem**

   If a nonlinear solver requiring a linear solver is chosen (e.g., the default Newton iteration), then the desired linear solver object for the backward problem must be created by calling the appropriate constructor function defined by the particular SUNLINSOL implementation.

   Note that it is not required to use the same linear solver module for both the forward and the backward problems; for example, the forward problem could be solved with the SUNLINSOL_DENSE linear solver module and the backward problem with SUNLINSOL_SPGMR linear solver module.

29. **Set linear solver interface optional inputs for the backward problem**

   Call IDASet* functions to change optional inputs specific to the linear solver interface. See §6.2.9 for details.

30. **Attach linear solver module for the backward problem**

   If a nonlinear solver requiring a linear solver is chosen for the backward problem (e.g., the default Newton iteration), then initialize the IDALS linear solver interface by attaching the linear solver object (and matrix object, if applicable) with the following call (for details see §4.5.3):

   ```c
   ier = IDASetLinearSolverB(...);
   ```

31. **Create nonlinear solver object for the backward problem (optional)**

   If using a non-default nonlinear solver for the backward problem, then create the desired nonlinear solver object by calling the appropriate constructor function defined by the particular SUNNONLINSOL implementation e.g., NLBSB = SUNNonlinSol_***(...) where *** is the name of the nonlinear solver (see Chapter 11 for details).

32. **Attach nonlinear solver module for the backward problem (optional)**
If using a non-default nonlinear solver for the backward problem, then initialize the nonlinear solver interface by attaching the nonlinear solver object by calling

\[ \text{ier} = \text{IDASetNonlinearSolverB(idaode_mem, NLSB)} ; \] (see §4.5.4 for details).

33. **Initialize quadrature calculation**

If additional quadrature equations must be evaluated, call IDAQuadInitB or IDAQuadInitBS (if quadrature depends also on the forward sensitivities) as shown in §6.2.11.1. These functions are wrappers around IDAQuadInit and can be used to initialize and allocate memory for quadrature integration. Optionally, call IDASetQuadB functions to change from their default values optional inputs that control the integration of quadratures during the backward phase.

34. **Integrate backward problem**

Call IDASolveB, a second wrapper around the idas main integration function IDASolve, to integrate the backward problem from \( t_B^0 \) (see §6.2.8). This function can be called either in IDA_NORMAL or IDA_ONE_STEP mode. Typically, IDASolveB will be called in IDA_NORMAL mode with an end time equal to the initial time \( t_0 \) of the forward problem.

35. **Extract quadrature variables**

If applicable, call IDAGetQuadB, a wrapper around IDAGetQuad, to extract the values of the quadrature variables at the time returned by the last call to IDASolveB. See §6.2.11.2.

36. **Deallocate memory**

Upon completion of the backward integration, call all necessary deallocation functions. These include appropriate destructors for the vectors \( y \) and \( y_B \), a call to IDAFree to free the idas memory block for the forward problem. If one or more additional adjoint sensitivity analyses are to be done for this problem, a call to IDAAdjFree (see §6.2.1) may be made to free and deallocate the memory allocated for the backward problems, followed by a call to IDAAdjInit.

37. **Free the nonlinear solver memory for the forward and backward problems**

38. **Free linear solver and matrix memory for the forward and backward problems**

39. **Finalize MPI, if used**

The above user interface to the adjoint sensitivity module in idas was motivated by the desire to keep it as close as possible in look and feel to the one for DAE IVP integration. Note that if steps (21)-(35) are not present, a program with the above structure will have the same functionality as one described in §4.4 for integration of DAEs, albeit with some overhead due to the checkpointing scheme.

If there are multiple backward problems associated with the same forward problem, repeat steps (21)-(35) above for each successive backward problem. In the process, each call to IDACreateB creates a new value of the identifier which.

6.2 User-callable functions for adjoint sensitivity analysis

6.2.1 Adjoint sensitivity allocation and deallocation functions

After the setup phase for the forward problem, but before the call to IDASolveF, memory for the combined forward-backward problem must be allocated by a call to the function IDAAdjInit. The form of the call to this function is

\[ \text{flag} = \text{IDAAdjInit(ida_mem, Nd, interpType)} ; \]
6.2 User-callable functions for adjoint sensitivity analysis

Description The function IDAAdjInit updates IDAS memory block by allocating the internal memory needed for backward integration. Space is allocated for the \( N_d \) interpolation data points, and a linked list of checkpoints is initialized.

Arguments

- **ida_mem** (void *) is the pointer to the IDAS memory block returned by a previous call to IDACreate.
- **Nd** (long int) is the number of integration steps between two consecutive checkpoints.
- **interpType** (int) specifies the type of interpolation used and can be IDA_POLYNOMIAL or IDA_HERMITE, indicating variable-degree polynomial and cubic Hermite interpolation, respectively (see §2.6.3).

Return value The return value **flag** (of type int) is one of:

- IDA_SUCCESS IDAAdjInit was successful.
- IDA_MEM_FAIL A memory allocation request has failed.
- IDA_MEM_NULL ida_mem was NULL.
- IDA_ILL_INPUT One of the parameters was invalid: \( N_d \) was not positive or **interpType** is not one of the IDA_POLYNOMIAL or IDA_HERMITE.

Notes The user must set \( N_d \) so that all data needed for interpolation of the forward problem solution between two checkpoints fits in memory. IDAAdjInit attempts to allocate space for \((2N_d+3)\) variables of type N_Vector.

If an error occurred, IDAAdjInit also sends a message to the error handler function.

F2003 Name FIDAAdjInit

**IDAAdjReInit**

Call flag = IDAAdjReInit(ida_mem);

Description The function IDAAdjReInit reinitializes the IDAS memory block for ASA, assuming that the number of steps between check points and the type of interpolation remain unchanged.

Arguments **ida_mem** (void *) is the pointer to the IDAS memory block returned by a previous call to IDACreate.

Return value The return value **flag** (of type int) is one of:

- IDA_SUCCESS IDAAdjReInit was successful.
- IDA_MEM_NULL ida_mem was NULL.
- IDA_NO_ADJ The function IDAAdjInit was not previously called.

Notes The list of check points (and associated memory) is deleted. The list of backward problems is kept. However, new backward problems can be added to this list by calling IDACreateB. If a new list of backward problems is also needed, then free the adjoint memory (by calling IDAAdjFree) and reinitialize ASA with IDAAdjInit.

The IDAS memory for the forward and backward problems can be reinitialized separately by calling IDAREInit and IDAREInitB, respectively.

F2003 Name FIDAAdjReInit

**IDAAdjFree**

Call IDAAdjFree(ida_mem);

Description The function IDAAdjFree frees the memory related to backward integration allocated by a previous call to IDAAdjInit.
Arguments  The only argument is the IDAS memory block pointer returned by a previous call to IDACreate.

Return value  The function IDAAdjFree has no return value.

Notes  This function frees all memory allocated by IDAAdjInit. This includes workspace memory, the linked list of checkpoints, memory for the interpolation data, as well as the IDAS memory for the backward integration phase. Unless one or more further calls to IDAAdjInit are to be made, IDAAdjFree should not be called by the user, as it is invoked automatically by IDAFree.

F2003 Name  FIDAAdjFree

### 6.2.2 Adjoint sensitivity optional input

At any time during the integration of the forward problem, the user can disable the checkpointing of the forward sensitivities by calling the following function:

```c
IAAdjSetNoSensi
```

Call  

```c
flag = IDAAdjSetNoSensi(ida_mem);
```

Description  The function IDAAdjSetNoSensi instructs IDASolveF not to save checkpointing data for forward sensitivities any more.

Arguments  ida_mem  (void *) pointer to the IDAS memory block.

Return value  The return flag (of type int) is one of:

- **IDA_SUCCESS**  The call to IDACreate was successful.
- **IDA_MEM_NULL**  The ida_mem was NULL.
- **IDA_NO_ADJ**  The function IDAAdjInit has not been previously called.

F2003 Name  FIDAAdjSetNoSensi

### 6.2.3 Forward integration function

The function IDASolveF is very similar to the IDAS function IDASolve (see §4.5.7) in that it integrates the solution of the forward problem and returns the solution \((y, \dot{y})\). At the same time, however, IDASolveF stores checkpoint data every \(Nd\) integration steps. IDASolveF can be called repeatedly by the user. Note that IDASolveF is used only for the forward integration pass within an Adjoint Sensitivity Analysis. It is not for use in Forward Sensitivity Analysis; for that, see Chapter 5. The call to this function has the form

```c
IDASolveF
```

Call  

```c
flag = IDASolveF(ida_mem, tout, &tret, yret, ypret, itask, &ncheck);
```

Description  The function IDASolveF integrates the forward problem over an interval in \(t\) and saves checkpointing data.

Arguments  ida_mem  (void *) pointer to the IDAS memory block.

tout  (realtype) the next time at which a computed solution is desired.

tret  (realtype) the time reached by the solver (output).

yret  (N_Vector) the computed solution vector \(y\).

ypret  (N_Vector) the computed solution vector \(\dot{y}\).

itask  (int) a flag indicating the job of the solver for the next step. The IDA_NORMAL task is to have the solver take internal steps until it has reached or just passed the user-specified tout parameter. The solver then interpolates in order to return an approximate value of \(y(tout)\) and \(\dot{y}(tout)\). The IDA_ONE_STEP option tells the solver to take just one internal step and return the solution at the point reached by that step.
ncheck (int) the number of (internal) checkpoints stored so far.

Return value On return, IDASolveF returns vectors \( y_{ret}, y_{pret} \) and a corresponding independent variable value \( t = t_{ret} \), such that \( y_{ret} \) is the computed value of \( y(t) \) and \( y_{pret} \) the value of \( \dot{y}(t) \). Additionally, it returns in \( ncheck \) the number of internal checkpoints saved; the total number of checkpoint intervals is \( ncheck + 1 \). The return value \( \text{flag} \) (of type int) will be one of the following. For more details see §4.5.7.

IDA_SUCCESS \hspace{10pt} \text{IDASolveF} succeeded.
IDA_TSTOP_RETURN \hspace{10pt} \text{IDASolveF} succeeded by reaching the optional stopping point.
IDA_ROOT_RETURN \hspace{10pt} \text{IDASolveF} succeeded and found one or more roots. In this case, \( t_{ret} \) is the location of the root. If \( nrtfn > 1 \), call IDAGetRootInfo to see which \( g_i \) were found to have a root.
IDA_NO_MALLOC \hspace{10pt} The function IDAInit has not been previously called.
IDA_Ill_INPUT \hspace{10pt} One of the inputs to IDASolveF is illegal.
IDA_TOO_MUCH_WORK \hspace{10pt} The solver took \( mxstep \) internal steps but could not reach \( tout \).
IDA_TOO_MUCH_ACC \hspace{10pt} The solver could not satisfy the accuracy demanded by the user for some internal step.
IDA_ERR_FAILURE \hspace{10pt} Error test failures occurred too many times during one internal time step or occurred with \( |h| = h_{\text{min}} \).
IDA_CONV_FAILURE \hspace{10pt} Convergence test failures occurred too many times during one internal time step or occurred with \( |h| = h_{\text{min}} \).
IDA_LSETUP_FAIL \hspace{10pt} The linear solver’s setup function failed in an unrecoverable manner.
IDA_LSOLVE_FAIL \hspace{10pt} The linear solver’s solve function failed in an unrecoverable manner.
IDA_NO_ADJ \hspace{10pt} The function IDAAdjInit has not been previously called.
IDA_MEM_FAIL \hspace{10pt} A memory allocation request has failed (in an attempt to allocate space for a new checkpoint).

Notes All failure return values are negative and therefore a test \( \text{flag} < 0 \) will trap all IDASolveF failures.

At this time, IDASolveF stores checkpoint information in memory only. Future versions will provide for a safeguard option of dumping checkpoint data into a temporary file as needed. The data stored at each checkpoint is basically a snapshot of the IDAS internal memory block and contains enough information to restart the integration from that time and to proceed with the same step size and method order sequence as during the forward integration.

In addition, IDASolveF also stores interpolation data between consecutive checkpoints so that, at the end of this first forward integration phase, interpolation information is already available from the last checkpoint forward. In particular, if no checkpoints were necessary, there is no need for the second forward integration phase.

It is illegal to change the integration tolerances between consecutive calls to IDASolveF, as this information is not captured in the checkpoint data.

F2003 Name FIDASolveF

6.2.4 Backward problem initialization functions

The functions IDACreateB and IDAInitB (or IDAInitBS) must be called in the order listed. They instantiate an IDAS solver object, provide problem and solution specifications, and allocate internal memory for the backward problem.
Using IDAS for Adjoint Sensitivity Analysis

**IDACreateB**

**Call**

```c
flag = IDACreateB(ida_mem, &which);
```

**Description**
The function **IDACreateB** instantiates an **idas** solver object for the backward problem.

**Arguments**

- `ida_mem` *(void *)* pointer to the **idas** memory block returned by **IDACreate**.
- `which` *(int)* contains the identifier assigned by **idas** for the newly created backward problem. Any call to **IDA*B** functions requires such an identifier.

**Return value**
The return flag *(of type int)* is one of:

- **IDA_SUCCESS** The call to **IDACreateB** was successful.
- **IDA_MEM NULL** The `ida_mem` was NULL.
- **IDA_NO_ADJ** The function **IDAAdjInit** has not been previously called.
- **IDA_MEM_FAIL** A memory allocation request has failed.

**Notes**
The memory allocated by **IDACreateB** is deallocated by the function **IDAAdjFree**.

**F2003 Name** FIDACreateB

**IDAInitB**

**Call**

```c
flag = IDAInitB(ida_mem, which, resB, tB0, yB0, ypB0);
```

**Description**
The function **IDAInitB** provides problem specification, allocates internal memory, and initializes the backward problem.

**Arguments**

- `ida_mem` *(void *)* pointer to the **idas** memory block returned by **IDACreate**.
- `which` *(int)* represents the identifier of the backward problem.
- `resB` *(IDAResFnB)* is the **C** function which computes $f_B$, the residual of the backward DAE problem. This function has the form `resB(t, y, yp, yB, ypB, resvalB, user_dataB)` *(for full details see §6.3.1)*.
- `tB0` *(realtype)* specifies the endpoint $T$ where final conditions are provided for the backward problem, normally equal to the endpoint of the forward integration.
- `yB0` *(N_Vector)* is the initial value *(at $t = tB0$)* of the backward solution.
- `ypB0` *(N_Vector)* is the initial derivative value *(at $t = tB0$)* of the backward solution.

**Return value**
The return flag *(of type int)* will be one of the following:

- **IDA_SUCCESS** The call to **IDAInitB** was successful.
- **IDA_NO_MALLOC** The function **IDAInit** has not been previously called.
- **IDA_MEM NULL** The `ida_mem` was NULL.
- **IDA_NO_ADJ** The function **IDAAdjInit** has not been previously called.
- **IDA_BAD_TBO** The final time `tB0` was outside the interval over which the forward problem was solved.
- **IDA_Ill_INPUT** The parameter `which` represented an invalid identifier, or one of `yB0`, `ypB0`, `resB` was NULL.

**Notes**
The memory allocated by **IDAInitB** is deallocated by the function **IDAAdjFree**.

**F2003 Name** FIDAInitB

For the case when backward problem also depends on the forward sensitivities, user must call **IDAInitBS** instead of **IDAInitB**. Only the third argument of each function differs between these functions.
6.2 User-callable functions for adjoint sensitivity analysis

### IDAInitBS

**Call**

```c
flag = IDAInitBS(ida_mem, which, resBS, tB0, yB0, ypB0);
```

**Description**
The function `IDAInitBS` provides problem specification, allocates internal memory, and initializes the backward problem.

**Arguments**

- `ida_mem` (void *) pointer to the IDAS memory block returned by `IDACreate`.
- `which` (int) represents the identifier of the backward problem.
- `resBS` (IDAResFnBS) is the C function which computes \( f_B \), the residual or the backward DAE problem. This function has the form `resBS(t, y, yp, yS, ypS, yB, ypB, resvalB, user_dataB)` (for full details see §6.3.2).
- `tB0` (realtype) specifies the endpoint \( T \) where final conditions are provided for the backward problem.
- `yB0` (N_Vector) is the initial value (at \( t = tB0 \)) of the backward solution.
- `ypB0` (N_Vector) is the initial derivative value (at \( t = tB0 \)) of the backward solution.

**Return value**
The return value `flag` (of type int) will be one of the following:

- `IDA_SUCCESS` The call to `IDAInitB` was successful.
- `IDA_NO_MALLOC` The function `IDAInit` has not been previously called.
- `IDA_MEM_NULL` The `ida_mem` memory block pointer was NULL.
- `IDA_NO_ADJ` The function `IDAAdjInit` has not been previously called.

**Notes**
The memory allocated by `IDAInitBS` is deallocated by the function `IDAAdjFree`.

**F2003 Name** `FIDAInitBS`

The function `IDAReInitB` reinitializes IDAS for the solution of a series of backward problems, each identified by a value of the parameter `which`. `IDAReInitB` is essentially a wrapper for `IDAReInit`, and so all details given for `IDAReInit` in §4.5.11 apply here. Also, `IDAReInitB` can be called to reinitialize a backward problem even if it has been initialized with the sensitivity-dependent version `IDAInitBS`. Before calling `IDAReInitB` for a new backward problem, call any desired solution extraction functions `IDAGet**` associated with the previous backward problem. The call to the `IDAReInitB` function has the form

### IDAReInitB

**Call**

```c
flag = IDAReInitB(ida_mem, which, tB0, yB0, ypB0);
```

**Description**
The function `IDAReInitB` reinitializes an IDAS backward problem.

**Arguments**

- `ida_mem` (void *) pointer to IDAS memory block returned by `IDACreate`.
- `which` (int) represents the identifier of the backward problem.
- `tB0` (realtype) specifies the endpoint \( T \) where final conditions are provided for the backward problem.
- `yB0` (N_Vector) is the initial value (at \( t = tB0 \)) of the backward solution.
- `ypB0` (N_Vector) is the initial derivative value (at \( t = tB0 \)) of the backward solution.

**Return value**
The return value `flag` (of type int) will be one of the following:

- `IDA_SUCCESS` The call to `IDAReInitB` was successful.
- `IDA_NO_MALLOC` The function `IDAInit` has not been previously called.
- `IDA_MEM_NULL` The `ida_mem` memory block pointer was NULL.
- `IDA_NO_ADJ` The function `IDAAdjInit` has not been previously called.
IDA_BAD_TBO  The final time $t_{B0}$ is outside the interval over which the forward problem was solved.

IDA_ILL_INPUT  The parameter which represented an invalid identifier, or one of $y_{B0}$, $y_{pB0}$ was NULL.

F2003 Name  FIDAREINITB

### 6.2.5  Tolerance specification functions for backward problem

One of the following two functions must be called to specify the integration tolerances for the backward problem. Note that this call must be made after the call to IDAINITB or IDAINITBS.

#### IDASStolerancesB

**Call**

```c
flag = IDASStolerances(ida_mem, which, reltolB, abstolB);
```

**Description**
The function IDASStolerancesB specifies scalar relative and absolute tolerances.

**Arguments**
- `ida_mem` (void *) pointer to the IDAS memory block returned by IDACreate.
- `which` (int) represents the identifier of the backward problem.
- `reltolB` (realtype) is the scalar relative error tolerance.
- `abstolB` (realtype) is the scalar absolute error tolerance.

**Return value**
The return `flag` (of type int) will be one of the following:

- `IDA_SUCCESS`  The call to IDASStolerancesB was successful.
- `IDA_MEM_NULL`  The IDAS memory block was not initialized through a previous call to IDACreate.
- `IDA_NO_MALLOC`  The allocation function IDAINIT has not been called.
- `IDA_NO_ADJ`  The function IDAAdjInit has not been previously called.
- `IDA_ILL_INPUT`  One of the input tolerances was negative.

F2003 Name  FIDASStolerancesB

#### IDASVtolerancesB

**Call**

```c
flag = IDASVtolerancesB(ida_mem, which, reltolB, abstol);
```

**Description**
The function IDASVtolerancesB specifies scalar relative tolerance and vector absolute tolerances.

**Arguments**
- `ida_mem` (void *) pointer to the IDAS memory block returned by IDACreate.
- `which` (int) represents the identifier of the backward problem.
- `reltol` (realtype) is the scalar relative error tolerance.
- `abstol` (N_Vector) is the vector of absolute error tolerances.

**Return value**
The return `flag` (of type int) will be one of the following:

- `IDA_SUCCESS`  The call to IDASVtolerancesB was successful.
- `IDA_MEM_NULL`  The IDAS memory block was not initialized through a previous call to IDACreate.
- `IDA_NO_MALLOC`  The allocation function IDAINIT has not been called.
- `IDA_NO_ADJ`  The function IDAAdjInit has not been previously called.
- `IDA_ILL_INPUT`  The relative error tolerance was negative or the absolute tolerance had a negative component.

**Notes**

This choice of tolerances is important when the absolute error tolerance needs to be different for each component of the DAE state vector $y$.

F2003 Name  FIDASVtolerancesB
6.2.6 Linear solver initialization functions for backward problem

All IDAS linear solver modules available for forward problems are available for the backward problem. They should be created as for the forward problem then attached to the memory structure for the backward problem using the following function.

\[ \text{f2003 Name: FIDASetLinearSolverB} \]

Call

\[
\text{flag = IDASetLinearSolverB(ida_mem, which, LS, A);}\
\]

Description

The function IDASetLinearSolverB attaches a generic SUNLINSOL object LS and corresponding template Jacobian SUNMATRIX object A (if applicable) to IDAS, initializing the IDALS linear solver interface for solution of the backward problem.

Arguments

- ida_mem (void *), pointer to the IDAS memory block.
- which (int), represents the identifier of the backward problem returned by IDACreateB.
- LS (SUNLinearSolver), SUNLINSOL object to use for solving linear systems for the backward problem.
- A (SUNMatrix), SUNMATRIX object for used as a template for the Jacobian for the backward problem (or NULL if not applicable).

Return value

The return value flag (of type int) is one of:

- IDALS_SUCCESS: The IDALS initialization was successful.
- IDALS_MEM_NULL: The ida_mem pointer is NULL.
- IDALS_Ill_INPUT: The IDALS interface is not compatible with the LS or A input objects or is incompatible with the current NVECTOR module.
- IDALS_MEM_FAIL: A memory allocation request failed.
- IDALS_NO_ADJ: The function IDAAdjInit has not been previously called.
- IDALS_Ill_INPUT: The parameter which represented an invalid identifier.

Notes

If LS is a matrix-based linear solver, then the template Jacobian matrix A will be used in the solve process, so if additional storage is required within the SUNMATRIX object (e.g. for factorization of a banded matrix), ensure that the input object is allocated with sufficient size (see the documentation of the particular SUNMATRIX type in Chapter 9 for further information).

The previous routines IDADlsSetLinearSolverB and IDASpilsSetLinearSolverB are now wrappers for this routine, and may still be used for backward-compatibility. However, these will be deprecated in future releases, so we recommend that users transition to the new routine name soon.

6.2.7 Initial condition calculation functions for backward problem

IDAS provide support for calculation of consistent initial conditions for certain backward index-one problems of semi-implicit form through the functions IDACalcICB and IDACalcICBS. Calling them is optional. It is only necessary when the initial conditions do not satisfy the adjoint system.

The above functions provide the same functionality for backward problems as IDACalcIC with parameter iopt = IDA_YA_YDP_INIT provides for forward problems (see §4.5.5): compute the algebraic components of \( y_B \) and differential components of \( \dot{y}_B \), given the differential components of \( y_B \). They require that the IDASIdB was previously called to specify the differential and algebraic components.

Both functions require forward solutions at the final time \( t_B0 \). IDACalcICBS also needs forward sensitivities at the final time \( t_B0 \).
Call  flag = IDACalcICB(ida_mem, which, tBout1, N_Vector yfin, N_Vector ypfin);

Description  The function IDACalcICB corrects the initial values \( y_{B0} \) and \( y_{pB0} \) at time \( t_{B0} \) for the backward problem.

Arguments  
- ida_mem (void *) pointer to the IDAS memory block.
- which (int) is the identifier of the backward problem.
- tBout1 (realtype) is the first value of \( t \) at which a solution will be requested (from IDASolveB). This value is needed here only to determine the direction of integration and rough scale in the independent variable \( t \).
- yfin (N_Vector) the forward solution at the final time \( t_{B0} \).
- ypfin (N_Vector) the forward solution derivative at the final time \( t_{B0} \).

Return value  The return value \( flag \) (of type int) can be any that is returned by IDACalcIC (see §4.5.5). However IDACalcICB can also return one of the following:

- IDA_NO_ADJ  IDAAdjInit has not been previously called.
- IDA_ILL_INPUT Parameter which represented an invalid identifier.

Notes  All failure return values are negative and therefore a test \( flag < 0 \) will trap all IDACalcICB failures.

Note that IDACalcICB will correct the values of \( y_B(t_{B0}) \) and \( y_B(t_{B0}) \) which were specified in the previous call to IDAInitB or IDAREInitB. To obtain the corrected values, call IDAGetconsistentICB (see §6.2.10.2).

F2003 Name  FIDACalcICB

In the case where the backward problem also depends on the forward sensitivities, user must call the following function to correct the initial conditions:

Call  flag = IDACalcICBS(ida_mem, which, tBout1, N_Vector yfin, N_Vector ypfin, N_Vector ySfin, N_Vector ypSfin);

Description  The function IDACalcICBS corrects the initial values \( y_{B0} \) and \( y_{pB0} \) at time \( t_{B0} \) for the backward problem.

Arguments  
- ida_mem (void *) pointer to the IDAS memory block.
- which (int) is the identifier of the backward problem.
- tBout1 (realtype) is the first value of \( t \) at which a solution will be requested (from IDASolveB). This value is needed here only to determine the direction of integration and rough scale in the independent variable \( t \).
- yfin (N_Vector) the forward solution at the final time \( t_{B0} \).
- ypfin (N_Vector) the forward solution derivative at the final time \( t_{B0} \).
- ySfin (N_Vector *) a pointer to an array of \( N_s \) vectors containing the sensitivities of the forward solution at the final time \( t_{B0} \).
- ypSfin (N_Vector *) a pointer to an array of \( N_s \) vectors containing the derivatives of the forward solution sensitivities at the final time \( t_{B0} \).

Return value  The return value \( flag \) (of type int) can be any that is returned by IDACalcIC (see §4.5.5). However IDACalcICBS can also return one of the following:

- IDA_NO_ADJ  IDAAdjInit has not been previously called.
- IDA_ILL_INPUT Parameter which represented an invalid identifier, sensitivities were not active during forward integration, or IDAInitBS (or IDAREInitBS) has not been previously called.
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Notes

All failure return values are negative and therefore a test flag < 0 will trap all IDACalcICBS failures.

Note that IDACalcICBS will correct the values of $y_B(t_B)$ and $\dot{y}_B(t_B)$ which were specified in the previous call to IDAInitBS or IDAReInitBS. To obtain the corrected values, call IDAGetConsistentICB (see §6.2.10.2).

F2003 Name  FIDACalcICBS

6.2.8 Backward integration function

The function IDASolveB performs the integration of the backward problem. It is essentially a wrapper for the IDAS main integration function IDASolve and, in the case in which checkpoints were needed, it evolves the solution of the backward problem through a sequence of forward-backward integration pairs between consecutive checkpoints. In each pair, the first run integrates the original IVP forward in time and stores interpolation data; the second run integrates the backward problem backward in time and performs the required interpolation to provide the solution of the IVP to the backward problem.

The function IDASolveB does not return the solution $y_B$ itself. To obtain that, call the function IDAGetB, which is also described below.

The IDASolveB function does not support rootfinding, unlike IDASolveF, which supports the finding of roots of functions of $(t, y, \dot{y})$. If rootfinding was performed by IDASolveF, then for the sake of efficiency, it should be disabled for IDASolveB by first calling IDARootInit with nrtfn = 0.

The call to IDASolveB has the form

```
IDASolveB
```

<table>
<thead>
<tr>
<th>Call</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>flag = IDASolveB(ida_mem, tBout, itaskB);</td>
<td>The function IDASolveB integrates the backward DAE problem.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Arguments</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>ida_mem (void *) pointer to the IDAS memory returned by IDACreate.</td>
<td></td>
</tr>
<tr>
<td>tBout (realtype) the next time at which a computed solution is desired.</td>
<td></td>
</tr>
<tr>
<td>itaskB (int) a flag indicating the job of the solver for the next step.</td>
<td>The IDA_NORMAL task is to have the solver take internal steps until it has reached or just passed the user-specified value tBout. The solver then interpolates in order to return an approximate value of $y_B(t_B)$. The IDA_ONE_STEP option tells the solver to take just one internal step in the direction of tBout and return.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Return value</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>The return value flag (of type int) will be one of the following. For more details see §4.5.7.</td>
<td></td>
</tr>
</tbody>
</table>

- IDA_SUCCESS       IDASolveB succeeded.
- IDA_MEM_NULL      The ida_mem was NULL.
- IDA_NO_ADJ        The function IDAAdjInit has not been previously called.
- IDA_NO_BCK        No backward problem has been added to the list of backward problems by a call to IDACreateB
- IDA_NO_FWD        The function IDASolveF has not been previously called.
- IDA_Ill_INPUT     One of the inputs to IDASolveB is illegal.
- IDA_BAD_ITASK     The itaskB argument has an illegal value.
- IDA_TOO_MUCH_WORK The solver took mxstep internal steps but could not reach tBout.
- IDA_TOO_MUCH_ACC  The solver could not satisfy the accuracy demanded by the user for some internal step.
- IDA_ERR_FAILURE   Error test failures occurred too many times during one internal time step.
- IDA_CONV_FAILURE  Convergence test failures occurred too many times during one internal time step.
IDA_LSETUP_FAIL  The linear solver’s setup function failed in an unrecoverable manner.
IDA_SOLVE_FAIL   The linear solver’s solve function failed in an unrecoverable manner.
IDA_BCKMEM_NULL  The IDAS memory for the backward problem was not created with a call to IDACreateB.
IDA_BAD_TBOUT    The desired output time tBout is outside the interval over which the forward problem was solved.
IDA_REIFWD_FAIL  Reinitialization of the forward problem failed at the first checkpoint (corresponding to the initial time of the forward problem).
IDA_FWD_FAIL     An error occurred during the integration of the forward problem.

Notes  All failure return values are negative and therefore a test flag < 0 will trap all IDASolveB failures.

In the case of multiple checkpoints and multiple backward problems, a given call to IDASolveB in IDA_ONE_STEP mode may not advance every problem one step, depending on the relative locations of the current times reached. But repeated calls will eventually advance all problems to tBout.

F2003 Name FIDASolveB
To obtain the solution yB to the backward problem, call the function IDAGetB as follows:

```
IDAGetB
```

Call   flag = IDAGetB(ida_mem, which, &tret, yB, ypB);

Description The function IDAGetB provides the solution yB of the backward DAE problem.

Arguments  ida_mem (void *) pointer to the IDAS memory returned by IDACreate.
   which (int) the identifier of the backward problem.
   tret (realtype) the time reached by the solver (output).
   yB (N_Vector) the backward solution at time tret.
   ypB (N_Vector) the backward solution derivative at time tret.

Return value The return value flag (of type int) will be one of the following.
   IDA_SUCCESS  IDAGetB was successful.
   IDA_MEM_NULL ida_mem is NULL.
   IDA_NO_ADJ   The function IDAAdjInit has not been previously called.
   IDA_IILL_INPUT The parameter which is an invalid identifier.

Notes The user must allocate space for yB and ypB.

To obtain the solution associated with a given backward problem at some other time within the last integration step, first obtain a pointer to the proper IDAS memory structure by calling IDAGetAdjIDABmem and then use it to call IDAGetDky.

F2003 Name FIDAGetB

6.2.9  Optional input functions for the backward problem

6.2.9.1  Main solver optional input functions

The adjoint module in IDAS provides wrappers for most of the optional input functions defined in §4.5.8.1. The only difference is that the user must specify the identifier which of the backward problem within the list managed by IDAS.

The optional input functions defined for the backward problem are:
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flag = IDASetNonlinearSolverB(ida_mem, which, NLSB);
flag = IDASetUserDataB(ida_mem, which, user_dataB);
flag = IDASetMaxOrdB(ida_mem, which, maxordB);
flag = IDASetMaxNumStepsB(ida_mem, which, mxstepsB);
flag = IDASetInitStepB(ida_mem, which, hinB);
flag = IDASetMaxStepB(ida_mem, which, hmaxB);
flag = IDASetSuppressAlgB(ida_mem, which, suppressalgB);
flag = IDASetIdB(ida_mem, which, idB);
flag = IDASetConstraintsB(ida_mem, which, constraintsB);

Their return value flag (of type int) can have any of the return values of their counterparts, but it can also be IDA_NO_ADJ if IDAAdjInit has not been called, or IDA_ILL_INPUT if which was an invalid identifier.

6.2.9.2 Linear solver interface optional input functions

When using matrix-based linear solver modules for the backward problem, i.e., a non-NULL SUNMATRIX object A was passed to IDASetLinearSolverB, the idals linear solver interface needs a function to compute an approximation to the Jacobian matrix. This can be attached through a call to either IDASetJacFnB or IDASetJacFnBS, with the second used when the backward problem depends on the forward sensitivities.

**IDASetJacFnB**

Call flag = IDASetJacFnB(ida_mem, which, jacB);

Description The function IDASetJacFnB specifies the Jacobian approximation function to be used for the backward problem.

Arguments

- **ida_mem** (void *) pointer to the IDAS memory block.
- **which** (int) represents the identifier of the backward problem.
- **jacB** (IDALsJacFnB) user-defined Jacobian approximation function.

Return value The return value flag (of type int) is one of

- IDALS_SUCCESS IDASetJacFnB succeeded.
- IDALS_MEM_NULL The ida_mem was NULL.
- IDALS_NO_ADJ The function IDAAdjInit has not been previously called.
- IDALS_LMEM_NULL The linear solver has not been initialized with a call to IDASetLinearSolverB.
- IDALS_ILL_INPUT The parameter which represented an invalid identifier.

Notes The function type IDALsJacFnB is described in §6.3.5.

The previous routine IDALsSetJacFnB is now a wrapper for this routine, and may still be used for backward-compatibility. However, this will be deprecated in future releases, so we recommend that users transition to the new routine name soon.

F2003 Name FIDASetJacFnB

**IDASetJacFnBS**

Call flag = IDASetJacFnBS(ida_mem, which, jacBS);

Description The function IDASetJacFnBS specifies the Jacobian approximation function to be used for the backward problem in the case where the backward problem depends on the forward sensitivities.

Arguments

- **ida_mem** (void *) pointer to the IDAS memory block.
- **which** (int) represents the identifier of the backward problem.
jacBS (IDALJacFnBS) user-defined Jacobian approximation function.

Return value The return value flag (of type int) is one of:
- IDALS_SUCCESS IDASetJacFnBS succeeded.
- IDALS_MEM_NULL The ida_mem was NULL.
- IDALS_NO_ADJ The function IDAAdjInit has not been previously called.
- IDALS_LMEM_NULL The linear solver has not been initialized with a call to IDASetLinearSolverBS.
- IDALS_IILL_INPUT The parameter which represented an invalid identifier.

Notes The function type IDALsJacFnBS is described in §6.3.5.

The previous routine IDAlsSetJacFnBS is now a wrapper for this routine, and may still be used for backward-compatibility. However, this will be deprecated in future releases, so we recommend that users transition to the new routine name soon.

F2003 Name FIDASetJacFnBS

When using a matrix-free linear solver module for the backward problem, the IDALS linear solver interface requires a function to compute an approximation to the product between the Jacobian matrix $J(t,y)$ and a vector $v$. This may be performed internally using a difference-quotient approximation, or it may be supplied by the user by calling one of the following two functions:

**IDASetJacTimesBS**

Call flag = IDASetJacTimesBS(ida_mem, which, jsetupBS, jtimesBS);

Description The function IDASetJacTimesBS specifies the Jacobian-vector product setup and evaluation functions to be used, in the case where the backward problem depends on the forward sensitivities.

Arguments ida_mem (void *) pointer to the IDAS memory block.
- which (int) the identifier of the backward problem.
- jsetupBS (IDALsJacTimesSetupFnBS) user-defined function to set up the Jacobian-vector product. Pass NULL if no setup is necessary.
- jtimesBS (IDALsJacTimesVecFnBS) user-defined Jacobian-vector product function.

Return value The return value flag (of type int) is one of:
- IDALS_SUCCESS The optional value has been successfully set.
- IDALS_MEM_NULL The ida_mem memory block pointer was NULL.
- IDALS_LMEM_NULL The IDALS linear solver has not been initialized.
- IDALS_NO_ADJ The function IDAAdjInit has not been previously called.
- IDALS_IILL_INPUT The parameter which represented an invalid identifier.

Notes The function types IDALsJacTimesVecFnBS and IDALsJacTimesSetupFnBS are described in §6.3.6.

The previous routine IDASpilsSetJacTimesBS is now a wrapper for this routine, and may still be used for backward-compatibility. However, this will be deprecated in future releases, so we recommend that users transition to the new routine name soon.

F2003 Name FIDASetJacTimesBS
which (int) the identifier of the backward problem.

jtsetupBS (IDALSJacTimesSetupFnBS) user-defined function to set up the Jacobian-vector product. Pass NULL if no setup is necessary.

jtimesBS (IDALSJacTimesVecFnBS) user-defined Jacobian-vector product function.

Return value The return value flag (of type int) is one of:

- IDALS_SUCCESS: The optional value has been successfully set.
- IDALS_MEM_NULL: The ida_mem memory block pointer was NULL.
- IDALS_LMEM_NULL: The IDALS linear solver has not been initialized.
- IDALS_NO_ADJ: The function IDAAdjInit has not been previously called.
- IDALS_ILL_INPUT: The parameter which represented an invalid identifier.

Notes The function types IDALSJacTimesVecFnBS and IDALSJacTimesSetupFnBS are described in §6.3.6.

The previous routine IDASpilsSetJacTimesBS is now a wrapper for this routine, and may still be used for backward-compatibility. However, this will be deprecated in future releases, so we recommend that users transition to the new routine name soon.

F2003 Name FIDASetJacTimesBS

Alternately, when using the default difference-quotient approximation to the Jacobian-vector product for the backward problem, the user may specify the factor to use in setting increments for the finite-difference approximation, via a call to IDASetIncrementFactorB:

```
IDASOptimizerB
```

Call flag = IDASetIncrementFactorB(ida_mem, which, dqincfacB);

Description The function IDASetIncrementFactorB specifies the factor in the increments used in the difference quotient approximations to matrix-vector products for the backward problem.

This routine can be used in both the cases where the backward problem does and does not depend on the forward sensitivities.

Arguments ida_mem (void *) pointer to the IDAS memory block.
which (int) the identifier of the backward problem.

dqincfacB (realtype) difference quotient approximation factor.

Return value The return value flag (of type int) is one of

- IDALS_SUCCESS: The optional value has been successfully set.
- IDALS_MEM_NULL: The ida_mem pointer is NULL.
- IDALS_LMEM_NULL: The IDALS linear solver has not been initialized.
- IDALS_NO_ADJ: The function IDAAdjInit has not been previously called.
- IDALS_ILL_INPUT: The value of eplifacB is negative.
- IDALS_ILL_INPUT: The parameter which represented an invalid identifier.

Notes The default value is 1.0.

The previous routine IDASpilsSetIncrementFactorB is now a wrapper for this routine, and may still be used for backward-compatibility. However, this will be deprecated in future releases, so we recommend that users transition to the new routine name soon.

F2003 Name FIDASetIncrementFactorB

When using an iterative linear solver for the backward problem, the user may supply a preconditioning operator to aid in solution of the system, or she/he may adjust the convergence tolerance factor for the iterative linear solver. These may be accomplished through calling the following functions:
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**IDASSetPreconditionerB**

Call

```
flag = IDASSetPreconditionerB(ida_mem, which, psetupB, psolveB);
```

Description

The function `IDASSetPreconditionerB` specifies the preconditioner setup and solve functions for the backward integration.

Arguments

- `ida_mem` (void *) pointer to the IDAS memory block.
- `which` (int) the identifier of the backward problem.
- `psetupB` (IDALsPrecSetupFnB) user-defined preconditioner setup function.
- `psolveB` (IDALsPrecSolveFnB) user-defined preconditioner solve function.

Return value

The return value `flag` (of type `int`) is one of:

- `IDALS_SUCCESS` The optional value has been successfully set.
- `IDALS_MEM_NULL` The `ida_mem` memory block pointer was NULL.
- `IDALS_LMEM_NULL` The IDALS linear solver has not been initialized.
- `IDALS_NO_ADJ` The function `IDAAdjInit` has not been previously called.
- `IDALS_ILL_INPUT` The parameter `which` represented an invalid identifier.

Notes

The function types `IDALsPrecSolveFnB` and `IDALsPrecSetupFnB` are described in §6.3.8 and §6.3.9, respectively. The `psetupB` argument may be NULL if no setup operation is involved in the preconditioner.

The previous routine `IDASpilsSetPreconditionerB` is now a wrapper for this routine, and may still be used for backward-compatibility. However, this will be deprecated in future releases, so we recommend that users transition to the new routine name soon.

F2003 Name: `FIDASetPreconditionerB`

**IDASSetPreconditionerBS**

Call

```
flag = IDASSetPreconditionerBS(ida_mem, which, psetupBS, psolveBS);
```

Description

The function `IDASSetPreconditionerBS` specifies the preconditioner setup and solve functions for the backward integration, in the case where the backward problem depends on the forward sensitivities.

Arguments

- `ida_mem` (void *) pointer to the IDAS memory block.
- `which` (int) the identifier of the backward problem.
- `psetupBS` (IDALsPrecSetupFnBS) user-defined preconditioner setup function.
- `psolveBS` (IDALsPrecSolveFnBS) user-defined preconditioner solve function.

Return value

The return value `flag` (of type `int`) is one of:

- `IDALS_SUCCESS` The optional value has been successfully set.
- `IDALS_MEM_NULL` The `ida_mem` memory block pointer was NULL.
- `IDALS_LMEM_NULL` The IDALS linear solver has not been initialized.
- `IDALS_NO_ADJ` The function `IDAAdjInit` has not been previously called.
- `IDALS_ILL_INPUT` The parameter `which` represented an invalid identifier.

Notes

The function types `IDALsPrecSolveFnBS` and `IDALsPrecSetupFnBS` are described in §6.3.8 and §6.3.9, respectively. The `psetupBS` argument may be NULL if no setup operation is involved in the preconditioner.

The previous routine `IDASpilsSetPreconditionerBS` is now a wrapper for this routine, and may still be used for backward-compatibility. However, this will be deprecated in future releases, so we recommend that users transition to the new routine name soon.

F2003 Name: `FIDASetPreconditionerBS`
6.2 User-callable functions for adjoint sensitivity analysis

**IDASetEpsLinB**

Call

flag = IDASetEpsLinB(ida_mem, which, eplifacB);

Description

The function IDASetEpsLinB specifies the factor by which the Krylov linear solver’s convergence test constant is reduced from the nonlinear iteration test constant. (See §2.1). This routine can be used in both the cases where the backward problem does and does not depend on the forward sensitivities.

Arguments

ida_mem (void *) pointer to the IDAS memory block.
which (int) the identifier of the backward problem.
eplifacB (realtype) linear convergence safety factor (\(\geq 0\)).

Return value

The return value flag (of type int) is one of

- IDALS_SUCCESS: The optional value has been successfully set.
- IDALS_MEM_NULL: The ida_mem pointer is NULL.
- IDALS_LMEM_NULL: The IDALS linear solver has not been initialized.
- IDALS_NO_ADJ: The function IDAAdjInit has not been previously called.
- IDALS_Ill_INPUT: The value of eplifacB is negative.
- IDALS_Ill_INPUT: The parameter which represented an invalid identifier.

Notes

The default value is 0.05.
Passing a value eplifacB = 0.0 also indicates using the default value.
The previous routine IDASpilsSetEpsLinB is now a wrapper for this routine, and may still be used for backward-compatibility. However, this will be deprecated in future releases, so we recommend that users transition to the new routine name soon.

F2003 Name FIDASetEpsLinB

### 6.2.10 Optional output functions for the backward problem

#### 6.2.10.1 Main solver optional output functions

The user of the adjoint module in IDAS has access to any of the optional output functions described in §4.5.10, both for the main solver and for the linear solver modules. The first argument of these IDAGet* and IDA*Get* functions is the pointer to the IDAS memory block for the backward problem. In order to call any of these functions, the user must first call the following function to obtain this pointer:

**IDAGetAdjIDABmem**

Call

ida_memB = IDAGetAdjIDABmem(ida_mem, which);

Description

The function IDAGetAdjIDABmem returns a pointer to the IDAS memory block for the backward problem.

Arguments

ida_mem (void *) pointer to the IDAS memory block created by IDACreate.
which (int) the identifier of the backward problem.

Return value

The return value, ida_memB (of type void *), is a pointer to the IDAS memory for the backward problem.

Notes

The user should not modify ida_memB in any way.

Optional output calls should pass ida_memB as the first argument; thus, for example, to get the number of integration steps: flag = IDAGetNumSteps(idas_memB, &nsteps).

F2003 Name FIDAGetAdjIDABmem

To get values of the forward solution during a backward integration, use the following function. The input value of \(t\) would typically be equal to that at which the backward solution has just been obtained with IDASolveB. In any case, it must be within the last checkpoint interval used by IDASolveB.
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**IDAGetAdjY**

Call

```c
flag = IDAGetAdjY(ida_mem, t, y, yp);
```

Description

The function `IDAGetAdjY` returns the interpolated value of the forward solution \( y \) and its derivative during a backward integration.

Arguments

- `ida_mem` (void *) pointer to the IDAS memory block created by `IDACreate`.
- `t` (realtype) value of the independent variable at which \( y \) is desired (input).
- `y` (N_Vector) forward solution \( y(t) \).
- `yp` (N_Vector) forward solution derivative \( \dot{y}(t) \).

Return value

The return value `flag` (of type int) is one of:

- `IDA_SUCCESS` `IDAGetAdjY` was successful.
- `IDA_MEM_NULL` `ida_mem` was NULL.
- `IDA_GETY_BADT` The value of \( t \) was outside the current checkpoint interval.

Notes

The user must allocate space for \( y \) and \( yp \).

F2003 Name `FIDAGetAdjY`

**IDAGetAdjCheckPointsInfo**

Call

```c
flag = IDAGetAdjCheckPointsInfo(ida_mem, IDAadjCheckPointRec *ckpnt);
```

Description

The function `IDAGetAdjCheckPointsInfo` loads an array of `ncheck+1` records of type `IDAadjCheckPointRec`. The user must allocate space for the array `ckpnt`.

Arguments

- `ida_mem` (void *) pointer to the IDAS memory block created by `IDACreate`.
- `ckpnt` (IDAadjCheckPointRec *) array of `ncheck+1` checkpoint records, each of type `IDAadjCheckPointRec`.

Return value

The return value is `IDA_SUCCESS` if successful, or `IDA_MEM_NULL` if `ida_mem` is NULL, or `IDA_NO_ADJ` if ASA was not initialized.

Notes

The members of each record `ckpnt[i]` are:

- `ckpnt[i].my_addr` (void *) address of current checkpoint in `ida_mem->ida_adj_mem`
- `ckpnt[i].next_addr` (void *) address of next checkpoint
- `ckpnt[i].t0` (realtype) start of checkpoint interval
- `ckpnt[i].t1` (realtype) end of checkpoint interval
- `ckpnt[i].nstep` (long int) step counter at checkpoint \( t0 \)
- `ckpnt[i].order` (int) method order at checkpoint \( t0 \)
- `ckpnt[i].step` (realtype) step size at checkpoint \( t0 \)

F2003 Name `FIDAGetAdjCheckPointsInfo`

6.2.10.2 Initial condition calculation optional output function

**IDAGetConsistentICB**

Call

```c
flag = IDAGetConsistentICB(ida_mem, which, yB0_mod, ypB0_mod);
```

Description

The function `IDAGetConsistentICB` returns the corrected initial conditions for backward problem calculated by `IDACalcICB`.

Arguments

- `ida_mem` (void *) pointer to the IDAS memory block.
- `which` is the identifier of the backward problem.
- `yB0_mod` (N_Vector) consistent initial vector.
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\( \text{ypB}_0 \text{\_mod (N\_Vector) consistent initial derivative vector.} \)

Return value The return value \text{flag} (of type \text{int}) is one of

- \text{IDA\_SUCCESS}: The optional output value has been successfully set.
- \text{IDA\_MEM\_NULL}: The \text{idamem} pointer is NULL.
- \text{IDA\_NO\_ADJ}: \text{IDAAdjInit} has not been previously called.
- \text{IDA\_ILL\_INPUT}: Parameter which did not refer a valid backward problem identifier.

Notes If the consistent solution vector or consistent derivative vector is not desired, pass NULL
for the corresponding argument.

The user must allocate space for \text{yB}_0 \text{\_mod and ypB}_0 \text{\_mod (if not NULL).}

F2003 Name FIDAGetConsistentICB

6.2.11 Backward integration of quadrature equations

Not only the backward problem but also the backward quadrature equations may or may not depend on
the forward sensitivities. Accordingly, one of the \text{IDAQuadInitB} or \text{IDAQuadInitBS} should be used to
allocate internal memory and to initialize backward quadratures. For any other operation (extraction,
onoptional input/output, reinitialization, deallocation), the same function is called regardless of whether
or not the quadratures are sensitivity-dependent.

6.2.11.1 Backward quadrature initialization functions

The function \text{IDAQuadInitB} initializes and allocates memory for the backward integration of quadra-
ture equations that do not depend on forward sensitivities. It has the following form:

\begin{verbatim}
IDAQuadInitB
Call
    flag = IDAQuadInitB(ida_mem, which, rhsQB, yQB0);
Description
    The function IDAQuadInitB provides required problem specifications, allocates internal
    memory, and initializes backward quadrature integration.
Arguments
    ida_mem (void *) pointer to the idas memory block.
    which (int) the identifier of the backward problem.
    rhsQB (IDAQuadRhsFnB) is the C function which computes \( fQB \), the residual of the
    backward quadrature equations. This function has the form \( rhsQB(t, y, yp, yB, ypB, rhsvalBQ, user_dataB) \)
    (see §6.3.3).
    yQB0 (N\_Vector) is the value of the quadrature variables at \( tB_0 \).
Return value
    The return value flag (of type int) will be one of the following:
    - IDA\_SUCCESS: The call to IDAQuadInitB was successful.
    - IDA\_MEM\_NULL: ida_mem was NULL.
    - IDA\_NO\_ADJ: The function IDAAdjInit has not been previously called.
    - IDA\_MEM\_FAIL: A memory allocation request has failed.
    - IDA\_ILL\_INPUT: The parameter which is an invalid identifier.
\end{verbatim}

F2003 Name FIDAQuadInitB

The function \text{IDAQuadInitBS} initializes and allocates memory for the backward integration of
quadrature equations that depend on the forward sensitivities.
**IDAQuadInitBS**

**Call**

```cpp
flag = IDAQuadInitBS(ida_mem, which, rhsQBS, yQBS0);
```

**Description**
The function `IDAQuadInitBS` provides required problem specifications, allocates internal memory, and initializes backward quadrature integration.

**Arguments**
- `ida_mem` *(void *)* pointer to the IDAS memory block.
- `which` *(int)* the identifier of the backward problem.
- `rhsQBS` *(IDAQuadRhsFnBS)* is the C function which computes \( f_{QBS} \), the residual of the backward quadrature equations. This function has the form `rhsQBS(t, y, yp, yS, ypS, yB, ypB, rhsvalBQS, user_dataB)` (see §6.3.4).
- `yQBS0` *(N_Vector)* is the value of the sensitivity-dependent quadrature variables at \( tB_0 \).

**Return value**
The return value `flag` (of type `int`) will be one of the following:
- `IDA_SUCCESS` The call to `IDAQuadInitBS` was successful.
- `IDA_MEM_NULL` `ida_mem` was NULL.
- `IDA_NO_ADJ` The function `IDAAdjInit` has not been previously called.
- `IDA_MEM_FAIL` A memory allocation request has failed.
- `IDA_IILL_INPUT` The parameter `which` is an invalid identifier.

**Notes**
`IDAQuadReInitB` can be used after a call to either `IDAQuadInitB` or `IDAQuadInitBS`.

**F2003 Name**
`FIDAQuadInitBS`

The integration of quadrature equations during the backward phase can be re-initialized by calling the following function. Before calling `IDAQuadReInitB` for a new backward problem, call any desired solution extraction functions `IDAGet***` associated with the previous backward problem.

**IDAQuadReInitB**

**Call**

```cpp
flag = IDAQuadReInitB(ida_mem, which, yQB0);
```

**Description**
The function `IDAQuadReInitB` re-initializes the backward quadrature integration.

**Arguments**
- `ida_mem` *(void *)* pointer to the IDAS memory block.
- `which` *(int)* the identifier of the backward problem.
- `yQB0` *(N_Vector)* is the value of the quadrature variables at \( tB_0 \).

**Return value**
The return value `flag` (of type `int`) will be one of the following:
- `IDA_SUCCESS` The call to `IDAQuadReInitB` was successful.
- `IDA_MEM_NULL` `ida_mem` was NULL.
- `IDA_NO_ADJ` The function `IDAAdjInit` has not been previously called.
- `IDA_MEM_FAIL` A memory allocation request has failed.
- `IDA_NO_QUAD` Quadrature integration was not activated through a previous call to `IDAQuadInitB`.
- `IDA_IILL_INPUT` The parameter `which` is an invalid identifier.

**Notes**
`IDAQuadReInitB` can be used after a call to either `IDAQuadInitB` or `IDAQuadInitBS`.

**F2003 Name**
`FIDAQuadReInitB`

### 6.2.11.2 Backward quadrature extraction function

To extract the values of the quadrature variables at the last return time of `IDASolveB`, IDAS provides a wrapper for the function `IDAGetQuad` (see §4.7.3). The call to this function has the form
6.3 User-supplied functions for adjoint sensitivity analysis

**IDAGetQuadB**

Call: \[ \text{flag} = \text{IDAGetQuadB}(\text{ida}_{\text{mem}}, \text{which}, \&\text{tret}, \text{yQB}); \]

Description: The function **IDAGetQuadB** returns the quadrature solution vector after a successful return from **IDASolveB**.

Arguments:
- **ida_{mem}**: (void *) pointer to the IDAS memory.
- **tret**: (realtype) the time reached by the solver (output).
- **yQB**: (N_Vector) the computed quadrature vector.

Return value: The return value **flag** of **IDAGetQuadB** is one of:
- **IDA_SUCCESS**: **IDAGetQuadB** was successful.
- **IDA_MEM_NULL**: **ida_{mem}** is NULL.
- **IDA_NO_ADJ**: The function **IDAAdjInit** has not been previously called.
- **IDA_NO_QUAD**: Quadrature integration was not initialized.
- **IDA_BAD_DKY**: **yQB** was NULL.
- **IDA_ILL_INPUT**: The parameter **which** is an invalid identifier.

Notes:
- The user must allocate space for **yQB**.
- To obtain the quadratures associated with a given backward problem at some other time within the last integration step, first obtain a pointer to the proper IDAS memory structure by calling **IDAGetAdjIDABmem** and then use it to call **IDAGetQuadDky**.

F2003 Name: **FIDAGetQuadB**

6.2.11.3 Optional input/output functions for backward quadrature integration

Optional values controlling the backward integration of quadrature equations can be changed from their default values through calls to one of the following functions which are wrappers for the corresponding optional input functions defined in \(\S\) 4.7.4. The user must specify the identifier **which** of the backward problem for which the optional values are specified.

\[ \text{flag} = \text{IDASetQuadErrConB}(\text{ida}_{\text{mem}}, \text{which}, \text{errconQ}); \]
\[ \text{flag} = \text{IDAQuadSSStolerancesB}(\text{ida}_{\text{mem}}, \text{which}, \text{reltolQ}, \text{abstolQ}); \]
\[ \text{flag} = \text{IDAQuadSVtolerancesB}(\text{ida}_{\text{mem}}, \text{which}, \text{reltolQ}, \text{abstolQ}); \]

Their return value **flag** (of type int) can have any of the return values of its counterparts, but it can also be **IDA_NO_ADJ** if the function **IDAAdjInit** has not been previously called or **IDA_ILL_INPUT** if the parameter **which** was an invalid identifier.

Access to optional outputs related to backward quadrature integration can be obtained by calling the corresponding **IDAGetQuadB** functions (see \(\S\) 4.7.5). A pointer **ida_{mem}** to the IDAS memory block for the backward problem, required as the first argument of these functions, can be obtained through a call to the functions **IDAGetAdjIDABmem** (see \(\S\) 6.2.10).

6.3 User-supplied functions for adjoint sensitivity analysis

In addition to the required DAE residual function and any optional functions for the forward problem, when using the adjoint sensitivity module in IDAS, the user must supply one function defining the backward problem DAE and, optionally, functions to supply Jacobian-related information and one or two functions that define the preconditioner (if applicable for the choice of SUNLINSOL object) for the backward problem. Type definitions for all these user-supplied functions are given below.

6.3.1 DAE residual for the backward problem

The user must provide a **resB** function of type **IDAResFnB** defined as follows:
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**IDAResFnB**

**Definition**

typedef int (*IDAResFnB)(realtype t, N_Vector y, N_Vector yp, N_Vector yB, N_Vector ypB, N_Vector resvalB, void *user dataB);

**Purpose**

This function evaluates the residual of the backward problem DAE system. This could be (2.20) or (2.25).

**Arguments**

- **t** is the current value of the independent variable.
- **y** is the current value of the forward solution vector.
- **yp** is the current value of the forward solution derivative vector.
- **yB** is the current value of the backward dependent variable vector.
- **ypB** is the current value of the backward dependent derivative vector.
- **resvalB** is the output vector containing the residual for the backward DAE problem.

**user dataB** is a pointer to user data, same as passed to IDASetUserDataB.

**Return value**

An **IDAResFnB** should return 0 if successful, a positive value if a recoverable error occurred (in which case IDAS will attempt to correct), or a negative value if an unrecoverable failure occurred (in which case the integration stops and IDASolveB returns IDA RESFUNC_FAIL).

**Notes**

Allocation of memory for **resvalB** is handled within IDAS.

The **y**, **yp**, **yB**, and **ypB** arguments are all of type **N_Vector**, but **yB**, **ypB**, and **resvalB** typically have different internal representations from **y** and **yp**. It is the user’s responsibility to access the vector data consistently (including the use of the correct accessor macros from each NVECTOR implementation). For the sake of computational efficiency, the vector functions in the two NVECTOR implementations provided with IDAS do not perform any consistency checks with respect to their **N_Vector** arguments (see §8.3 and §8.4).

The **user dataB** pointer is passed to the user’s **resB** function every time it is called and can be the same as the **user data** pointer used for the forward problem.

Before calling the user’s **resB** function, IDAS needs to evaluate (through interpolation) the values of the states from the forward integration. If an error occurs in the interpolation, IDAS triggers an unrecoverable failure in the residual function which will halt the integration and IDASolveB will return IDA RESFUNC_FAIL.

**6.3.2 DAE residual for the backward problem depending on the forward sensitivities**

The user must provide a **resBS** function of type **IDAResFnBS** defined as follows:

**IDAResFnBS**

**Definition**

typedef int (*IDAResFnBS)(realtype t, N_Vector y, N_Vector yp, N_Vector *yS, N_Vector *ypS, N_Vector yB, N_Vector ypB, N_Vector resvalB, void *user dataB);

**Purpose**

This function evaluates the residual of the backward problem DAE system. This could be (2.20) or (2.25).

**Arguments**

- **t** is the current value of the independent variable.
- **y** is the current value of the forward solution vector.
- **yp** is the current value of the forward solution derivative vector.
- **yS** is a pointer to an array of **Ns** vectors containing the sensitivities of the forward solution.
6.3 User-supplied functions for adjoint sensitivity analysis

ypS a pointer to an array of Ns vectors containing the derivatives of the forward
sensitivities.

yB is the current value of the backward dependent variable vector.

ypB is the current value of the backward dependent derivative vector.

resvalB is the output vector containing the residual for the backward DAE problem.

user_dataB is a pointer to user data, same as passed to IDASetUserDataB.

Return value An IDASResFnBS should return 0 if successful, a positive value if a recoverable error
occurred (in which case IDAS will attempt to correct), or a negative value if an unre-
coverable error occurred (in which case the integration stops and IDASolveB returns
IDA_RESFUNC_FAIL).

Notes Allocation of memory for resvalB is handled within IDAS.

The y, yp, yB, ypB, and resvalB arguments are all of type N_Vector, but yB, ypB, and
resvalB typically have different internal representations from y and yp. Likewise
for each yS[i] and ypS[i]. It is the user’s responsibility to access the vector data
consistently (including the use of the correct accessor macros from each NVECTOR im-
plementation). For the sake of computational efficiency, the vector functions in the two
NVECTOR implementations provided with IDAS do not perform any consistency checks
with respect to their N_Vector arguments (see §8.3 and §8.4).

The user_dataB pointer is passed to the user’s resBS function every time it is called
and can be the same as the user_data pointer used for the forward problem.

Before calling the user’s resBS function, IDAS needs to evaluate (through interpolation)
the values of the states from the forward integration. If an error occurs in the inter-
polation, IDAS triggers an unrecoverable failure in the residual function which will halt
the integration and IDASolveB will return IDA_RESFUNC_FAIL.

6.3.3 Quadrature right-hand side for the backward problem

The user must provide an fQB function of type IDAQuadRhsFnB defined by

```
typedef int (*IDAQuadRhsFnB)(realtype t, N_Vector y, N_Vector yp,
                          N_Vector yB, N_Vector ypB,
                          N_Vector rhsvalB, void *user_data);
```

Definition This function computes the quadrature equation right-hand side for the backward prob-
lem.

Arguments t is the current value of the independent variable.
y is the current value of the forward solution vector.
yp is the current value of the forward solution derivative vector.
yB is the current value of the backward dependent variable vector.
ypB is the current value of the backward dependent derivative vector.
rhsvalB is the output vector containing the residual for the backward quadrature
equations.

user_dataB is a pointer to user data, same as passed to IDASetUserDataB.

Return value An IDAQuadRhsFnB should return 0 if successful, a positive value if a recoverable er-
or occurred (in which case IDAS will attempt to correct), or a negative value if it
failed unrecoverably (in which case the integration is halted and IDASolveB returns
IDA_QRHSFUNC_FAIL).
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Notes  
Allocation of memory for rhsvalBQ is handled within IDAS.

The y, yp, yB, ypB, and rhsvalBQ arguments are all of type N_Vector, but they typically all have different internal representations. It is the user’s responsibility to access the vector data consistently (including the use of the correct accessor macros from each NVECTOR implementation). For the sake of computational efficiency, the vector functions in the two NVECTOR implementations provided with IDAS do not perform any consistency checks with respect to their N_Vector arguments (see §8.3 and §8.4).

The user_dataB pointer is passed to the user’s fQB function every time it is called and can be the same as the user_data pointer used for the forward problem.

Before calling the user’s fQB function, IDAS needs to evaluate (through interpolation) the values of the states from the forward integration. If an error occurs in the interpolation, IDAS triggers an unrecoverable failure in the quadrature right-hand side function which will halt the integration and IDASolveB will return IDA_QRHSFUNC_FAIL.

6.3.4 Sensitivity-dependent quadrature right-hand side for the backward problem

The user must provide an fQBS function of type IDAQuadRhsFnBS defined by

```c
typedef int (*IDAQuadRhsFnBS)(realtype t, N_Vector y, N_Vector yp,
                               N_Vector *yS, N_Vector *ypS,
                               N_Vector yB, N_Vector ypB,
                               N_Vector rhsvalBQS, void *user_dataB);
```

Purpose  
This function computes the quadrature equation residual for the backward problem.

Arguments  
- t is the current value of the independent variable.
- y is the current value of the forward solution vector.
- yp is the current value of the forward solution derivative vector.
- yS is a pointer to an array of Ns vectors containing the sensitivities of the forward solution.
- ypS is a pointer to an array of Ns vectors containing the derivatives of the forward sensitivities.
- yB is the current value of the backward dependent variable vector.
- ypB is the current value of the backward dependent derivative vector.
- rhsvalBQS is the output vector containing the residual for the backward quadrature equations.
- user_dataB is a pointer to user data, same as passed to IDASetUserDataB.

Return value  
An IDAQuadRhsFnBS should return 0 if successful, a positive value if a recoverable error occurred (in which case IDAS will attempt to correct), or a negative value if it failed unrecoverably (in which case the integration is halted and IDASolveB returns IDA_QRHSFUNC_FAIL).

Notes  
Allocation of memory for rhsvalBQS is handled within IDAS.

The y, yp, yB, ypB, and rhsvalBQS arguments are all of type N_Vector, but they typically do not all have the same internal representations. Likewise for each yS[i] and ypS[i].

It is the user’s responsibility to access the vector data consistently (including the use of the correct accessor macros from each NVECTOR implementation). For the sake of computational efficiency, the vector functions in the two NVECTOR implementations provided with IDAS do not perform any consistency checks with respect to their N_Vector arguments (see §8.3 and §8.4).
The user data pointer is passed to the user's fQBS function every time it is called and can be the same as the user data pointer used for the forward problem.

Before calling the user's fQBS function, IDAS needs to evaluate (through interpolation) the values of the states from the forward integration. If an error occurs in the interpolation, IDAS triggers an unrecoverable failure in the quadrature right-hand side function which will halt the integration and IDASolveB will return IDA_QRHSFUNC_FAIL.

### 6.3.5 Jacobian construction for the backward problem (matrix-based linear solvers)

If a matrix-based linear solver module is is used for the backward problem (i.e., IDASSetLinearSolverB is called with non-NULL SUNMATRIX argument in the step described in §6.1), the user may provide a function of type IDALsJacFnB or IDALsJacFnBS (see §6.2.9), defined as follows:

**IDALsJacFnB**

**Definition**

```c
#define IDALsJacFnB(ret, arg1, arg2, arg3, arg4, arg5, arg6, arg7, arg8, arg9, arg10, arg11)

typedef int (*IDALsJacFnB)(realtype tt, realtype cjB, N_Vector yy, N_Vector yp, N_Vector yB, N_Vector ypB, N_Vector resvalB, SUNMatrix JacB, void *user_dataB, N_Vector tmp1B, N_Vector tmp2B, N_Vector tmp3B);
```

**Purpose**

This function computes the Jacobian of the backward problem (or an approximation to it).

**Arguments**

- `tt` is the current value of the independent variable.
- `cjB` is the scalar in the system Jacobian, proportional to the inverse of the step size ($\alpha$ in Eq. (2.6)).
- `yy` is the current value of the forward solution vector.
- `yp` is the current value of the forward solution derivative vector.
- `yB` is the current value of the backward dependent variable vector.
- `ypB` is the current value of the backward dependent derivative vector.
- `resvalB` is the current value of the residual for the backward problem.
- `JacB` is the output approximate Jacobian matrix.
- `user_dataB` is a pointer to user data — the parameter passed to IDASSetUserDataB.
- `tmp1B`, `tmp2B`, `tmp3B` are pointers to memory allocated for variables of type `N_Vector` which can be used by the IDALsJacFnB function as temporary storage or work space.

**Return value**

An IDALsJacFnB should return 0 if successful, a positive value if a recoverable error occurred (in which case IDAS will attempt to correct, while IDALS sets last_flag to IDALS_JACFUNC_RECVR), or a negative value if it failed unrecoverably (in which case the integration is halted, IDASolveB returns IDA_LSETUP_FAIL and IDALS sets last_flag to IDALS_JACFUNC_UNRECVR).

**Notes**

A user-supplied Jacobian function must load the matrix JacB with an approximation to the Jacobian matrix at the point $(tt,yy,yB)$, where $yy$ is the solution of the original IVP at time $tt$, and $yB$ is the solution of the backward problem at the same time. Information regarding the structure of the specific SUNMATRIX structure (e.g. number of rows, upper/lower bandwidth, sparsity type) may be obtained through using the implementation-specific SUNMATRIX interface functions (see Chapter 9 for details).
With direct linear solvers (i.e., linear solvers with type \texttt{SUNLINEARSOLVER\_DIRECT}), the Jacobian matrix $J(t, y)$ is zeroed out prior to calling the user-supplied Jacobian function so only nonzero elements need to be loaded into \texttt{JacB}.

Before calling the user’s \texttt{IDALsJacFnB}, IDAS needs to evaluate (through interpolation) the values of the states from the forward integration. If an error occurs in the interpolation, IDAS triggers an unrecoverable failure in the Jacobian function which will halt the integration (\texttt{IDASolveB} returns \texttt{IDAL\_SETUP\_FAIL} and IDALS sets \texttt{last\_flag} to \texttt{IDALS\_JACFUNC\_UNRECVR}).

The previous function type \texttt{IDAD1sJacFnB} is identical to \texttt{IDALsJacFnB}, and may still be used for backward-compatibility. However, this will be deprecated in future releases, so we recommend that users transition to the new function type name soon.

### IDALsJacFnBS

**Definition**

```c
typedef int (*IDALsJacFnBS)(realtype tt, realtype cjB,
                           N_Vector yy, N_Vector yp,
                           N_Vector *yS, N_Vector *ypS,
                           N_Vector yB, N_Vector ypB,
                           N_Vector resvalB,
                           SUNMatrix JacB, void *user_dataB,
                           N_Vector tmp1B, N_Vector tmp2B,
                           N_Vector tmp3B);
```

**Purpose**

This function computes the Jacobian of the backward problem (or an approximation to it), in the case where the backward problem depends on the forward sensitivities.

**Arguments**

- **tt** is the current value of the independent variable.
- **cjB** is the scalar in the system Jacobian, proportional to the inverse of the step size ($\alpha$ in Eq. (2.6)).
- **yy** is the current value of the forward solution vector.
- **yp** is the current value of the forward solution derivative vector.
- **yS** is a pointer to an array of $Ns$ vectors containing the sensitivities of the forward solution.
- **ypS** is a pointer to an array of $Ns$ vectors containing the derivatives of the forward solution sensitivities.
- **yB** is the current value of the backward dependent variable vector.
- **ypB** is the current value of the backward dependent derivative vector.
- **resvalB** is the current value of the residual for the backward problem.
- **JacB** is the output approximate Jacobian matrix.
- **user\_dataB** is a pointer to user data — the parameter passed to \texttt{IDAS\_Set\_UserDataB}.
- **tmp1B**
- **tmp2B**
- **tmp3B** are pointers to memory allocated for variables of type \texttt{N\_Vector} which can be used by \texttt{IDAL\_Jac\_Fn\_BS} as temporary storage or work space.

**Return value**

An \texttt{IDALsJacFnBS} should return 0 if successful, a positive value if a recoverable error occurred (in which case IDAS will attempt to correct, while IDALS sets \texttt{last\_flag} to \texttt{IDALS\_JACFUNC\_RECVR}), or a negative value if it failed unrecoverably (in which case the integration is halted, \texttt{IDAS\_SolveB} returns \texttt{IDAL\_SETUP\_FAIL} and IDALS sets \texttt{last\_flag} to \texttt{IDALS\_JACFUNC\_UNRECVR}).

**Notes**

A user-supplied dense Jacobian function must load the matrix \texttt{JacB} with an approximation to the Jacobian matrix at the point $(tt, yy, yS, yB)$, where $yy$ is the solution of the original IVP at time $tt$, $yS$ is the array of forward sensitivities at time $tt$, and
yB is the solution of the backward problem at the same time. Information regarding the structure of the specific SUNMATRIX structure (e.g., number of rows, upper/lower bandwidth, sparsity type) may be obtained through using the implementation-specific SUNMATRIX interface functions (see Chapter 9 for details).

With direct linear solvers (i.e., linear solvers with type SUNLINEARSOLVER_DIRECT, the Jacobian matrix \( J(t, y) \) is zeroed out prior to calling the user-supplied Jacobian function so only nonzero elements need to be loaded into \( \text{JacB} \).

Before calling the user’s IDALSJacFnBS, IDAS needs to evaluate (through interpolation) the values of the states from the forward integration. If an error occurs in the interpolation, IDAS triggers an unrecoverable failure in the Jacobian function which will halt the integration (IDASolveB returns IDA_LSETUP_FAIL and IDALS sets last_flag to IDALS_JACFUNC_UNRECVR).

The previous function type IDADlsJacFnBS is identical to IDALSJacFnBS, and may still be used for backward-compatibility. However, this will be deprecated in future releases, so we recommend that users transition to the new function type name soon.

### 6.3.6 Jacobian-vector product for the backward problem (matrix-free linear solvers)

If a matrix-free linear solver is selected for the backward problem (i.e., IDASSetLinearSolverB is called with NULL-valued SUNMATRIX argument in the steps described in §6.1), the user may provide a function of type IDALSJacTimesVecFnB or IDALSJacTimesVecFnBS in the following form, to compute matrix-vector products \( Jv \). If such a function is not supplied, the default is a difference quotient approximation to these products.

```c
typedef int (*IDALSJacTimesVecFnB)(realtype t, N_Vector yy, N_Vector yp, N_Vector yB, N_Vector ypB, N_Vector resvalB, N_Vector vB, N_Vector JvB, realtype cjB, void *user_dataB, N_Vector tmp1B, N_Vector tmp2B);
```

**Definition**

This function computes the action of the backward problem Jacobian \( JB \) on a given vector \( vB \).

**Purpose**

- \( t \) is the current value of the independent variable.
- \( yy \) is the current value of the forward solution vector.
- \( yp \) is the current value of the forward solution derivative vector.
- \( yB \) is the current value of the backward dependent variable vector.
- \( ypB \) is the current value of the backward dependent derivative vector.
- \( resvalB \) is the current value of the residual for the backward problem.
- \( vB \) is the vector by which the Jacobian must be multiplied.
- \( JvB \) is the computed output vector, \( JB*vB \).
- \( cjB \) is the scalar in the system Jacobian, proportional to the inverse of the step size (\( \alpha \) in Eq. (2.6)).
- `user_dataB` is a pointer to user data — the same as the `user_dataB` parameter passed to IDASSetUserDataB.
- `tmp1B` and `tmp2B` are pointers to memory allocated for variables of type N_Vector which can be used by IDALSJacTimesVecFnB as temporary storage or work space.
Using IDAS for Adjoint Sensitivity Analysis

Return value The return value of a function of type IDALsJtimesVecFnB should be 0 if successful or nonzero if an error was encountered, in which case the integration is halted.

Notes A user-supplied Jacobian-vector product function must load the vector $Jv_B$ with the product of the Jacobian of the backward problem at the point $(t, y, y_B)$ and the vector $v_B$. Here, $y$ is the solution of the original IVP at time $t$ and $y_B$ is the solution of the backward problem at the same time. The rest of the arguments are equivalent to those passed to a function of type IDALsJacTimesVecFn (see §4.6.6). If the backward problem is the adjoint of $\dot{y} = f(t, y)$, then this function is to compute $-(\partial f/\partial y)^T v_B$.

The previous function type IDALsJacTimesVecFn is identical to IDALsJacTimesVecFnB, and may still be used for backward-compatibility. However, this will be deprecated in future releases, so we recommend that users transition to the new function type name soon.

**IDALsJacTimesVecFnBS**

Definition

```c
typedef int (*IDALsJacTimesVecFnBS)(realtype t,
   N_Vector yy, N_Vector yp,
   N_Vector *yyS, N_Vector *ypS,
   N_Vector yB, N_Vector ypB,
   N_Vector resvalB,
   N_Vector vB, N_Vector JvB,
   realtype cjB, void *user_dataB,
   N_Vector tmp1B, N_Vector tmp2B);
```

Purpose This function computes the action of the backward problem Jacobian $J_B$ on a given vector $v_B$, in the case where the backward problem depends on the forward sensitivities.

Arguments

- $t$ is the current value of the independent variable.
- $yy$ is the current value of the forward solution vector.
- $yp$ is the current value of the forward solution derivative vector.
- $yyS$ a pointer to an array of $N_s$ vectors containing the sensitivities of the forward solution.
- $ypS$ a pointer to an array of $N_s$ vectors containing the derivatives of the forward sensitivities.
- $yB$ is the current value of the backward dependent variable vector.
- $ypB$ is the current value of the backward dependent derivative vector.
- $resvalB$ is the current value of the residual for the backward problem.
- $vB$ is the vector by which the Jacobian must be multiplied.
- $JvB$ is the computed output vector, $J_B v_B$.
- $cjB$ is the scalar in the system Jacobian, proportional to the inverse of the step size ($\alpha$ in Eq. (2.6)).
- $user_dataB$ is a pointer to user data — the same as the $user_dataB$ parameter passed to IDASetUserDataB.
- $tmp1B$
- $tmp2B$ are pointers to memory allocated for variables of type $N_Vector$ which can be used by IDALsJacTimesVecFnBS as temporary storage or work space.

Return value The return value of a function of type IDALsJtimesVecFnBS should be 0 if successful or nonzero if an error was encountered, in which case the integration is halted.

Notes A user-supplied Jacobian-vector product function must load the vector $Jv_B$ with the product of the Jacobian of the backward problem at the point $(t, y, y_B)$ and the vector $v_B$. Here, $y$ is the solution of the original IVP at time $t$ and $y_B$ is the solution of the backward problem at the same time. The rest of the arguments are equivalent to those passed to a function of type IDALsJacTimesVecFn (see §4.6.6).
6.3 User-supplied functions for adjoint sensitivity analysis

The previous function type IDASpilsJacTimesVecFnBS is identical to IDALsJacTimesVecFnBS, and may still be used for backward-compatibility. However, this will be deprecated in future releases, so we recommend that users transition to the new function type name soon.

6.3.7 Jacobian-vector product setup for the backward problem (matrix-free linear solvers)

If the user’s Jacobian-times-vector requires that any Jacobian-related data be preprocessed or evaluated, then this needs to be done in a user-supplied function of type IDALsJacTimesSetupFnB or IDALsJacTimesSetupFnBS, defined as follows:

```c
typedef int (*IDALsJacTimesSetupFnB)(realtype tt,
    N_Vector yy, N_Vector yp,
    N_Vector yB, N_Vector ypB,
    N_Vector resvalB,
    realtype cjB, void *user_dataB);
```

**Purpose**
This function preprocesses and/or evaluates Jacobian data needed by the Jacobian-times-vector routine for the backward problem.

**Arguments**
- `tt` is the current value of the independent variable.
- `yy` is the current value of the dependent variable vector, \( y(t) \).
- `yp` is the current value of \( \dot{y}(t) \).
- `yB` is the current value of the backward dependent variable vector.
- `ypB` is the current value of the backward dependent derivative vector.
- `resvalB` is the current value of the residual for the backward problem.
- `cjB` is the scalar in the system Jacobian, proportional to the inverse of the step size (\( \alpha \) in Eq. (2.6)).
- `user_dataB` is a pointer to user data — the same as the `user_dataB` parameter passed to IDASetUserDataB.

**Return value**
The value returned by the Jacobian-vector setup function should be 0 if successful, positive for a recoverable error (in which case the step will be retried), or negative for an unrecoverable error (in which case the integration is halted).

**Notes**
Each call to the Jacobian-vector setup function is preceded by a call to the backward problem residual user function with the same \( (t, y, yp, yB, ypB) \) arguments. Thus, the setup function can use any auxiliary data that is computed and saved during the evaluation of the DAE residual.

If the user’s IDALsJacTimesVecFnB function uses difference quotient approximations, it may need to access quantities not in the call list. These include the current stepsize, the error weights, etc. To obtain these, the user will need to add a pointer to `ida_mem` to `user_dataB` and then use the `IDAGet*` functions described in §4.5.10.2. The unit roundoff can be accessed as `UNIT_ROUNDOFF` defined in `sundials_types.h`.

The previous function type IDASpilsJacTimesSetupFnB is identical to IDALsJacTimesSetupFnB, and may still be used for backward-compatibility. However, this will be deprecated in future releases, so we recommend that users transition to the new function type name soon.
Definition: 
```c
typedef int (*IDALsJacTimesSetupFnBS)(realtype tt,
   N_Vector yy, N_Vector yp,
   N_Vector *yyS, N_Vector *ypS,
   N_Vector yB, N_Vector ypB,
   N_Vector resvalB,
   realtype cjB, void *user_dataB);
```

Purpose: This function preprocesses and/or evaluates Jacobian data needed by the Jacobian-times-vector routine for the backward problem, in the case that the backward problem depends on the forward sensitivities.

Arguments:
- `tt` is the current value of the independent variable.
- `yy` is the current value of the dependent variable vector, $y(t)$.
- `yp` is the current value of $\dot{y}(t)$.
- `yyS` is a pointer to an array of $N_s$ vectors containing the sensitivities of the forward solution.
- `ypS` is a pointer to an array of $N_s$ vectors containing the derivatives of the forward sensitivities.
- `yB` is the current value of the backward dependent variable vector.
- `ypB` is the current value of the backward dependent derivative vector.
- `resvalB` is the current value of the residual for the backward problem.
- `cjB` is the scalar in the system Jacobian, proportional to the inverse of the step size ($\alpha$ in Eq. (2.6)).
- `user_dataB` is a pointer to user data — the same as the `user_dataB` parameter passed to `IDASetUserDataB`.

Return value: The value returned by the Jacobian-vector setup function should be 0 if successful, positive for a recoverable error (in which case the step will be retried), or negative for an unrecoverable error (in which case the integration is halted).

Notes: Each call to the Jacobian-vector setup function is preceded by a call to the backward problem residual user function with the same $(t, y, \dot{y}, y_S, y_{SP}, y_B, y_{PB})$ arguments. Thus, the setup function can use any auxiliary data that is computed and saved during the evaluation of the DAE residual.

If the user’s `IDALsJacTimesVecFnB` function uses difference quotient approximations, it may need to access quantities not in the call list. These include the current stepsize, the error weights, etc. To obtain these, the user will need to add a pointer to `ida_mem` to `user_dataB` and then use the `IDAGet*` functions described in §4.5.10.2. The unit roundoff can be accessed as `UNIT_ROUNDOFF` defined in `sundials_types.h`.

The previous function type `IDASpilsJacTimesSetupFnBS` is identical to `IDALsJacTimesSetupFnBS`, and may still be used for backward-compatibility. However, this will be deprecated in future releases, so we recommend that users transition to the new function type name soon.

6.3.8 Preconditioner solve for the backward problem (iterative linear solvers)

If preconditioning is used during integration of the backward problem, then the user must provide a function to solve the linear system $Pz = r$, where $P$ is a left preconditioner matrix. This function must have one of the following two forms:

```c
IDALsPrecSolveFnB
```
6.3 User-supplied functions for adjoint sensitivity analysis

Definition

typedef int (*IDALsPrecSolveFnB)(realtype t,
    N_Vector yy, N_Vector yp,
    N_Vector yB, N_Vector ypB,
    N_Vector resvalB,
    N_Vector rvecB, N_Vector zvecB,
    realtype cjB, realtype deltaB,
    void *user_dataB);

Purpose

This function solves the preconditioning system \( Pz = r \) for the backward problem.

Arguments

- \( t \) is the current value of the independent variable.
- \( yy \) is the current value of the forward solution vector.
- \( yp \) is the current value of the forward solution derivative vector.
- \( yB \) is the current value of the backward dependent variable vector.
- \( ypB \) is the current value of the backward dependent derivative vector.
- \( resvalB \) is the current value of the residual for the backward problem.
- \( rvecB \) is the right-hand side vector \( r \) of the linear system to be solved.
- \( zvecB \) is the computed output vector.
- \( cjB \) is the scalar in the system Jacobian, proportional to the inverse of the step size \( (\alpha \text{ in Eq. (2.6)}) \).
- \( deltaB \) is an input tolerance to be used if an iterative method is employed in the solution.
- \( user_dataB \) is a pointer to user data — the same as the \( user_dataB \) parameter passed to the function \( IDASetUserDataB \).

Return value

The return value of a preconditioner solve function for the backward problem should be 0 if successful, positive for a recoverable error (in which case the step will be retried), or negative for an unrecoverable error (in which case the integration is halted).

Notes

The previous function type \( IDASpilsPrecSolveFnB \) is identical to \( IDALsPrecSolveFnB \), and may still be used for backward-compatibility. However, this will be deprecated in future releases, so we recommend that users transition to the new function type name soon.

---

Definition

typedef int (*IDALsPrecSolveFnBS)(realtype t,
    N_Vector yy, N_Vector yp,
    N_Vector *yyS, N_Vector *ypS,
    N_Vector yB, N_Vector ypB,
    N_Vector resvalB,
    N_Vector rvecB, N_Vector zvecB,
    realtype cjB, realtype deltaB,
    void *user_dataB);

Purpose

This function solves the preconditioning system \( Pz = r \) for the backward problem, for the case in which the backward problem depends on the forward sensitivities.

Arguments

- \( t \) is the current value of the independent variable.
- \( yy \) is the current value of the forward solution vector.
- \( yp \) is the current value of the forward solution derivative vector.
- \( yyS \) a pointer to an array of \( Ns \) vectors containing the sensitivities of the forward solution.
- \( ypS \) a pointer to an array of \( Ns \) vectors containing the derivatives of the forward sensitivities.
- \( yB \) is the current value of the backward dependent variable vector.
ypB is the current value of the backward dependent derivative vector.
resvalB is the current value of the residual for the backward problem.
rvecB is the right-hand side vector $r$ of the linear system to be solved.
zvecB is the computed output vector.
cjB is the scalar in the system Jacobian, proportional to the inverse of the step size ($\alpha$ in Eq. (2.6)).
deltaB is an input tolerance to be used if an iterative method is employed in the solution.
user_dataB is a pointer to user data — the same as the user_dataB parameter passed to the function IDASSetUserDataB.

Return value The return value of a preconditioner solve function for the backward problem should be 0 if successful, positive for a recoverable error (in which case the step will be retried), or negative for an unrecoverable error (in which case the integration is halted).

Notes The previous function type IDASpilsPrecSolveFnBS is identical to IDALsPrecSolveFnBS, and may still be used for backward-compatibility. However, this will be deprecated in future releases, so we recommend that users transition to the new function type name soon.

6.3.9 Preconditioner setup for the backward problem (iterative linear solvers)

If the user’s preconditioner requires that any Jacobian-related data be preprocessed or evaluated, then this needs to be done in a user-supplied function of one of the following two types:

**IDALsPrecSetupFnB**

Definition typedef int (*IDALsPrecSetupFnB)(realtype t,
N_Vector yy, N_Vector yp,
N_Vector yB, N_Vector ypB,
N_Vector resvalB,
realtype cjB, void *user_dataB);

Purpose This function preprocesses and/or evaluates Jacobian-related data needed by the preconditioner for the backward problem.

Arguments The arguments of an IDALsPrecSetupFnB are as follows:
t is the current value of the independent variable.
yy is the current value of the forward solution vector.
yp is the current value of the forward solution vector.
yB is the current value of the backward dependent variable vector.
ypB is the current value of the backward dependent derivative vector.
resvalB is the current value of the residual for the backward problem.
cjB is the scalar in the system Jacobian, proportional to the inverse of the step size ($\alpha$ in Eq. (2.6)).
user_dataB is a pointer to user data — the same as the user_dataB parameter passed to the function IDASSetUserDataB.

Return value The return value of a preconditioner setup function for the backward problem should be 0 if successful, positive for a recoverable error (in which case the step will be retried), or negative for an unrecoverable error (in which case the integration is halted).

Notes The previous function type IDASpilsPrecSetupFnB is identical to IDALsPrecSetupFnB, and may still be used for backward-compatibility. However, this will be deprecated in future releases, so we recommend that users transition to the new function type name soon.
6.4 Using the band-block-diagonal preconditioner for backward problems

As on the forward integration phase, the efficiency of Krylov iterative methods for the solution of linear systems can be greatly enhanced through preconditioning. The band-block-diagonal preconditioner module IDABBDPRE, provides interface functions through which it can be used on the backward integration phase.

The adjoint module in IDAS offers an interface to the band-block-diagonal preconditioner module IDABBDPRE described in section §4.8. This generates a preconditioner that is a block-diagonal matrix with each block being a band matrix and can be used with one of the Krylov linear solvers and with the MPI-parallel vector module NVECTOR_PARALLEL.

In order to use the IDABBDPRE module in the solution of the backward problem, the user must define one or two additional functions, described at the end of this section.
6.4.1 Usage of IDABBDPRE for the backward problem

The idabbdpre module is initialized by calling the following function, after an iterative linear solver for the backward problem has been attached to IDAS by calling IDASSetLinearSolverB (see §6.2.6).

```c
Call flag = IDABBDPrecInitB(ida_mem, which, NlocalB, mudqB, mldqB, mukeepB, mlkeepB, dqrelyB, GresB, GcommB);
```

**Description** The function IDABBDPrecInitB initializes and allocates memory for the idabbdpre preconditioner for the backward problem.

**Arguments**
- `ida_mem` (void *) pointer to the IDAS memory block.
- `which` (int) the identifier of the backward problem.
- `NlocalB` (sunindextype) local vector dimension for the backward problem.
- `mudqB` (sunindextype) upper half-bandwidth to be used in the difference-quotient Jacobian approximation.
- `mldqB` (sunindextype) lower half-bandwidth to be used in the difference-quotient Jacobian approximation.
- `mukeepB` (sunindextype) upper half-bandwidth of the retained banded approximate Jacobian block.
- `mlkeepB` (sunindextype) lower half-bandwidth of the retained banded approximate Jacobian block.
- `dqrelyB` (realtype) the relative increment in components of \( y_B \) used in the difference quotient approximations. The default is \( dqrelyB = \sqrt{\text{unit roundoff}} \), which can be specified by passing \( dqrely = 0.0 \).
- `GresB` (IDABBDLocalFnB) the C function which computes \( G_B(t, y, \dot{y}, y_B, \dot{y}_B) \), the function approximating the residual of the backward problem.
- `GcommB` (IDABBDCommFnB) the optional C function which performs all interprocess communication required for the computation of \( G_B \).

**Return value** If successful, IDABBDPrecInitB creates, allocates, and stores (internally in the IDAS solver block) a pointer to the newly created idabbdpre memory block. The return value `flag` (of type int) is one of:
- **IDALS_SUCCESS** The call to IDABBDPrecInitB was successful.
- **IDALS_MEM_FAIL** A memory allocation request has failed.
- **IDALS_MEM_NULL** The `ida_mem` argument was NULL.
- **IDALS_LMEM_NULL** No linear solver has been attached.
- **IDALS_ILL_INPUT** An invalid parameter has been passed.

**F2003 Name** FIDABBDPrecInitB

To reinitialize the idabbdpre preconditioner module for the backward problem, possibly with a change in `mudqB`, `mldqB`, or `dqrelyB`, call the following function:

```c
Call flag = IDABBDPrecReInitB(ida_mem, which, mudqB, mldqB, dqrelyB);
```

**Description** The function IDABBDPrecReInitB reinitializes the idabbdpre preconditioner for the backward problem.

**Arguments**
- `ida_mem` (void *) pointer to the IDAS memory block returned by IDACreate.
- `which` (int) the identifier of the backward problem.
- `mudqB` (sunindextype) upper half-bandwidth to be used in the difference-quotient Jacobian approximation.
6.4 Using the band-block-diagonal preconditioner for backward problems

\texttt{mldqB} (\texttt{sunindextype}) lower half-bandwidth to be used in the difference-quotient Jacobian approximation.

\texttt{dqrelyB} (\texttt{realtype}) the relative increment in components of \texttt{yB} used in the difference quotient approximations.

Return value The return value \texttt{flag} (of type \texttt{int}) is one of:

- \texttt{IDALS_SUCCESS} The call to \texttt{IDABBDPrecReInitB} was successful.
- \texttt{IDALS_MEM_FAIL} A memory allocation request has failed.
- \texttt{IDALS_MEM_NULL} The \texttt{ida_mem} argument was \texttt{NULL}.
- \texttt{IDALS_PMEM_NULL} The \texttt{IDABBDPrecInitB} has not been previously called.
- \texttt{IDALS_LMEM_NULL} No linear solver has been attached.
- \texttt{IDALS_ILL_INPUT} An invalid parameter has been passed.

F’2003 Name \texttt{FIDABBDPrecReInitB}

For more details on IDABBDPRE see §4.8.

6.4.2 User-supplied functions for IDABBDPRE

To use the IDABBDPRE module, the user must supply one or two functions which the module calls to construct the preconditioner: a required function \texttt{GresB} (of type \texttt{IDABBDLocalFnB}) which approximates the residual of the backward problem and which is computed locally, and an optional function \texttt{GcommB} (of type \texttt{IDABBDCommFnB}) which performs all interprocess communication necessary to evaluate this approximate residual (see §4.8). The prototypes for these two functions are described below.

\textbf{IDABBDLocalFnB}

\begin{verbatim}
typedef int (*IDABBDLocalFnB)(sunindextype NlocalB, realtype t,
    N_Vector y, N_Vector yp,
    N_Vector yB, N_Vector ypB,
    N_Vector gB, void *user_dataB);
\end{verbatim}

\textbf{Definition}

\textbf{Purpose} This \texttt{GresB} function loads the vector \texttt{gB}, an approximation to the residual of the backward problem, as a function of \texttt{t}, \texttt{y}, \texttt{yp}, and \texttt{yB} and \texttt{ypB}.

\textbf{Arguments} \texttt{NlocalB} is the local vector length for the backward problem.
\texttt{t} is the value of the independent variable.
\texttt{y} is the current value of the forward solution vector.
\texttt{yp} is the current value of the forward solution derivative vector.
\texttt{yB} is the current value of the backward dependent variable vector.
\texttt{ypB} is the current value of the backward dependent derivative vector.
\texttt{gB} is the output vector, \(G_B(t, y, \dot{y}, y_B, \dot{y}_B)\).
\texttt{user_dataB} is a pointer to user data — the same as the \texttt{user_dataB} parameter passed to \texttt{IDASetUserDataB}.

\textbf{Return value} An \texttt{IDABBDLocalFnB} should return 0 if successful, a positive value if a recoverable error occurred (in which case IDAS will attempt to correct), or a negative value if it failed unrecoverably (in which case the integration is halted and IDASolveB returns \texttt{IDA_LSETUP_FAIL}).

\textbf{Notes} This routine must assume that all interprocess communication of data needed to calculate \texttt{gB} has already been done, and this data is accessible within \texttt{user_dataB}.

Before calling the user’s \texttt{IDABBDLocalFnB}, IDAS needs to evaluate (through interpolation) the values of the states from the forward integration. If an error occurs in the interpolation, IDAS triggers an unrecoverable failure in the preconditioner setup function which will halt the integration (IDASolveB returns \texttt{IDA_LSETUP_FAIL}).
typedef int (*IDABBDCommFnB)(sunindextype NlocalB, realtype t,
    N_Vector y, N_Vector yp,
    N_Vector yB, N_Vector ypB,
    void *user_dataB);

Purpose This GcommB function performs all interprocess communications necessary for the execution of the GresB function above, using the input vectors y, yp, yB and ypB.

Arguments NlocalB is the local vector length.
    t is the value of the independent variable.
    y is the current value of the forward solution vector.
    yp is the current value of the forward solution derivative vector.
    yB is the current value of the backward dependent variable vector.
    ypB is the current value of the backward dependent derivative vector.
    user_dataB is a pointer to user data — the same as the user_dataB parameter passed to IDASetUserDataB.

Return value An IDABBDCommFnB should return 0 if successful, a positive value if a recoverable error occurred (in which case IDAS will attempt to correct), or a negative value if it failed unrecoverably (in which case the integration is halted and IDASolveB returns IDA_LSETUP_FAIL).

Notes The GcommB function is expected to save communicated data in space defined within the structure user_dataB.

Each call to the GcommB function is preceded by a call to the function that evaluates the residual of the backward problem with the same t, y, yp, yB and ypB arguments. If there is no additional communication needed, then pass GcommB = NULL to IDABBDPrecInitB.
Chapter 7

Using IDAS for Fortran Applications

A Fortran 2003 module (fidas_mod) is provided to support the use of idas, for the solution of DAE systems and performing forward sensitivity analysis or adjoint sensitivity analysis in a mixed Fortran/C setting. While idas is written in C, it is assumed here that the user’s calling program and user-supplied problem-defining routines are written in Fortran.

7.1 IDAS Fortran 2003 Interface Module

The fidas_mod Fortran module defines interfaces to most idas C functions using the intrinsic iso_c_binding module which provides a standardized mechanism for interoprating with C. All interfaced functions are named after the corresponding C function, but with a leading ‘F’. For example, the idas function IDACreate is interfaced as FIDACreate. Thus, the steps to use idas and the function calls in Fortran 2003 are identical (ignoring language differences) to those in C. The C functions with Fortran 2003 interfaces indicate this in their description in Chapters 4, 5, and 6. The Fortran 2003 idas interface module can be accessed by the use statement, i.e. use fidas_mod, and linking to the library libsundials_fidas_mod.lib in addition to libsundials_idas.lib.

The Fortran 2003 interface modules were generated with SWIG Fortran, a fork of SWIG [36]. Users who are interested in the SWIG code used in the generation process should contact the SUNDIALS development team.

7.1.1 SUNDIALS Fortran 2003 Interface Modules

All of the generic sundials modules provide Fortran 2003 interface modules. Many of the generic module implementations provide Fortran 2003 interfaces (a complete list of modules with Fortran 2003 interfaces is given in Table 7.1). A module can be accessed with the use statement, e.g. use fnvector_openmp_mod, and linking to the Fortran 2003 library in addition to the C library, e.g. libsundials_fnvecopenmp_mod.lib and libsundials_nvecopenmp.lib.

The Fortran 2003 interfaces leverage the iso_c_binding module and the bind(C) attribute to closely follow the SUNDIALS C API (ignoring language differences). The generic sundials structures, e.g. N_Vector, are interfaced as Fortran derived types, and function signatures are matched but with an F prepending the name, e.g. FN_VConst instead of N_VConst. Constants are named exactly as they are in the C API. Accordingly, using SUNDIALS via the Fortran 2003 interfaces looks just like using it in C. Some caveats stemming from the language differences are discussed in the section 7.1.3. A discussion on the topic of equivalent data types in C and Fortran 2003 is presented in section 7.1.2.

Further information on the Fortran 2003 interfaces specific to modules is given in the NVVECTOR, SUNMATRIX, SUNLINSOL, and SUNNONLINSOL alongside the C documentation (chapters 8, 9, 10, and
11 respectively). For details on where the Fortran 2003 module (.mod) files and libraries are installed see Appendix A.

Table 7.1: Summary of Fortran 2003 interfaces for shared SUNDIALS modules.

<table>
<thead>
<tr>
<th>Module</th>
<th>Fortran 2003 Module Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>NVECTOR</td>
<td>fsundials_nvector_mod</td>
</tr>
<tr>
<td>NVECTOR_SERIAL</td>
<td>fnvector_serial_mod</td>
</tr>
<tr>
<td>NVECTOR_PARALLEL</td>
<td>fnvector_parallel_mod</td>
</tr>
<tr>
<td>NVECTOR_OPENMP</td>
<td>fnvector_openmp_mod</td>
</tr>
<tr>
<td>NVECTOR_PTHREADS</td>
<td>fnvector_pthreads_mod</td>
</tr>
<tr>
<td>NVECTOR_PARHYPER</td>
<td>Not interfaced</td>
</tr>
<tr>
<td>NVECTOR_PETSC</td>
<td>Not interfaced</td>
</tr>
<tr>
<td>NVECTOR_CUDA</td>
<td>Not interfaced</td>
</tr>
<tr>
<td>NVECTOR_RAJA</td>
<td>Not interfaced</td>
</tr>
<tr>
<td>NVECTOR_MANYVECTOR</td>
<td>fnvector_manyvector_mod</td>
</tr>
<tr>
<td>NVECTOR_MPI_MANYVECTOR</td>
<td>fnvector_mpi_manyvector_mod</td>
</tr>
<tr>
<td>SUNMatrix</td>
<td>fsundials_matrix_mod</td>
</tr>
<tr>
<td>SUNMATRIX_BAND</td>
<td>fsunmatrix_band_mod</td>
</tr>
<tr>
<td>SUNMATRIX_DENSE</td>
<td>fsunmatrix_dense_mod</td>
</tr>
<tr>
<td>SUNMATRIX_SPARSE</td>
<td>fsunmatrix_sparse_mod</td>
</tr>
<tr>
<td>SUNLinearSolver</td>
<td>fsundials_linear_solver_mod</td>
</tr>
<tr>
<td>SUNLINSOL_BAND</td>
<td>fsuninsol_band_mod</td>
</tr>
<tr>
<td>SUNLINSOL_DENSE</td>
<td>fsuninsol_dense_mod</td>
</tr>
<tr>
<td>SUNLINSOL_LAPACKBAND</td>
<td>Not interfaced</td>
</tr>
<tr>
<td>SUNLINSOL_LAPACKDENSE</td>
<td>Not interfaced</td>
</tr>
<tr>
<td>SUNLINSOL_KLU</td>
<td>fsuninsol_klu_mod</td>
</tr>
<tr>
<td>SUNLINSOL_SUPERLUMT</td>
<td>Not interfaced</td>
</tr>
<tr>
<td>SUNLINSOL_SUPERLUDIST</td>
<td>Not interfaced</td>
</tr>
<tr>
<td>SUNLINSOL_SPGMR</td>
<td>fsuninsol_spgrmr_mod</td>
</tr>
<tr>
<td>SUNLINSOL_SPFGMR</td>
<td>fsuninsol_spfgmr_mod</td>
</tr>
<tr>
<td>SUNLINSOL_SPBCGS</td>
<td>fsuninsol_spbcgs_mod</td>
</tr>
<tr>
<td>SUNLINSOL_SPTFQMR</td>
<td>fsuninsol_sptfqrmod</td>
</tr>
<tr>
<td>SUNLINSOL_PCG</td>
<td>fsuninsol_pcg_mod</td>
</tr>
<tr>
<td>SUNNonlinearSolver</td>
<td>fsundials_nonlinear_solver_mod</td>
</tr>
<tr>
<td>SUNNONLINSOL_NEWTON</td>
<td>fsunnonlinsol_newton_mod</td>
</tr>
<tr>
<td>SUNNONLINSOL_FIXEDPOINT</td>
<td>fsunnonlinsol_fixedpoint_mod</td>
</tr>
</tbody>
</table>

7.1.2 Data Types

Generally, the Fortran 2003 type that is equivalent to the C type is what one would expect. Primitive types map to the iso_c_binding type equivalent. SUNDIALS generic types map to a Fortran derived type. However, the handling of pointer types is not always clear as they can depend on the parameter direction. Table 7.2 presents a summary of the type equivalencies with the parameter direction in mind.

Currently, the Fortran 2003 interfaces are only compatible with SUNDIALS builds where the realtype is double precision and the sunindextype size is 64-bits.
### 7.1 IDAS Fortran 2003 Interface Module

#### 7.1.3 Notable Fortran/C usage differences

While the Fortran 2003 interface to SUNDIALS closely follows the C API, some differences are inevitable due to the differences between Fortran and C. In this section, we note the most critical differences. Additionally, section 7.1.2 discusses equivalencies of data types in the two languages.

#### 7.1.3.1 Creating generic SUNDIALS objects

In the C API a generic SUNDIALS object, such as an `N_Vector`, is actually a pointer to an underlying C struct. However, in the Fortran 2003 interface, the derived type is bound to the C struct, not the pointer to the struct. E.g., `type(N_Vector)` is bound to the C struct `generic N_Vector` not the `N_Vector` type. The consequence of this is that creating and declaring SUNDIALS objects in Fortran is nuanced. This is illustrated in the code snippets below:

**C code:**

```c
N_Vector x;
x = N_VNew_Serial(N);
```

**Fortran code:**

```fortran
N_Vector x;
x = N_VNew_Serial(N);
```

---

<table>
<thead>
<tr>
<th>C type</th>
<th>Parameter Direction</th>
<th>Fortran 2003 type</th>
</tr>
</thead>
<tbody>
<tr>
<td>double</td>
<td>in, inout, out, return</td>
<td>real(c_double)</td>
</tr>
<tr>
<td>int</td>
<td>in, inout, out, return</td>
<td>integer(c_int)</td>
</tr>
<tr>
<td>long</td>
<td>in, inout, out, return</td>
<td>integer(c_long)</td>
</tr>
<tr>
<td>booleantype</td>
<td>in, inout, out, return</td>
<td>integer(c_int)</td>
</tr>
<tr>
<td>realtype</td>
<td>in, inout, out, return</td>
<td>real(c_double)</td>
</tr>
<tr>
<td>sunindextype</td>
<td>in, inout, out, return</td>
<td>integer(c_long)</td>
</tr>
<tr>
<td>double*</td>
<td>in, inout, out</td>
<td>real(c_double), dimension(*)</td>
</tr>
<tr>
<td>int*</td>
<td>return</td>
<td>integer(c_int), dimension(*)</td>
</tr>
<tr>
<td>long*</td>
<td>in, inout, out</td>
<td>integer(c_long), dimension(*)</td>
</tr>
<tr>
<td>realtype*</td>
<td>in, inout, out</td>
<td>real(c_double), dimension(*)</td>
</tr>
<tr>
<td>sunindextype*</td>
<td>in, inout, out</td>
<td>integer(c_long), dimension(*)</td>
</tr>
<tr>
<td>realtype[]</td>
<td>in, inout, out</td>
<td>integer(c_long), pointer, dimension(*)</td>
</tr>
<tr>
<td>sunindextype[]</td>
<td>in, inout, out</td>
<td>integer(c_long), dimension(*)</td>
</tr>
<tr>
<td>N_Vector</td>
<td>in, inout, out</td>
<td>type(N_Vector)</td>
</tr>
<tr>
<td>SUNMatrix</td>
<td>in, inout, out</td>
<td>type(SUNMatrix)</td>
</tr>
<tr>
<td>SUNMatrix</td>
<td>return</td>
<td>type(SUNMatrix), pointer</td>
</tr>
<tr>
<td>SUNLinearSolver</td>
<td>in, inout, out</td>
<td>type(SUNLinearSolver)</td>
</tr>
<tr>
<td>SUNLinearSolver</td>
<td>return</td>
<td>type(SUNLinearSolver), pointer</td>
</tr>
<tr>
<td>SUNNonlinearSolver</td>
<td>in, inout, out</td>
<td>type(SUNNonlinearSolver)</td>
</tr>
<tr>
<td>FILE*</td>
<td>in, inout, out, return</td>
<td>type(c_ptr)</td>
</tr>
<tr>
<td>void*</td>
<td>in, inout, out, return</td>
<td>type(c_ptr)</td>
</tr>
<tr>
<td>T**</td>
<td>in, inout, out, return</td>
<td>type(c_ptr)</td>
</tr>
<tr>
<td>T***</td>
<td>in, inout, out, return</td>
<td>type(c_ptr)</td>
</tr>
<tr>
<td>T****</td>
<td>in, inout, out, return</td>
<td>type(c_ptr)</td>
</tr>
</tbody>
</table>

---

### Table 7.2: C/Fortran 2003 Equivalent Types

<table>
<thead>
<tr>
<th>C type</th>
<th>Parameter Direction</th>
<th>Fortran 2003 type</th>
</tr>
</thead>
<tbody>
<tr>
<td>double</td>
<td>in, inout, out, return</td>
<td>real(c_double)</td>
</tr>
<tr>
<td>int</td>
<td>in, inout, out, return</td>
<td>integer(c_int)</td>
</tr>
<tr>
<td>long</td>
<td>in, inout, out, return</td>
<td>integer(c_long)</td>
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<td>booleantype</td>
<td>in, inout, out, return</td>
<td>integer(c_int)</td>
</tr>
<tr>
<td>realtype</td>
<td>in, inout, out, return</td>
<td>real(c_double)</td>
</tr>
<tr>
<td>sunindextype</td>
<td>in, inout, out, return</td>
<td>integer(c_long)</td>
</tr>
<tr>
<td>double*</td>
<td>in, inout, out</td>
<td>real(c_double), dimension(*)</td>
</tr>
<tr>
<td>int*</td>
<td>return</td>
<td>integer(c_int), dimension(*)</td>
</tr>
<tr>
<td>long*</td>
<td>in, inout, out</td>
<td>integer(c_long), dimension(*)</td>
</tr>
<tr>
<td>realtype*</td>
<td>in, inout, out</td>
<td>real(c_double), dimension(*)</td>
</tr>
<tr>
<td>sunindextype*</td>
<td>in, inout, out</td>
<td>integer(c_long), dimension(*)</td>
</tr>
<tr>
<td>realtype[]</td>
<td>in, inout, out</td>
<td>integer(c_long), pointer, dimension(*)</td>
</tr>
<tr>
<td>sunindextype[]</td>
<td>in, inout, out</td>
<td>integer(c_long), dimension(*)</td>
</tr>
<tr>
<td>N_Vector</td>
<td>in, inout, out</td>
<td>type(N_Vector)</td>
</tr>
<tr>
<td>SUNMatrix</td>
<td>in, inout, out</td>
<td>type(SUNMatrix)</td>
</tr>
<tr>
<td>SUNMatrix</td>
<td>return</td>
<td>type(SUNMatrix), pointer</td>
</tr>
<tr>
<td>SUNLinearSolver</td>
<td>in, inout, out</td>
<td>type(SUNLinearSolver)</td>
</tr>
<tr>
<td>SUNLinearSolver</td>
<td>return</td>
<td>type(SUNLinearSolver), pointer</td>
</tr>
<tr>
<td>SUNNonlinearSolver</td>
<td>in, inout, out</td>
<td>type(SUNNonlinearSolver)</td>
</tr>
<tr>
<td>FILE*</td>
<td>in, inout, out, return</td>
<td>type(c_ptr)</td>
</tr>
<tr>
<td>void*</td>
<td>in, inout, out, return</td>
<td>type(c_ptr)</td>
</tr>
<tr>
<td>T**</td>
<td>in, inout, out, return</td>
<td>type(c_ptr)</td>
</tr>
<tr>
<td>T***</td>
<td>in, inout, out, return</td>
<td>type(c_ptr)</td>
</tr>
<tr>
<td>T****</td>
<td>in, inout, out, return</td>
<td>type(c_ptr)</td>
</tr>
</tbody>
</table>
type(N_Vector), pointer :: x
x => FN_VNew_Serial(N)

Note that in the Fortran declaration, the vector is a type(N_Vector), pointer, and that the pointer assignment operator is then used.

7.1.3.2 Arrays and pointers

Unlike in the C API, in the Fortran 2003 interface, arrays and pointers are treated differently when they are return values versus arguments to a function. Additionally, pointers which are meant to be out parameters, not arrays, in the C API must still be declared as a rank-1 array in Fortran. The reason for this is partially due to the Fortran 2003 standard for C bindings, and partially due to the tool used to generate the interfaces. Regardless, the code snippets below illustrate the differences.

C code:
N_Vector x
realtype* xdata;
long int leniw, lenrw;

x = N_VNew_Serial(N);

/* capturing a returned array/pointer */
xdata = N_VGetArrayPointer(x)

/* passing array/pointer to a function */
N_VSetArrayPointer(xdata, x)

/* pointers that are out-parameters */
N_VSpace(x, &leniw, &lenrw);

Fortran code:
type(N_Vector), pointer :: x
real(c_double), pointer :: xdataptr(:)
real(c_double) :: xdata(N)
integer(c_long) :: leniw(1), lenrw(1)

x => FN_VNew_Serial(x)

! capturing a returned array/pointer
xdataptr => FN_VGetArrayPointer(x)

! passing array/pointer to a function
call FN_VSetArrayPointer(xdata, x)

! pointers that are out-parameters
call FN_VSpace(x, leniw, lenrw)

7.1.3.3 Passing procedure pointers and user data

Since functions/subroutines passed to SUNDIALS will be called from within C code, the Fortran procedure must have the attribute bind(C). Additionally, when providing them as arguments to a Fortran 2003 interface routine, it is required to convert a procedure's Fortran address to C with the Fortran intrinsic c_funloc.

Typically when passing user data to a SUNDIALS function, a user may simply cast some custom data structure as a void*. When using the Fortran 2003 interfaces, the same thing can be achieved.
Note, the custom data structure does not have to be bind(C) since it is never accessed on the C side.

C code:

MyUserData* udata;
void *cvode_mem;

ierr = CVodeSetUserData(cvode_mem, udata);

Fortran code:

type(MyUserData) :: udata

type(c_ptr) :: cvode_mem

ierr = FCVodeSetUserData(cvode_mem, c_loc(udata))

On the other hand, Fortran users may instead choose to store problem-specific data, e.g. problem parameters, within modules, and thus do not need the SUNDIALS-provided user_data pointers to pass such data back to user-supplied functions. These users should supply the c_null_ptr input for user_data arguments to the relevant SUNDIALS functions.

7.1.3.4 Passing NULL to optional parameters

In the SUNDIALS C API some functions have optional parameters that a caller can pass NULL to. If the optional parameter is of a type that is equivalent to a Fortran type(c_ptr) (see section 7.1.2), then a Fortran user can pass the intrinsic c_null_ptr. However, if the optional parameter is of a type that is not equivalent to type(c_ptr), then a caller must provide a Fortran pointer that is dissociated. This is demonstrated in the code example below.

C code:

SUNLinearSolver LS;
N_Vector x, b;

! SUNLinSolSolve expects a SUNMatrix or NULL
! as the second parameter.
ierr = SUNLinSolSolve(LS, NULL, x, b);

Fortran code:

type(SUNLinearSolver), pointer :: LS

type(SUNMatrix), pointer :: A

type(N_Vector), pointer :: x, b

A => null()

! SUNLinSolSolve expects a type(SUNMatrix), pointer
! as the second parameter. Therefore, we cannot
! pass a c_null_ptr, rather we pass a disassociated A.
ierr = FSUNLinSolSolve(LS, A, x, b)

7.1.3.5 Working with N_Vector arrays

Arrays of N_Vector objects are interfaced to Fortran 2003 as opaque type(c_ptr). As such, it is not possible to directly index an array of N_Vector objects returned by the N_Vector "VectorArray" operations, or packages with sensitivity capabilities. Instead, SUNDIALS provides a utility function FN_VGetVecAtIndexVectorArray that can be called for accessing a vector in a vector array. The
example below demonstrates this:

C code:

```c
N_Vector x;
N_Vector* vecs;

vecs = N_VCloneVectorArray(count, x);
for (int i=0; i < count; ++i)
    N_VConst(vecs[i]);
```

Fortran code:

```fortran
type(N_Vector), pointer :: x, xi
type(c_ptr) :: vecs

vecs = FN_VCloneVectorArray(count, x)
do index, count
    xi => FN_VGetVecAtIndexVectorArray(vecs, index)
    call FN_VConst(xi)
enddo
```

SUNDIALS also provides the functions `FN_VSetVecAtIndexVectorArray` and `FN_VNewVectorArray` for working with `N_Vector` arrays. These functions are particularly useful for users of the Fortran interface to the `NVECTOR_MANYVECTOR` or `NVECTOR_MIRROR_VISIBILITY` when creating the subvector array. Both of these functions along with `FN_VGetVecAtIndexVectorArray` are further described in Chapter 8.1.5.

### 7.1.3.6 Providing file pointers

Expert SUNDIALS users may notice that there are a few advanced functions in the SUNDIALS C API that take a `FILE *` argument. Since there is no portable way to convert between a Fortran file descriptor and a C file pointer, SUNDIALS provides two utility functions for creating a `FILE *` and destroying it. These functions are defined in the module `fsundials_futils_mod`.

```c
FSUNDIALSFileOpen
```

**Call**

```c
fp = FSUNDIALSFileOpen(filename, mode)
```

**Description**
The function allocates a `FILE *` by calling the C function `fopen`.

**Arguments**

- `filename` (character(kind=C_CHAR, len=*)) - the path to the file to open
- `mode` (character(kind=C_CHAR, len=*)) - the mode string given to `fopen` It should begin with one of the following characters:
  - “r” - open text file for reading
  - “r+” - open text file for reading and writing
  - “w” - truncate text file to zero length or create it for writing
  - “w+” - open text file for reading or writing, create it if it does not exist
  - “a” - open for appending, see documentation of “fopen” for your system/compiler
  - “a+” - open for reading and appending, see documentation for “fopen” for your system/compiler

**Return value**
This returns a `type(C_PTR)` which is a `FILE*` in C. If it is `NULL`, then there was an error opening the file.
7.1 IDAS Fortran 2003 Interface Module

FSUNDIALSFileClose
Call call FSUNDIALSFileClose(fp)
Description The function deallocates a FILE* by calling the C function fclose.
Arguments fp (type(C_PTR)) - the file pointer (type FILE* in C)
Return value None

7.1.4 Important notes on portability

The SUNDIALS Fortran 2003 interface should be compatible with any compiler supporting the Fortran 2003 ISO standard. However, it has only been tested and confirmed to be working with GNU Fortran 4.9+ and Intel Fortran 18.0.1+.

Upon compilation of SUNDIALS, Fortran module (.mod) files are generated for each Fortran 2003 interface. These files are highly compiler specific, and thus it is almost always necessary to compile a consuming application with the same compiler used to generate the modules.
Chapter 8

Description of the NVVECTOR module

The SUNDIALS solvers are written in a data-independent manner. They all operate on generic vectors (of type N_Vector) through a set of operations defined by the particular NVVECTOR implementation. Users can provide their own specific implementation of the NVVECTOR module, or use one of the implementations provided with SUNDIALS. The generic NVVECTOR is described below and the implementations provided with SUNDIALS are described in the following sections.

8.1 The NVVECTOR API

The generic NVVECTOR API can be broken down into five groups of functions: the core vector operations, the fused vector operations, the vector array operations, the local reduction operations, and finally some utility functions. The first four groups are defined by a particular NVVECTOR implementation. The utility functions are defined by the generic NVVECTOR itself.

8.1.1 NVVECTOR core functions

\texttt{N_VGetVectorID}

Call \hspace{1em} id = N_VGetVectorID(w);

Description \hspace{1em} Returns the vector type identifier for the vector \( w \). It is used to determine the vector implementation type (e.g. serial, parallel,\ldots) from the abstract \texttt{N_Vector} interface.

Arguments \hspace{1em} \( w \) (N_Vector) a NVVECTOR object

Return value \hspace{1em} This function returns an \texttt{N_Vector_ID}. Possible values are given in Table 8.1.

F2003 Name \hspace{1em} FN_VGetVectorID

\texttt{N_VClone}

Call \hspace{1em} v = N_VClone(w);

Description \hspace{1em} Creates a new \texttt{N_Vector} of the same type as an existing vector \( w \) and sets the \textit{ops} field. It does not copy the vector, but rather allocates storage for the new vector.

Arguments \hspace{1em} \( w \) (N_Vector) a NVVECTOR object

Return value \hspace{1em} This function returns an \texttt{N_Vector} object. If an error occurs, then this routine will return \texttt{NULL}.

F2003 Name \hspace{1em} FN_VClone
N_VCloneEmpty

Call \quad \texttt{v = N\_VCloneEmpty(w);} \\
Description \quad \text{Creates a new N\_Vector of the same type as an existing vector w and sets the ops field. It does not allocate storage for data.}

Arguments \quad \texttt{w (N\_Vector) a NVECTOR object}

Return value \quad \text{This function returns an N\_Vector object. If an error occurs, then this routine will return NULL.}

F2003 Name \quad \texttt{FN\_VCloneEmpty}

N_VDestroy

Call \quad \texttt{N\_VDestroy(v);} \\
Description \quad \text{Destroys the N\_Vector v and frees memory allocated for its internal data.}

Arguments \quad \texttt{v (N\_Vector) a NVECTOR object to destroy}

Return value \quad \text{None}

F2003 Name \quad \texttt{FN\_VDestroy}

N_VSpace

Call \quad \texttt{N\_VSpace(v, &lrw, &liw);} \\
Description \quad \text{Returns storage requirements for one N\_Vector. lrw contains the number of realtype words and liw contains the number of integer words, This function is advisory only, for use in determining a user's total space requirements; it could be a dummy function in a user-supplied NVECTOR module if that information is not of interest.}

Arguments \quad \texttt{v (N\_Vector) a NVECTOR object}

\texttt{lrw (sunindextype*) out parameter containing the number of realtype words}

\texttt{liw (sunindextype*) out parameter containing the number of integer words}

Return value \quad \text{None}

F2003 Name \quad \texttt{FN\_VSpace}

F2003 Call \quad \texttt{integer(c\_long) :: lrw(1), liw(1)}
\quad \texttt{call FN\_VSpace\_Serial(v, lrw, liw)}

N_VGetArrayPointer

Call \quad \texttt{vdata = N\_VGetArrayPointer(v);} \\
Description \quad \text{Returns a pointer to a realtype array from the N\_Vector v. Note that this assumes that the internal data in N\_Vector is a contiguous array of realtype. This routine is only used in the solver-specific interfaces to the dense and banded (serial) linear solvers, the sparse linear solvers (serial and threaded), and in the interfaces to the banded (serial) and band-block-diagonal (parallel) preconditioner modules provided with SUNDIALS.}

Arguments \quad \texttt{v (N\_Vector) a NVECTOR object}

Return value \quad \text{realtype*}

F2003 Name \quad \texttt{FN\_VGetArrayPointer}
8.1 The NVeCtOr API

**N_VSetArrayPointer**

Call: `N_VSetArrayPointer(vdata, v);`

Description: Overwrites the pointer to the data in an N_Vector with a given realtype*. Note that this assumes that the internal data in N_Vector is a contiguous array of realtype. This routine is only used in the interfaces to the dense (serial) linear solver, hence need not exist in a user-supplied NVECTOR module for a parallel environment.

Arguments:  
- v (N_Vector) a NVECTOR object

Return value: None

F2003 Name: `FN_VSetArrayPointer`

**N_VGetCommunicator**

Call: `N_VGetCommunicator(v);`

Description: Returns a pointer to the MPI_Comm object associated with the vector (if applicable). For MPI-unaware vector implementations, this should return NULL.

Arguments:  
- v (N_Vector) a NVECTOR object

Return value: A void * pointer to the MPI_Comm object if the vector is MPI-aware, otherwise NULL.

F2003 Name: `FN_VGetCommunicator`

**N_VGetLength**

Call: `N_VGetLength(v);`

Description: Returns the global length (number of ‘active’ entries) in the NVECTOR v. This value should be cumulative across all processes if the vector is used in a parallel environment. If v contains additional storage, e.g., for parallel communication, those entries should not be included.

Arguments:  
- v (N_Vector) a NVECTOR object

Return value: sunindextype

F2003 Name: `FN_VGetLength`

**N_VLinearSum**

Call: `N_VLinearSum(a, x, b, y, z);`

Description: Performs the operation \( z = ax + by \), where a and b are realtype scalars and x and y are of type N_Vector: \( z_i = ax_i + by_i, \) \( i = 0, \ldots, n - 1 \).

Arguments:  
- a (realtype) constant that scales x
- x (N_Vector) a NVECTOR object
- b (realtype) constant that scales y
- y (N_Vector) a NVECTOR object
- z (N_Vector) a NVECTOR object containing the result

Return value: None

F2003 Name: `FN_VLinearSum`
Description of the NVVECTOR module

**N_VConst**

Call \( \text{N_VConst}(c, z); \)

Description Sets all components of the \( \text{N}_\text{Vector} \ z \) to \text{realtype} \( c \): \( z_i = c, \ i = 0, \ldots, n - 1. \)

Arguments \( c \) (realtype) constant to set all components of \( z \) to 
\( z \) (N_Vector) a NVVECTOR object containing the result

Return value None

F2003 Name FN_VConst

**N_VProd**

Call \( \text{N_VProd}(x, y, z); \)

Description Sets the \( \text{N}_\text{Vector} \ z \) to be the component-wise product of the \( \text{N}_\text{Vector} \) inputs \( x \) and \( y \): 
\( z_i = x_i y_i, \ i = 0, \ldots, n - 1. \)

Arguments \( x \) (N_Vector) a NVVECTOR object 
\( y \) (N_Vector) a NVVECTOR object 
\( z \) (N_Vector) a NVVECTOR object containing the result

Return value None

F2003 Name FN_VProd

**N_VDiv**

Call \( \text{N_VDiv}(x, y, z); \)

Description Sets the \( \text{N}_\text{Vector} \ z \) to be the component-wise ratio of the \( \text{N}_\text{Vector} \) inputs \( x \) and \( y \): 
\( z_i = x_i / y_i, \ i = 0, \ldots, n - 1. \) The \( y_i \) may not be tested for 0 values. It should only be called with a \( y \) that is guaranteed to have all nonzero components.

Arguments \( x \) (N_Vector) a NVVECTOR object 
\( y \) (N_Vector) a NVVECTOR object 
\( z \) (N_Vector) a NVVECTOR object containing the result

Return value None

F2003 Name FN_VDiv

**N_VScale**

Call \( \text{N_VScale}(c, x, z); \)

Description Scales the \( \text{N}_\text{Vector} \ x \) by the \text{realtype} scalar \( c \) and returns the result in \( z \): 
\( z_i = cx_i, \ i = 0, \ldots, n - 1. \)

Arguments \( c \) (realtype) constant that scales the vector \( x \) 
\( x \) (N_Vector) a NVVECTOR object 
\( z \) (N_Vector) a NVVECTOR object containing the result

Return value None

F2003 Name FN_VScale
The NVECTOR API

N_VAbs
Call N_VAbs(x, z);
Description Sets the components of the N_Vector z to be the absolute values of the components of
the N_Vector x: \( y_i = |x_i|, \ i = 0, \ldots, n - 1 \).
Arguments x (N_Vector) a NVECTOR object
z (N_Vector) a NVECTOR object containing the result
Return value None
F2003 Name FN_VAbs

N_VInv
Call N_VInv(x, z);
Description Sets the components of the N_Vector z to be the inverses of the components of the
N_Vector x: \( z_i = \frac{1}{x_i}, \ i = 0, \ldots, n - 1 \). This routine may not check for division by 0.
It should be called only with an x which is guaranteed to have all nonzero components.
Arguments x (N_Vector) a NVECTOR object to
z (N_Vector) a NVECTOR object containing the result
Return value None
F2003 Name FN_VInv

N_VAddConst
Call N_VAddConst(x, b, z);
Description Adds the realtype scalar b to all components of x and returns the result in the N_Vector
z: \( z_i = x_i + b, \ i = 0, \ldots, n - 1 \).
Arguments x (N_Vector) a NVECTOR object
b (realtype) constant added to all components of x
z (N_Vector) a NVECTOR object containing the result
Return value None
F2003 Name FN_VAddConst

N_VDotProd
Call d = N_VDotProd(x, y);
Description Returns the value of the ordinary dot product of x and y: \( d = \sum_{i=0}^{n-1} x_i y_i \).
Arguments x (N_Vector) a NVECTOR object with y
y (N_Vector) a NVECTOR object with x
Return value realtype
F2003 Name FN_VDotProd

N_VMaxNorm
Call m = N_VMaxNorm(x);
Description Returns the maximum norm of the N_Vector x: \( m = \max_i |x_i| \).
Arguments x (N_Vector) a NVECTOR object
Return value realtype
F2003 Name FN_VMaxNorm
**N_VWrmsNorm**

**Call**  
\[ m = \text{N_VWrmsNorm}(x, w) \]

**Description**  
Returns the weighted root-mean-square norm of the N_Vector \( x \) with realtype weight vector \( w \):  
\[ m = \sqrt{\frac{\sum_{i=0}^{n-1} (x_i w_i)^2}{n}}. \]

**Arguments**  
- \( x \) (N_Vector) a NVECTOR object
- \( w \) (N_Vector) a NVECTOR object containing weights

**Return value**  
realtype

**F2003 Name**  
FN_VWrmsNorm

---

**N_VWrmsNormMask**

**Call**  
\[ m = \text{N_VWrmsNormMask}(x, w, id); \]

**Description**  
Returns the weighted root mean square norm of the N_Vector \( x \) with realtype weight vector \( w \) built using only the elements of \( x \) corresponding to positive elements of the N_Vector \( id \):  
\[ m = \sqrt{\frac{\sum_{i=0}^{n-1} (x_i w_i H(id_i))^2}{n}}, \text{ where } H(\alpha) = \begin{cases} 1 & \alpha > 0 \\ 0 & \alpha \leq 0 \end{cases} \]

**Arguments**  
- \( x \) (N_Vector) a NVECTOR object
- \( w \) (N_Vector) a NVECTOR object containing weights
- \( id \) (N_Vector) mask vector

**Return value**  
realtype

**F2003 Name**  
FN_VWrmsNormMask

---

**N_VMin**

**Call**  
\[ m = \text{N_VMin}(x); \]

**Description**  
Returns the smallest element of the N_Vector \( x \):  
\[ m = \min_i x_i. \]

**Arguments**  
- \( x \) (N_Vector) a NVECTOR object

**Return value**  
realtype

**F2003 Name**  
FN_VMin

---

**N_VWL2Norm**

**Call**  
\[ m = \text{N_VWL2Norm}(x, w); \]

**Description**  
Returns the weighted Euclidean \( \ell_2 \) norm of the N_Vector \( x \) with realtype weight vector \( w \):  
\[ m = \sqrt{\sum_{i=0}^{n-1} (x_i w_i)^2}. \]

**Arguments**  
- \( x \) (N_Vector) a NVECTOR object
- \( w \) (N_Vector) a NVECTOR object containing weights

**Return value**  
realtype

**F2003 Name**  
FN_VWL2Norm

---

**N_VL1Norm**

**Call**  
\[ m = \text{N_VL1Norm}(x); \]

**Description**  
Returns the \( \ell_1 \) norm of the N_Vector \( x \):  
\[ m = \sum_{i=0}^{n-1} |x_i|. \]

**Arguments**  
- \( x \) (N_Vector) a NVECTOR object to obtain the norm of

**Return value**  
realtype

**F2003 Name**  
FN_VL1Norm
8.1 The NVECTOR API

N_VCompare
Call N_VCompare(c, x, z);
Description Compares the components of the N_Vector x to the realtype scalar c and returns an N_Vector z such that: \( z_i = 1.0 \) if \( |x_i| \geq c \) and \( z_i = 0.0 \) otherwise.
Arguments c (realtype) constant that each component of x is compared to
x (N_Vector) a NVECTOR object
z (N_Vector) a NVECTOR object containing the result
Return value None
F2003 Name FN_VCompare

N_VInvTest
Call t = N_VInvTest(x, z);
Description Sets the components of the N_Vector z to be the inverses of the components of the N_Vector x, with prior testing for zero values: \( z_i = 1.0/x_i, \ i = 0, \ldots, n-1 \).
Arguments x (N_Vector) a NVECTOR object
z (N_Vector) an output NVECTOR object
Return value Returns a booleantype with value SUNTRUE if all components of x are nonzero (successful inversion) and returns SUNFALSE otherwise.
F2003 Name FN_VInvTest

N_VConstrMask
Call t = N_VConstrMask(c, x, m);
Description Performs the following constraint tests: \( x_i > 0 \) if \( c_i = 2 \), \( x_i \geq 0 \) if \( c_i = 1 \), \( x_i \leq 0 \) if \( c_i = -1 \), \( x_i < 0 \) if \( c_i = -2 \). There is no constraint on \( x_i \) if \( c_i = 0 \). This routine returns a boolean assigned to SUNFALSE if any element failed the constraint test and assigned to SUNTRUE if all passed. It also sets a mask vector m, with elements equal to 1.0 where the constraint test failed, and 0.0 where the test passed. This routine is used only for constraint checking.
Arguments c (realtype) scalar constraint value
x (N_Vector) a NVECTOR object
m (N_Vector) output mask vector
Return value Returns a booleantype with value SUNFALSE if any element failed the constraint test, and SUNTRUE if all passed.
F2003 Name FN_VConstrMask

N_VMinQuotient
Call minq = N_VMinQuotient(num, denom);
Description This routine returns the minimum of the quotients obtained by term-wise dividing num, by denom. A zero element in denom will be skipped. If no such quotients are found, then the large value BIG_REAL (defined in the header file sundials_types.h) is returned.
Arguments num (N_Vector) a NVECTOR object used as the numerator
denom (N_Vector) a NVECTOR object used as the denominator
Return value realtype
F2003 Name FN_VMinQuotient
8.1.2 NVECTOR fused functions

Fused and vector array operations are intended to increase data reuse, reduce parallel communication on distributed memory systems, and lower the number of kernel launches on systems with accelerators. If a particular NVECTOR implementation defines a fused or vector array operation as NULL, the generic NVECTOR module will automatically call standard vector operations as necessary to complete the desired operation. In all SUNDIALS-provided NVECTOR implementations, all fused and vector array operations are disabled by default. However, these implementations provide additional user-callable functions to enable/disable any or all of the fused and vector array operations. See the following sections for the implementation specific functions to enable/disable operations.

\[ z_i = \sum_{j=0}^{n_v-1} c_j x_{j,i}, \quad i = 0, \ldots, n - 1, \]

where \( c \) is an array of \( n_v \) scalars, \( X \) is an array of \( n_v \) vectors, and \( z \) is the output vector.

**Arguments**
- `nv` (int) the number of scalars and vectors in \( c \), \( Y \), and \( Z \)
- `c` (realtype*) an array of \( n_v \) scalars used to scale the corresponding vector in \( X \)
- `X` (N_Vector*) an array of \( n_v \) NVECTOR objects to be scaled and combined
- `z` (N_Vector) a NVECTOR object containing the result

**Return value** Returns an int with value 0 for success and a non-zero value otherwise.

**Notes** If the output vector \( z \) is one of the vectors in \( X \), then it must be the first vector in the vector array.

F2003 Name FN_VLinearCombination

F2003 Call

```fortran
real(c_double) :: c(nv)
type(c_ptr), target :: X(nv)

type(N_Vector), pointer :: z

ier = FN_VLinearCombination(nv, c, X, z)
```

\[ z_{j,i} = c_j x_i + y_{j,i}, \quad j = 0, \ldots, n_v - 1, \quad i = 0, \ldots, n - 1, \]

where \( c \) is an array of \( n_v \) scalars, \( x \) is the vector to be scaled and added to each vector in the vector array of \( n_v \) vectors \( Y \), and \( Z \) is a vector array of \( n_v \) output vectors.

**Arguments**
- `nv` (int) the number of scalars and vectors in \( c \), \( Y \), and \( Z \)
- `c` (realtype*) an array of \( n_v \) scalars
- `x` (N_Vector) a NVECTOR object to be scaled and added to each vector in \( Y \)
- `Y` (N_Vector*) an array of \( n_v \) NVECTOR objects where each vector \( j \) will have the vector \( x \) scaled by \( c_j \) added to it
- `Z` (N_Vector) an output array of \( n_v \) NVECTOR objects

**Return value** Returns an int with value 0 for success and a non-zero value otherwise.

F2003 Name FN_VScaleAddMulti
8.1 The NVECTOR API

F2003 Call

real(c_double) :: c(nv)
type(c_ptr), target :: Y(nv), Z(nv)
type(N_Vector), pointer :: x
ier = FN_VScaleAddMulti(nv, c, x, Y, Z)

N_VDotProdMulti

Call

ier = N_VDotProdMulti(nv, x, Y, d);

Description

This routine computes the dot product of a vector with \( n_v \) other vectors:

\[
d_j = \sum_{i=0}^{n_v-1} x_i y_{j,i}, \quad j = 0, \ldots, n_v - 1,
\]

where \( d \) is an array of \( n_v \) scalars containing the dot products of the vector \( x \) with each of the \( n_v \) vectors in the vector array \( Y \).

Arguments

- \( n_v \) (int) the number of vectors in \( Y \)
- \( x \) (N_Vector) a NVector object to be used in a dot product with each of the vectors in \( Y \)
- \( Y \) (N_Vector*) an array of \( n_v \) NVectors objects to use in a dot product with \( x \)
- \( d \) (realtype*) an output array of \( n_v \) dot products

Return value

Returns an int with value 0 for success and a non-zero value otherwise.

F2003 Name

FN_VDotProdMulti

F2003 Call

real(c_double) :: d(nv)
type(c_ptr), target :: Y(nv)
type(N_Vector), pointer :: x
ier = FN_VDotProdMulti(nv, x, Y, d)

8.1.3 NVectors vector array functions

N_VLinearSumVectorArray

Call

ier = N_VLinearSumVectorArray(nv, a, X, b, Y, Z);

Description

This routine computes the linear sum of two vector arrays containing \( n_v \) vectors of \( n \) elements:

\[
z_{j,i} = a x_{j,i} + b y_{j,i}, \quad i = 0, \ldots, n - 1 \quad j = 0, \ldots, n_v - 1,
\]

where \( a \) and \( b \) are scalars and \( X, Y, \) and \( Z \) are arrays of \( n_v \) vectors.

Arguments

- \( n_v \) (int) the number of vectors in the vector arrays
- \( a \) (realtype) constant to scale each vector in \( X \) by
- \( X \) (N_Vector*) an array of \( n_v \) NVectors objects
- \( Y \) (N_Vector*) an array of \( n_v \) NVectors objects
- \( Z \) (N_Vector*) an output array of \( n_v \) NVectors objects

Return value

Returns an int with value 0 for success and a non-zero value otherwise.

F2003 Name

FN_VLinearSumVectorArray
180 Description of the NVECTOR module

N_VScaleVectorArray
Call
   ier = N_VScaleVectorArray(nv, c, X, Z);
Description
This routine scales each vector of n elements in a vector array of nv vectors by a potentially different constant:

\[ z_{j,i} = c_j x_{j,i}, \quad i = 0, \ldots, n - 1 \quad j = 0, \ldots, n_v - 1, \]

where c is an array of n_v scalars and X and Z are arrays of n_v vectors.

Arguments
- nv (int) the number of vectors in the vector arrays
- c (realtype) constant to scale each vector in X by
- X (N_Vector*) an array of n_v NVECTOR objects
- Z (N_Vector*) an output array of n_v NVECTOR objects

Return value
Returns an int with value 0 for success and a non-zero value otherwise.

F2003 Name FN_VScaleVectorArray

N_VConstVectorArray
Call
   ier = N_VConstVectorArray(nv, c, X);
Description
This routine sets each element in a vector of n elements in a vector array of nv vectors to the same value:

\[ z_{j,i} = c, \quad i = 0, \ldots, n - 1 \quad j = 0, \ldots, n_v - 1, \]

where c is a scalar and X is an array of n_v vectors.

Arguments
- nv (int) the number of vectors in X
- c (realtype) constant to set every element in every vector of X to
- X (N_Vector*) an array of n_v NVECTOR objects

Return value
Returns an int with value 0 for success and a non-zero value otherwise.

F2003 Name FN_VConstVectorArray

N_VWrmsNormVectorArray
Call
   ier = N_VWrmsNormVectorArray(nv, X, W, m);
Description
This routine computes the weighted root mean square norm of n_v vectors with n elements:

\[ m_j = \left( \frac{1}{n} \sum_{i=0}^{n-1} (x_{j,i}w_{j,i})^2 \right)^{1/2}, \quad j = 0, \ldots, n_v - 1, \]

where m contains the n_v norms of the vectors in the vector array X with corresponding weight vectors W.

Arguments
- nv (int) the number of vectors in the vector arrays
- X (N_Vector*) an array of n_v NVECTOR objects
- W (N_Vector*) an array of n_v NVECTOR objects
- m (realtype*) an output array of n_v norms

Return value
Returns an int with value 0 for success and a non-zero value otherwise.

F2003 Name FN_VWrmsNormVectorArray
8.1 The NVECTOR API

N_VWrmsNormMaskVectorArray

Call

\[ \text{ier} = \text{N\_VWrmsNormMaskVectorArray}(\text{nv}, \text{X}, \text{W}, \text{id}, \text{m}); \]

Description

This routine computes the masked weighted root mean square norm of \( n_v \) vectors with \( n \) elements:

\[
m_j = \left( \frac{1}{n} \sum_{i=0}^{n-1} (x_{j,i}w_{j,i}H(id_i))^2 \right)^{1/2}, \quad j = 0, \ldots, n_v - 1,
\]

where \( H(id_i) = 1 \) for \( id_i > 0 \) and is zero otherwise. \( m \) contains the \( n_v \) norms of the vectors in the vector array \( X \) with corresponding weight vectors \( W \) and mask vector \( \text{id} \).

Arguments

\( \text{nv} \) (int) the number of vectors in the vector arrays
\( \text{X} \) (N_Vector*) an array of \( n_v \) NVECTOR objects
\( \text{W} \) (N_Vector*) an array of \( n_v \) NVECTOR objects
\( \text{id} \) (N_Vector) the mask vector
\( \text{m} \) (realtype*) an output array of \( n_v \) norms

Return value

Returns an int with value 0 for success and a non-zero value otherwise.

F2003 Name

FN_VWrmsNormMaskVectorArray

N_VScaleAddMultiVectorArray

Call

\[ \text{ier} = \text{N\_VScaleAddMultiVectorArray}(\text{nv}, \text{ns}, \text{c}, \text{X}, \text{YY}, \text{ZZ}); \]

Description

This routine scales and adds a vector in a vector array of \( n_v \) vectors to the corresponding vector in \( n_s \) vector arrays:

\[
z_{j,i} = \sum_{k=0}^{n_s-1} c_kx_{k,j,i}, \quad i = 0, \ldots, n - 1 \quad j = 0, \ldots, n_v - 1,
\]

where \( c \) is an array of \( n_s \) scalars, \( X \) is a vector array of \( n_v \) vectors to be scaled and added to the corresponding vector in each of the \( n_s \) vector arrays in the array of vector arrays \( YY \) and stored in the output array of vector arrays \( ZZ \).

Arguments

\( \text{nv} \) (int) the number of vectors in the vector arrays
\( \text{ns} \) (int) the number of scalars in \( c \) and vector arrays in \( YY \) and \( ZZ \)
\( \text{c} \) (realtype*) an array of \( n_s \) scalars
\( \text{X} \) (N_Vector*) an array of \( n_v \) NVECTOR objects
\( \text{YY} \) (N_Vector**) an array of \( n_s \) NVECTOR arrays
\( \text{ZZ} \) (N_Vector**) an output array of \( n_s \) NVECTOR arrays

Return value

Returns an int with value 0 for success and a non-zero value otherwise.

N_VLinearCombinationVectorArray

Call

\[ \text{ier} = \text{N\_VLinearCombinationVectorArray}(\text{nv}, \text{ns}, \text{c}, \text{XX}, \text{Z}); \]

Description

This routine computes the linear combination of \( n_s \) vector arrays containing \( n_v \) vectors with \( n \) elements:

\[
z_{j,i} = \sum_{k=0}^{n_s-1} c_kx_{k,j,i}, \quad i = 0, \ldots, n - 1 \quad j = 0, \ldots, n_v - 1,
\]

where \( c \) is an array of \( n_s \) scalars (type realtype*). \( XX \) (type N_Vector**) is an array of \( n_s \) vector arrays each containing \( n_v \) vectors to be summed into the output vector array of \( n_v \) vectors \( Z \) (type N_Vector*). If the output vector array \( Z \) is one of the vector arrays in \( XX \), then it must be the first vector array in \( XX \).
Arguments  

nv (int) the number of vectors in the vector arrays  
ns (int) the number of scalars in c and vector arrays in YY and ZZ  
c (realtype*) an array of ns scalars  
XX (N_Vector**) an array of ns NVVECTOR arrays  
Z (N_Vector*) an output array NVVECTOR objects

Return value  
Returns an int with value 0 for success and a non-zero value otherwise.

### 8.1.4 NVVECTOR local reduction functions

Local reduction operations are intended to reduce parallel communication on distributed memory systems, particularly when NVVECTOR objects are combined together within a NVVECTOR_MPIANYVECTOR object (see Section 8.14). If a particular NVVECTOR implementation defines a local reduction operation as NULL, the NVVECTOR_MPIANYVECTOR module will automatically call standard vector reduction operations as necessary to complete the desired operation. All SUNDIALS-provided NVVECTOR implementations include these local reduction operations, which may be used as templates for user-defined NVVECTOR implementations.

#### N_VDotProdLocal

**Call**

```c
N_VDotProdLocal(x, y);
```

**Description**  
This routine computes the MPI task-local portion of the ordinary dot product of x and y:

\[ d = \sum_{i=0}^{n_{\text{local}}-1} x_i y_i, \]

where \( n_{\text{local}} \) corresponds to the number of components in the vector on this MPI task (or \( n_{\text{local}} = n \) for MPI-unaware applications).

**Arguments**  

x (N_Vector) a NVVECTOR object  
y (N_Vector) a NVVECTOR object

**Return value**  
realtype

F2003 Name  
FN_VDotProdLocal

#### N_VMaxNormLocal

**Call**

```c
N_VMaxNormLocal(x);
```

**Description**  
This routine computes the MPI task-local portion of the maximum norm of the N_Vector x:

\[ m = \max_{0 \leq i < n_{\text{local}}} |x_i|, \]

where \( n_{\text{local}} \) corresponds to the number of components in the vector on this MPI task (or \( n_{\text{local}} = n \) for MPI-unaware applications).

**Arguments**  

x (N_Vector) a NVVECTOR object

**Return value**  
realtype

F2003 Name  
FN_VMaxNormLocal

#### N_VMinLocal

**Call**

```c
N_VMinLocal(x);
```

**Description**  
This routine computes the MPI task-local portion of the minimum norm of the N_Vector x:

\[ m = \min_{0 \leq i < n_{\text{local}}} |x_i|, \]

where \( n_{\text{local}} \) corresponds to the number of components in the vector on this MPI task (or \( n_{\text{local}} = n \) for MPI-unaware applications).

**Arguments**  

x (N_Vector) a NVVECTOR object

**Return value**  
realtype

F2003 Name  
FN_VMinLocal
8.1 The NVECTOR API

Description This routine computes the smallest element of the MPI task-local portion of the N_Vector $x$:

$$m = \min_{0 \leq i < n_{local}} x_i,$$

where $n_{local}$ corresponds to the number of components in the vector on this MPI task (or $n_{local} = n$ for MPI-unaware applications).

Arguments $x$ (N_Vector) a NVECTOR object

Return value realltype

F2003 Name FN_VMinLocal

N_VL1NormLocal

Call $n = \text{N_VL1NormLocal}(x);$

Description This routine computes the MPI task-local portion of the $\ell_1$ norm of the N_Vector $x$:

$$n = \sum_{i=0}^{n_{local}-1} |x_i|,$$

where $n_{local}$ corresponds to the number of components in the vector on this MPI task (or $n_{local} = n$ for MPI-unaware applications).

Arguments $x$ (N_Vector) a NVECTOR object

Return value realltype

F2003 Name FN_VL1NormLocal

N_VWSqrSumLocal

Call $s = \text{N_VWSqrSumLocal}(x,w);$ 

Description This routine computes the MPI task-local portion of the weighted squared sum of the N_Vector $x$ with weight vector $w$:

$$s = \sum_{i=0}^{n_{local}-1} (x_i w_i)^2,$$

where $n_{local}$ corresponds to the number of components in the vector on this MPI task (or $n_{local} = n$ for MPI-unaware applications).

Arguments $x$ (N_Vector) a NVECTOR object  
$w$ (N_Vector) a NVECTOR object containing weights

Return value realltype

F2003 Name FN_VWSqrSumLocal

N_VWSqrSumMaskLocal

Call $s = \text{N_VWSqrSumMaskLocal}(x,w,id);$ 

Description This routine computes the MPI task-local portion of the weighted squared sum of the N_Vector $x$ with weight vector $w$ built using only the elements of $x$ corresponding to positive elements of the N_Vector $id$:

$$m = \sum_{i=0}^{n_{local}-1} (x_i w_i H(id_i))^2, \text{ where } H(\alpha) = \begin{cases} 1 & \alpha > 0 \\ 0 & \alpha \leq 0 \end{cases}$$

and $n_{local}$ corresponds to the number of components in the vector on this MPI task (or $n_{local} = n$ for MPI-unaware applications).
**Description of the NVECTOR module**

Arguments

- `x` (*N_Vector*) a NVECTOR object
- `w` (*N_Vector*) a NVECTOR object containing weights
- `id` (*N_Vector*) a NVECTOR object used as a mask

Return value: `realtype`

**F2003 Name:** `FN_VWSqrSumMaskLocal`

**N_VInvTestLocal**

Call: `t = N_VInvTestLocal(x, z);`

Description: Sets the MPI task-local components of the `N_Vector` `z` to be the inverses of the components of the `N_Vector` `x`, with prior testing for zero values:

\[ z_i = \frac{1.0}{x_i}, \quad i = 0, \ldots, n_{local} - 1, \]

where `n_{local}` corresponds to the number of components in the vector on this MPI task (or `n_{local} = n` for MPI-unaware applications).

Arguments

- `x` (*N_Vector*) a NVECTOR object
- `z` (*N_Vector*) an output NVECTOR object

Return value: Returns a `booleantype` with the value `SUNTRUE` if all task-local components of `x` are nonzero (successful inversion) and with the value `SUNFALSE` otherwise.

**F2003 Name:** `FN_VInvTestLocal`

**N_VConstrMaskLocal**

Call: `t = N_VConstrMaskLocal(c, x, m);`

Description: Performs the following constraint tests:

- \( x_i > 0 \) if \( c_i = 2 \),
- \( x_i \geq 0 \) if \( c_i = 1 \),
- \( x_i \leq 0 \) if \( c_i = -1 \),
- \( x_i < 0 \) if \( c_i = -2 \), and
- no test if \( c_i = 0 \),

for all MPI task-local components of the vectors. It sets a mask vector `m`, with elements equal to 1.0 where the constraint test failed, and 0.0 where the test passed. This routine is used only for constraint checking.

Arguments

- `c` (*realtype*) scalar constraint value
- `x` (*N_Vector*) a NVECTOR object
- `m` (*N_Vector*) output mask vector

Return value: Returns a `booleantype` with the value `SUNFALSE` if any task-local element failed the constraint test and the value `SUNTRUE` if all passed.

**F2003 Name:** `FN_VConstrMaskLocal`

**N_VMinQuotientLocal**

Call: `minq = N_VMinQuotientLocal(num, denom);`

Description: This routine returns the minimum of the quotients obtained by term-wise dividing \( num_i \) by \( denom_i \), for all MPI task-local components of the vectors. A zero element in `denom` will be skipped. If no such quotients are found, then the large value `BIG_REAL` (defined in the header file `sundials_types.h`) is returned.
Arguments  
num (N_Vector) a NVECTOR object used as the numerator  
denom (N_Vector) a NVECTOR object used as the denominator  

Return value  
realtype  

F2003 Name  FN_VMinQuotientLocal

8.1.5 NVECTOR utility functions

To aid in the creation of custom NVECTOR modules the generic NVECTOR module provides three utility functions N_VNewEmpty, N_VCopyOps and N_VFreeEmpty. When used in custom NVECTOR constructors and clone routines these functions will ease the introduction of any new optional vector operations to the NVECTOR API by ensuring only required operations need to be set and all operations are copied when cloning a vector.

To aid the use of arrays of NVECTOR objects, the generic NVECTOR module also provides the utility functions N_VCloneVectorArray, N_VCloneVectorArrayEmpty, and N_VDestroyVectorArray.

**N_VNewEmpty**

Call  
v = N_VNewEmpty();  
Description The function N_VNewEmpty allocates a new generic NVECTOR object and initializes its content pointer and the function pointers in the operations structure to NULL.  
Arguments None  
Return value This function returns an N_Vector object. If an error occurs when allocating the object, then this routine will return NULL.

F2003 Name FN_VNewEmpty

**N_VCopyOps**

Call  
retval = N_VCopyOps(w, v);  
Description The function N_VCopyOps copies the function pointers in the ops structure of w into the ops structure of v.  
Arguments w (N_Vector) the vector to copy operations from  
v (N_Vector) the vector to copy operations to  
Return value This returns 0 if successful and a non-zero value if either of the inputs are NULL or the ops structure of either input is NULL.

F2003 Name FN_VCopyOps

**N_VFreeEmpty**

Call  
N_VFreeEmpty(v);  
Description This routine frees the generic N_Vector object, under the assumption that any implementation-specific data that was allocated within the underlying content structure has already been freed. It will additionally test whether the ops pointer is NULL, and, if it is not, it will free it as well.  
Arguments v (N_Vector)  
Return value None  
F2003 Name FN_VFreeEmpty
**N_VCloneEmptyVectorArray**

Call

```c
vecarray = N_VCloneEmptyVectorArray(count, w);
```

Description Creates an array of count variables of type N_Vector, each of the same type as the existing N_Vector w. It achieves this by calling the implementation-specific N_VCloneEmpty operation.

Arguments

- `count` (int) the size of the vector array
- `w` (N_Vector) the vector to clone

Return value Returns an array of count N_Vector objects if successful, or NULL if an error occurred while cloning.

**N_VCloneVectorArray**

Call

```c
vecarray = N_VCloneVectorArray(count, w);
```

Description Creates an array of count variables of type N_Vector, each of the same type as the existing N_Vector w. It achieves this by calling the implementation-specific N_VClone operation.

Arguments

- `count` (int) the size of the vector array
- `w` (N_Vector) the vector to clone

Return value Returns an array of count N_Vector objects if successful, or NULL if an error occurred while cloning.

**N_VDestroyVectorArray**

Call

```c
N_VDestroyVectorArray(count, w);
```

Description Destroys (frees) an array of variables of type N_Vector. It depends on the implementation-specific N_VDestroy operation.

Arguments

- `vs` (N_Vector*) the array of vectors to destroy
- `count` (int) the size of the vector array

Return value None

**N_VNewVectorArray**

Call

```c
vecarray = N_VNewVectorArray(count);
```

Description Returns an empty N_Vector array large enough to hold count N_Vector objects. This function is primarily meant for users of the Fortran 2003 interface.

Arguments

- `count` (int) the size of the vector array

Return value Returns a N_Vector* if successful, Returns NULL if an error occurred.

Notes Users of the Fortran 2003 interface to the N_VManyVector or N_VMPIManyVector will need this to create an array to hold the subvectors. Note that this function does restrict the the max number of subvectors usable with the N_VManyVector and N_VMPIManyVector to the max size of an int despite the ManyVector implementations accepting a subvector count larger than this value.

F2003 Name FN_VNewVectorArray
Table 8.1: Vector Identifications associated with vector kernels supplied with SUNDIALS.

<table>
<thead>
<tr>
<th>Vector ID</th>
<th>Vector type</th>
<th>ID Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>SUNDIALS_NVEC_SERIAL</td>
<td>Serial</td>
<td>0</td>
</tr>
<tr>
<td>SUNDIALS_NVEC_PARALLEL</td>
<td>Distributed memory parallel (MPI)</td>
<td>1</td>
</tr>
<tr>
<td>SUNDIALS_NVEC_OPENMP</td>
<td>OpenMP shared memory parallel</td>
<td>2</td>
</tr>
<tr>
<td>SUNDIALS_NVEC_PTHREADS</td>
<td>PThreads shared memory parallel</td>
<td>3</td>
</tr>
<tr>
<td>SUNDIALS_NVEC_PARHYPP</td>
<td>hypre ParHyp parallel vector</td>
<td>4</td>
</tr>
<tr>
<td>SUNDIALS_NVEC_PETSC</td>
<td>PETSc parallel vector</td>
<td>5</td>
</tr>
<tr>
<td>SUNDIALS_NVEC_CUDA</td>
<td>CUDA parallel vector</td>
<td>6</td>
</tr>
<tr>
<td>SUNDIALS_NVEC_RAJA</td>
<td>RAJA parallel vector</td>
<td>7</td>
</tr>
<tr>
<td>SUNDIALS_NVEC_OPENMPDEV</td>
<td>OpenMP parallel vector with device offloading</td>
<td>8</td>
</tr>
<tr>
<td>SUNDIALS_NVEC_TRILINOS</td>
<td>Trilinos Tpetra vector</td>
<td>9</td>
</tr>
<tr>
<td>SUNDIALS_NVEC_MANYVECTOR</td>
<td>“ManyVector” vector</td>
<td>10</td>
</tr>
<tr>
<td>SUNDIALS_NVEC_MPI MANYVECTOR</td>
<td>MPI-enabled “ManyVector” vector</td>
<td>11</td>
</tr>
<tr>
<td>SUNDIALS_NVEC_MPIPLUSX</td>
<td>MPI+X vector</td>
<td>12</td>
</tr>
<tr>
<td>SUNDIALS_NVEC_CUSTOM</td>
<td>User-provided custom vector</td>
<td>13</td>
</tr>
</tbody>
</table>

[N_VGetVecAtIndexVectorArray](#)

Call  
\[ v = \text{N_VGetVecAtIndexVectorArray}(\text{vecs}, \text{index}); \]

Description  
Returns the \text{N_Vector} object stored in the vector array at the provided index. This function is primarily meant for users of the Fortran 2003 interface.

Arguments  
- \text{vecs} (\text{N Vector}*) the array of vectors to index
- \text{index} (\text{int}) the index of the vector to return

Return value  
Returns the \text{N Vector} object stored in the vector array at the provided index. Returns NULL if an error occurred.

F2003 Name  
\text{FN_VGetVecAtIndexVectorArray}

[N_VSetVecAtIndexVectorArray](#)

Call  
\[ \text{N_VSetVecAtIndexVectorArray}(\text{vecs}, \text{index}, v); \]

Description  
Sets the \text{N Vector} object stored in the vector array at the provided index. This function is primarily meant for users of the Fortran 2003 interface.

Arguments  
- \text{vecs} (\text{N Vector}*) the array of vectors to index
- \text{index} (\text{int}) the index of the vector to return
- \text{v} (\text{N Vector}) the vector to store at the index

Return value  
None

F2003 Name  
\text{FN_VSetVecAtIndexVectorArray}

### 8.1.6 NVECTOR identifiers

Each NVECTOR implementation included in SUNDIALS has a unique identifier specified in enumeration and shown in Table 8.1.

### 8.1.7 The generic NVECTOR module implementation

The generic N Vector type is a pointer to a structure that has an implementation-dependent content field containing the description and actual data of the vector, and an ops field pointing to a structure with generic vector operations. The type N Vector is defined as
typedef struct _generic_N_Vector *N_Vector;

struct _generic_N_Vector {
    void *content;
    struct _generic_N_Vector_Ops *ops;
};

The _generic_N_Vector_Ops structure is essentially a list of pointers to the various actual vector operations, and is defined as

struct _generic_N_Vector_Ops {
    N_Vector_ID (*nvgetvectorid)(N_Vector);
    N_Vector (*nvclone)(N_Vector);
    N_Vector (*nvcloneempty)(N_Vector);
    void (*nvdestroy)(N_Vector);
    void (*nvspace)(N_Vector, sunindextype *, sunindextype *);
    realtype* (*nvgetarraypointer)(N_Vector);
    void (*nvsetarraypointer)(realtype *, N_Vector);
    void* (*nvgetcommunicator)(N_Vector);
    sunindextype (*nvgetlength)(N_Vector);
    void (*nvlinearsum)(realtype, N_Vector, realtype, N_Vector, N_Vector);
    void (*nvconst)(realtype, N_Vector);
    void (*nvprod)(N_Vector, N_Vector, N_Vector);
    void (*nvdiv)(N_Vector, N_Vector, N_Vector);
    void (*nvscale)(realtype, N_Vector, N_Vector);
    void (*nvabs)(N_Vector, N_Vector);
    void (*nvinv)(N_Vector, N_Vector);
    void (*nvaddconst)(N_Vector, realtype, N_Vector);
    realtype (*nvdotprod)(N_Vector, N_Vector);
    realtype (*nvmaxnorm)(N_Vector);
    realtype (*nvwrmsnorm)(N_Vector, N_Vector);
    realtype (*nvwrmsnormmask)(N_Vector, N_Vector, N_Vector);
    realtype (*nvmin)(N_Vector);
    realtype (*nvwl2norm)(N_Vector, N_Vector);
    realtype (*nvcompare)(realtype, N_Vector, N_Vector);
    booleantype (*nvinvtest)(N_Vector, N_Vector);
    booleantype (*nvconstrmask)(N_Vector, N_Vector);
    realtype (*nvminquotient)(N_Vector, N_Vector);
    int (*nvlinearcombination)(int, realtype*, N_Vector*, N_Vector);
    int (*nvscaleaddmulti)(int, realtype*, N_Vector, N_Vector*, N_Vector*);
    int (*nvdotprodmulti)(int, N_Vector, N_Vector*, realtype*);
    int (*nvlinearsumvectorarray)(int, realtype, N_Vector*, N_Vector*, N_Vector*);
    int (*nvwrmsnormvectorarray)(int, N_Vector*, N_Vector*, realtype*);
    int (*nvscaleaddmultivectorarray)(int, int, realtype*, N_Vector*, N_Vector*, N_Vector*);
    int (*nvlinearcombinationvectorarray)(int, int, realtype*, N_Vector*, N_Vector*, N_Vector*);
    realtype (*nvdotprodlocal)(N_Vector, N_Vector);
    realtype (*nvmaxnormlocal)(N_Vector);
The generic nvector module defines and implements the vector operations acting on an *N* Vector. These routines are nothing but wrappers for the vector operations defined by a particular nvector implementation, which are accessed through the *ops* field of the *N* Vector structure. To illustrate this point we show below the implementation of a typical vector operation from the generic nvector module, namely N_VScale, which performs the scaling of a vector *x* by a scalar *c*:

```c
void N_VScale(realtype c, N_Vector x, N_Vector z) {
    z->ops->nvscale(c, x, z);
}
```

Section 8.1.1 defines a complete list of all standard vector operations defined by the generic nvector module. Sections 8.1.2, 8.1.3 and 8.1.4 list optional fused, vector array and local reduction operations, respectively.

The Fortran 2003 interface provides a *bind(C)* derived-type for the _generic_N_Vector and the _generic_N_Vector_Ops structures. Their definition is given below.

```fortran
type, bind(C), public :: N_Vector
    type(C_PTR), public :: content
    type(C_PTR), public :: ops
end type N_Vector

type, bind(C), public :: N_Vector_Ops
    type(C_FUNPTR), public :: nvgetvectorid
    type(C_FUNPTR), public :: nvclone
    type(C_FUNPTR), public :: nvcloneempty
    type(C_FUNPTR), public :: nvdestroy
    type(C_FUNPTR), public :: nvspace
    type(C_FUNPTR), public :: nvgetarraypointer
    type(C_FUNPTR), public :: nvsetarraypointer
    type(C_FUNPTR), public :: nvgetcommunicator
    type(C_FUNPTR), public :: nvgetlength
    type(C_FUNPTR), public :: nvlinsrsum
    type(C_FUNPTR), public :: nvconst
    type(C_FUNPTR), public :: nvprod
    type(C_FUNPTR), public :: nvddiv
    type(C_FUNPTR), public :: nvscale
    type(C_FUNPTR), public :: nvabs
    type(C_FUNPTR), public :: nvinv
    type(C_FUNPTR), public :: nvaddconst
    type(C_FUNPTR), public :: nvdotprod
    type(C_FUNPTR), public :: nvmaxnorm
    type(C_FUNPTR), public :: nvwrmsnorm
    type(C_FUNPTR), public :: nvwrmsnormmask
    type(C_FUNPTR), public :: nvmin
    type(C_FUNPTR), public :: nvwl2norm
```
type(C_FUNPTR), public :: nvl1norm
(type(C_FUNPTR), public :: nvcompare
(type(C_FUNPTR), public :: nvinvtest
(type(C_FUNPTR), public :: nvconstmask
(type(C_FUNPTR), public :: nvminquotient
(type(C_FUNPTR), public :: nvlinearcombination
(type(C_FUNPTR), public :: nsvscaleaddmulti
(type(C_FUNPTR), public :: nvdotprodmulti
(type(C_FUNPTR), public :: nsvlinearsumvectorarray
(type(C_FUNPTR), public :: nvscalevectorarray
(type(C_FUNPTR), public :: nvconstvectorarray
(type(C_FUNPTR), public :: nvwrmsnormvectorarray
(type(C_FUNPTR), public :: nvwrmsnormmaskvectorarray
(type(C_FUNPTR), public :: nsvscaleaddmultivectorarray
(type(C_FUNPTR), public :: nvlinearcombinationvectorarray
(type(C_FUNPTR), public :: nvdotprodlocal
(type(C_FUNPTR), public :: nvmaxnrmlocal
(type(C_FUNPTR), public :: nvminlocal
(type(C_FUNPTR), public :: nvl1normlocal
(type(C_FUNPTR), public :: nvinvtestlocal
(type(C_FUNPTR), public :: nvminquotientlocal
(type(C_FUNPTR), public :: nwsqsrmultlocal
(type(C_FUNPTR), public :: nvwssqsummasklocal
end type N_Vector_Ops

8.1.8 Implementing a custom NVVECTOR

A particular implementation of the NVVECTOR module must:

- Specify the content field of N_Vector.
- Define and implement the vector operations. Note that the names of these routines should be unique to that implementation in order to permit using more than one NVVECTOR module (each with different N_Vector internal data representations) in the same code.

- Define and implement user-callable constructor and destructor routines to create and free an N_Vector with the new content field and with ops pointing to the new vector operations.

- Optionally, define and implement additional user-callable routines acting on the newly defined N_Vector (e.g., a routine to print the content for debugging purposes).

- Optionally, provide accessor macros as needed for that particular implementation to be used to access different parts in the content field of the newly defined N_Vector.

It is recommended that a user-supplied NVVECTOR implementation returns the SUNDIALS_NVEC_CUSTOM identifier from the N_VGetVectorID function.

To aid in the creation of custom NVVECTOR modules the generic NVVECTOR module provides two utility functions N_VNewEmpty and N_VCopyOps. When used in custom NVVECTOR constructors and clone routines these functions will ease the introduction of any new optional vector operations to the NVVECTOR API by ensuring only required operations need to be set and all operations are copied when cloning a vector.

8.1.8.1 Support for complex-valued vectors

While Sundials itself is written under an assumption of real-valued data, it does provide limited support for complex-valued problems. However, since none of the built-in NVVECTOR modules supports
complex-valued data, users must provide a custom NVECTOR implementation for this task. Many of the NVECTOR routines described in Sections 8.1.1-8.1.4 above naturally extend to complex-valued vectors; however, some do not. To this end, we provide the following guidance:

- `N_VMin` and `N_VMinLocal` should return the minimum of all real components of the vector, i.e., $m = \min_i \text{real}(x_i)$.
- `N_VConst` (and similarly `N_VConstVectorArray`) should set the real components of the vector to the input constant, and set all imaginary components to zero, i.e., $z_i = c + 0j$, $i = 0, \ldots, n - 1$.
- `N_VAddConst` should only update the real components of the vector with the input constant, leaving all imaginary components unchanged.
- `N_VWrmNorm`, `N_VWrmNormMask`, `N_VWSqrSumLocal` and `N_VWSqrSumMaskLocal` should assume that all entries of the weight vector $w$ and the mask vector $id$ are real-valued.
- `N_VDotProd` should mathematically return a complex number for complex-valued vectors; as this is not possible with SUNDIALS' current realtype, this routine should be set to NULL in the custom NVECTOR implementation.
- `N_VCompare`, `N_VConstrMask`, `N_VMinQuotient`, `N_VConstrMaskLocal` and `N_VMinQuotientLocal` are ill-defined due to the lack of a clear ordering in the complex plane. These routines should be set to NULL in the custom NVECTOR implementation.

While many SUNDIALS solver modules may be utilized on complex-valued data, others cannot. Specifically, although both SUNNONLIN_SOL_NEWTON and SUNNONLIN_SOL_FIXEDPOINT may be used with any of the IVP solvers (CVODE, CVODES, IDA, IDAS and ARKODE) for complex-valued problems, the Anderson-acceleration feature SUNNONLIN_SOL_FIXEDPOINT cannot be used due to its reliance on `N_VDotProd`. By this same logic, the Anderson acceleration feature within KINSOL also will not work with complex-valued vectors.

Similarly, although each package’s linear solver interface (e.g., CVLS) may be used on complex-valued problems, none of the built-in SUNMATRIX or SUNLINSOL modules work. Hence a complex-valued user should provide a custom SUNLINSOL (and optionally a custom SUNMATRIX) implementation for solving linear systems, and then attach this module as normal to the package’s linear solver interface.

Finally, constraint-handling features of each package cannot be used for complex-valued data, due to the issue of ordering in the complex plane discussed above with `N_VCompare`, `N_VConstrMask`, `N_VMinQuotient`, `N_VConstrMaskLocal` and `N_VMinQuotientLocal`.

We provide a simple example of a complex-valued example problem, including a custom complex-valued Fortran 2003 NVECTOR module, in the files `examples/arkode/F2003_custom/ark_analytic_complex_f2003.f90`, `examples/arkode/F2003_custom/invector_complex_mod.f90`, and `examples/arkode/F2003_custom/test_invector_complex_mod.f90`.

### 8.2 NVECTOR functions used by IDAS

In Table 8.2 below, we list the vector functions used in the NVECTOR module used by the IDAS package. The table also shows, for each function, which of the code modules uses the function. The IDAS column shows function usage within the main integrator module, while the remaining columns show function usage within the IDAS linear solvers interface, the IDABBDPRE preconditioner module, and the IDAA module.

At this point, we should emphasize that the IDAS user does not need to know anything about the usage of vector functions by the IDAS code modules in order to use IDAS. The information is presented as an implementation detail for the interested reader.

Special cases (numbers match markings in table):
1. These routines are only required if an internal difference-quotient routine for constructing dense or band Jacobian matrices is used.

2. This routine is optional, and is only used in estimating space requirements for IDAS modules for user feedback.

3. The optional function \texttt{NVDotProdMulti} is only used when Classical Gram-Schmidt is enabled with \texttt{SPGMR} or \texttt{SPFGMR}. The remaining operations from Tables 8.1.2 and 8.1.3 not listed above are unused and a user-supplied NVECTOR module for IDAS could omit these operations.

4. This routine is only used when an iterative or matrix iterative SUNLINSOL module is supplied to IDAS.

Of the functions listed in Table 8.1.1, \texttt{NDotProd}, \texttt{NVL2Norm}, \texttt{NVL1Norm}, \texttt{NInvTest}, and \texttt{NGetCommunicator} are not used by IDAS. Therefore a user-supplied NVECTOR module for IDAS could omit these functions (although some may be needed by SUNNONLINSOL or SUNLINSOL modules).

### 8.3 The NVECTOR\textunderscore SERIAL implementation

The serial implementation of the NVECTOR module provided with SUNDIALS, NVECTOR\textunderscore SERIAL, defines the \texttt{content} field of \texttt{NVector} to be a structure containing the length of the vector, a pointer to the beginning of a contiguous data array, and a boolean flag \texttt{own\_data} which specifies the ownership of \texttt{data}.

```c
struct _N_VectorContent_Serial {
    sunindextype length;
    booleantype own_data;
    realtype *data;
};
```

The header file to include when using this module is \texttt{nvector\_serial.h}. The installed module library to link to is \texttt{libsundials_nvecserial.lib} where \texttt{.lib} is typically \texttt{.so} for shared libraries and \texttt{.a} for static libraries.

#### 8.3.1 NVECTOR\textunderscore SERIAL accessor macros

The following macros are provided to access the content of an NVECTOR\textunderscore SERIAL vector. The suffix \texttt{S} in the names denotes the serial version.

- **NV\_CONTENT\_S**
  
  This routine gives access to the contents of the serial vector \texttt{NVector}.
  
  The assignment \texttt{v\_cont = NV\_CONTENT\_S(v)} sets \texttt{v\_cont} to be a pointer to the serial \texttt{NVector} content structure.
  
  Implementation:
  ```c
  #define NV\_CONTENT\_S(v) ((N\_VectorContent\_Serial)(v->content))
  ```

- **NV\_OWN\_DATA\_S, NV\_DATA\_S, NV\_LENGTH\_S**
  
  These macros give individual access to the parts of the content of a serial \texttt{NVector}.
  
  The assignment \texttt{v\_data = NV\_DATA\_S(v)} sets \texttt{v\_data} to be a pointer to the first component of the data for the \texttt{NVector} \texttt{v}. The assignment \texttt{NV\_DATA\_S(v) = v\_data} sets the component array of \texttt{v} to be \texttt{v\_data} by storing the pointer \texttt{v\_data}.
  
  The assignment \texttt{v\_len = NV\_LENGTH\_S(v)} sets \texttt{v\_len} to be the length of \texttt{v}. On the other hand, the call \texttt{NV\_LENGTH\_S(v) = len\_v} sets the length of \texttt{v} to be \texttt{len\_v}.
  
  Implementation:
#define NV_OWN_DATA_S(v) ( NV_CONTENT_S(v)->own_data )
#define NV_DATA_S(v) ( NV_CONTENT_S(v)->data )
#define NV_LENGTH_S(v) ( NV_CONTENT_S(v)->length )

• NV_Ith_S

This macro gives access to the individual components of the data array of an N_Vector.

The assignment r = NV_Ith_S(v,i) sets r to be the value of the i-th component of v. The
assignment NV_Ith_S(v,i) = r sets the value of the i-th component of v to be r.

Here i ranges from 0 to n \( - 1 \) for a vector of length n.

Implementation:
#define NV_Ith_S(v,i) ( NV_DATA_S(v)[i] )

## 8.3.2 NVECTOR_SERIAL functions

The NVECTOR_SERIAL module defines serial implementations of all vector operations listed in Tables 8.1.1, 8.1.2, 8.1.3 and 8.1.4. Their names are obtained from those in these tables by appending the suffix _Serial (e.g. N_VDestroy_Serial). All the standard vector operations listed in 8.1.1 with the suffix _Serial appended are callable via the FORTRAN 2003 interface by prepending an ‘F’ (e.g. FN_VDestroy_Serial).

The module NVECTOR_SERIAL provides the following additional user-callable routines:

### N_VNew_Serial

Prototype N_Vector N_VNew_Serial(sunindextype vec_length);

Description This function creates and allocates memory for a serial N_Vector. Its only argument is the vector length.

F2003 Name This function is callable as FN_VNew_Serial when using the Fortran 2003 interface module.

### N_VNewEmpty_Serial

Prototype N_Vector N_VNewEmpty_Serial(sunindextype vec_length);

Description This function creates a new serial N_Vector with an empty (NULL) data array.

F2003 Name This function is callable as FN_VNewEmpty_Serial when using the Fortran 2003 interface module.

### N_VMake_Serial

Prototype N_Vector N_VMake_Serial(sunindextype vec_length, realtype *v_data);

Description This function creates and allocates memory for a serial vector with user-provided data array.

   (This function does not allocate memory for v_data itself.)

F2003 Name This function is callable as FN_VMake_Serial when using the Fortran 2003 interface module.

### N_VCloneVectorArray_Serial

Prototype N_Vector *N_VCloneVectorArray_Serial(int count, N_Vector w);

Description This function creates (by cloning) an array of count serial vectors.

F2003 Name This function is callable as FN_VCloneVectorArray_Serial when using the Fortran 2003 interface module.
Description of the NVVECTOR module

**N_VCloneVectorArrayEmpty_Serial**

Prototype: ```c
N_Vector *N_VCloneVectorArrayEmpty_Serial(int count, N_Vector w);
```  
Description: This function creates (by cloning) an array of `count` serial vectors, each with an empty (NULL) data array.  
F2003 Name: This function is callable as `FN_VCloneVectorArrayEmpty_Serial` when using the Fortran 2003 interface module.

**N_VDestroyVectorArray_Serial**

Prototype: ```c
void N_VDestroyVectorArray_Serial(N_Vector *vs, int count);
```  
Description: This function frees memory allocated for the array of `count` variables of type `N_Vector` created with `N_VCloneVectorArray_Serial` or with `N_VCloneVectorArrayEmpty_Serial`.  
F2003 Name: This function is callable as `FN_VDestroyVectorArray_Serial` when using the Fortran 2003 interface module.

**N_VPrint_Serial**

Prototype: ```c
void N_VPrint_Serial(N_Vector v);
```  
Description: This function prints the content of a serial vector to stdout.  
F2003 Name: This function is callable as `FN_VPrint_Serial` when using the Fortran 2003 interface module.

**N_VPrintFile_Serial**

Prototype: ```c
void N_VPrintFile_Serial(N_Vector v, FILE *outfile);
```  
Description: This function prints the content of a serial vector to outfile.  
F2003 Name: This function is callable as `FN_VPrintFile_Serial` when using the Fortran 2003 interface module.

By default all fused and vector array operations are disabled in the `nvектор_serial` module. The following additional user-callable routines are provided to enable or disable fused and vector array operations for a specific vector. To ensure consistency across vectors it is recommended to first create a vector with `N_VNew_Serial`, enable/disable the desired operations for that vector with the functions below, and create any additional vectors from that vector using `N_VClone`. This guarantees the new vectors will have the same operations enabled/disabled as cloned vectors inherit the same enable/disable options as the vector they are cloned from while vectors created with `N_VNew_Serial` will have the default settings for the `nvектор_serial` module.

**N_VEnableFusedOps_Serial**

Prototype: ```c
int N_VEnableFusedOps_Serial(N_Vector v, booleantype tf);
```  
Description: This function enables (SUNTRUE) or disables (SUNFALSE) all fused and vector array operations in the serial vector. The return value is 0 for success and -1 if the input vector or its ops structure are NULL.  
F2003 Name: This function is callable as `FN_VEnableFusedOps_Serial` when using the Fortran 2003 interface module.
### 8.3 The NVECTOR_SERIAL Implementation

<table>
<thead>
<tr>
<th>Function Name</th>
<th>Prototype</th>
<th>Description</th>
<th>F2003 Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>N_VEnableLinearCombination_Serial</td>
<td>int N_VEnableLinearCombination_Serial(N_Vector v, booleantype tf);</td>
<td>This function enables (SUNTRUE) or disables (SUNFALSE) the linear combination fused operation in the serial vector. The return value is 0 for success and -1 if the input vector or its ops structure are NULL.</td>
<td>This function is callable as FN_VEnableLinearCombination_Serial when using the Fortran 2003 interface module.</td>
</tr>
<tr>
<td>N_VEnableScaleAddMulti_Serial</td>
<td>int N_VEnableScaleAddMulti_Serial(N_Vector v, booleantype tf);</td>
<td>This function enables (SUNTRUE) or disables (SUNFALSE) the scale and add a vector to multiple vectors fused operation in the serial vector. The return value is 0 for success and -1 if the input vector or its ops structure are NULL.</td>
<td>This function is callable as FN_VEnableScaleAddMulti_Serial when using the Fortran 2003 interface module.</td>
</tr>
<tr>
<td>N_VEnableDotProdMulti_Serial</td>
<td>int N_VEnableDotProdMulti_Serial(N_Vector v, booleantype tf);</td>
<td>This function enables (SUNTRUE) or disables (SUNFALSE) the multiple dot products fused operation in the serial vector. The return value is 0 for success and -1 if the input vector or its ops structure are NULL.</td>
<td>This function is callable as FN_VEnableDotProdMulti_Serial when using the Fortran 2003 interface module.</td>
</tr>
<tr>
<td>N_VEnableLinearSumVectorArray_Serial</td>
<td>int N_VEnableLinearSumVectorArray_Serial(N_Vector v, booleantype tf);</td>
<td>This function enables (SUNTRUE) or disables (SUNFALSE) the linear sum operation for vector arrays in the serial vector. The return value is 0 for success and -1 if the input vector or its ops structure are NULL.</td>
<td>This function is callable as FN_VEnableLinearSumVectorArray_Serial when using the Fortran 2003 interface module.</td>
</tr>
<tr>
<td>N_VEnableScaleVectorArray_Serial</td>
<td>int N_VEnableScaleVectorArray_Serial(N_Vector v, booleantype tf);</td>
<td>This function enables (SUNTRUE) or disables (SUNFALSE) the scale operation for vector arrays in the serial vector. The return value is 0 for success and -1 if the input vector or its ops structure are NULL.</td>
<td>This function is callable as FN_VEnableScaleVectorArray_Serial when using the Fortran 2003 interface module.</td>
</tr>
<tr>
<td>N_VEnableConstVectorArray_Serial</td>
<td>int N_VEnableConstVectorArray_Serial(N_Vector v, booleantype tf);</td>
<td>This function enables (SUNTRUE) or disables (SUNFALSE) the const operation for vector arrays in the serial vector. The return value is 0 for success and -1 if the input vector or its ops structure are NULL.</td>
<td>This function is callable as FN_VEnableConstVectorArray_Serial when using the Fortran 2003 interface module.</td>
</tr>
</tbody>
</table>
F2003 Name This function is callable as FN_VEnableConstVectorArray_Serial when using the Fortran 2003 interface module.

**N_VEnableWrmsNormVectorArray_Serial**

Prototype  

```c
int N_VEnableWrmsNormVectorArray_Serial(N_Vector v, booleantype tf);
```

Description  

This function enables (SUNTRUE) or disables (SUNFALSE) the WRMS norm operation for vector arrays in the serial vector. The return value is 0 for success and -1 if the input vector or its ops structure are NULL.

F2003 Name This function is callable as FN_VEnableWrmsNormVectorArray_Serial when using the Fortran 2003 interface module.

**N_VEnableWrmsNormMaskVectorArray_Serial**

Prototype  

```c
int N_VEnableWrmsNormMaskVectorArray_Serial(N_Vector v, booleantype tf);
```

Description  

This function enables (SUNTRUE) or disables (SUNFALSE) the masked WRMS norm operation for vector arrays in the serial vector. The return value is 0 for success and -1 if the input vector or its ops structure are NULL.

F2003 Name This function is callable as FN_VEnableWrmsNormMaskVectorArray_Serial when using the Fortran 2003 interface module.

**N_VEnableScaleAddMultiVectorArray_Serial**

Prototype  

```c
int N_VEnableScaleAddMultiVectorArray_Serial(N_Vector v, booleantype tf);
```

Description  

This function enables (SUNTRUE) or disables (SUNFALSE) the scale and add a vector array to multiple vector arrays operation in the serial vector. The return value is 0 for success and -1 if the input vector or its ops structure are NULL.

**N_VEnableLinearCombinationVectorArray_Serial**

Prototype  

```c
int N_VEnableLinearCombinationVectorArray_Serial(N_Vector v, booleantype tf);
```

Description  

This function enables (SUNTRUE) or disables (SUNFALSE) the linear combination operation for vector arrays in the serial vector. The return value is 0 for success and -1 if the input vector or its ops structure are NULL.

Notes

- When looping over the components of an N_Vector v, it is more efficient to first obtain the component array via \( v\_data = NV\_DATA\_S(v) \) and then access \( v\_data[i] \) within the loop than it is to use \( NV\_Ith\_S(v,i) \) within the loop.

  ![Warning]

- N_VNewEmpty_Serial, N_VMake_Serial, and N_VCloneVectorArrayEmpty_Serial set the field `own_data = SUNFALSE`. N_VDestroy_Serial and N_VDestroyVectorArray_Serial will not attempt to free the pointer `data` for any N_Vector with `own_data` set to SUNFALSE. In such a case, it is the user’s responsibility to deallocate the `data` pointer.

  ![Warning]

- To maximize efficiency, vector operations in the NVECTOR_SERIAL implementation that have more than one N_Vector argument do not check for consistent internal representation of these vectors. It is the user’s responsibility to ensure that such routines are called with N_Vector arguments that were all created with the same internal representations.
8.3.3 NVECTOR_SERIAL Fortran interfaces

The nvector_serial module provides a FORTRAN 2003 module as well as FORTRAN 77 style interface functions for use from FORTRAN applications.

FORTRAN 2003 interface module

The fnvector_serial_mod FORTRAN module defines interfaces to all NVECTOR_SERIAL C functions using the intrinsic iso_c_binding module which provides a standardized mechanism for interoperating with C. As noted in the C function descriptions above, the interface functions are named after the corresponding C function, but with a leading ‘F’. For example, the function N_VNew_Serial is interfaced as FN_VNew_Serial.

The FORTRAN 2003 NVECTOR_SERIAL interface module can be accessed with the use statement, i.e. use fnvector_serial_mod, and linking to the library libsundials_fnvectorserial_mod.lib in addition to the C library. For details on where the library and module file fnvector_serial_mod.mod are installed see Appendix A. We note that the module is accessible from the FORTRAN 2003 SUNDIALS integrators without separately linking to the libsundials_fnvectorserial_mod library.

FORTRAN 77 interface functions

For solvers that include a FORTRAN 77 interface module, the NVECTOR_SERIAL module also includes a FORTRAN-callable function FNVINITS(code, NEQ, IER), to initialize this NVECTOR_SERIAL module. Here code is an input solver id (1 for cvode, 2 for IDA, 3 for KINSOL, 4 for ARKODE); NEQ is the problem size (declared so as to match C type long int); and IER is an error return flag equal 0 for success and -1 for failure.

8.4 The NVECTOR_PARALLEL implementation

The NVECTOR_PARALLEL implementation of the NVECTOR module provided with SUNDIALS is based on MPI. It defines the content field of N_Vector to be a structure containing the global and local lengths of the vector, a pointer to the beginning of a contiguous local data array, an MPI communicator, and a boolean flag own_data indicating ownership of the data array data.

```c
struct _N_VectorContent_Parallel {
    sunindextype local_length;
    sunindextype global_length;
    booleantype own_data;
    realtype *data;
    MPI_Comm comm;
};
```

The header file to include when using this module is nvector_parallel.h. The installed module library to link to is libsundials_nvecparallel.lib where .lib is typically .so for shared libraries and .a for static libraries.

8.4.1 NVECTOR_PARALLEL accessor macros

The following macros are provided to access the content of a NVECTOR_PARALLEL vector. The suffix _P in the names denotes the distributed memory parallel version.

- **NV_CONTENT_P**
  
  This macro gives access to the contents of the parallel vector N_Vector.
  
  The assignment v_cont = NV_CONTENT_P(v) sets v_cont to be a pointer to the N_Vector content structure of type struct _N_VectorContent_Parallel.

  Implementation:
#define NV_CONTENT_P(v) ( (N_VectorContent_Parallel)(v->content) )

  These macros give individual access to the parts of the content of a parallel N_Vector.
  The assignment v_data = NV_DATA_P(v) sets v_data to be a pointer to the first component of the local data for the N_Vector v. The assignment NV_DATA_P(v) = v_data sets the component array of v to be v_data by storing the pointer v_data.
  The assignment v llen = NV_LOCLENGTH_P(v) sets v llen to be the length of the local part of v. The call NV_LENGTH_P(v) = llen_v sets the local length of v to be llen_v.
  The assignment v glen = NV_GLOBLENGTH_P(v) sets v glen to be the global length of the vector v. The call NV_GLOBLENGTH_P(v) = glen_v sets the global length of v to be glen_v.

Implementation:
#define NV_OWN_DATA_P(v) ( NV_CONTENT_P(v)->own_data )
#define NV_DATA_P(v) ( NV_CONTENT_P(v)->data )
#define NV_LOCLENGTH_P(v) ( NV_CONTENT_P(v)->local_length )
#define NV_GLOBLENGTH_P(v) ( NV_CONTENT_P(v)->global_length )

- NV_COMM_P
  This macro provides access to the MPI communicator used by the NVECTOR_PARALLEL vectors.
  Implementation:
#define NV_COMM_P(v) ( NV_CONTENT_P(v)->comm )

- NV_Ith_P
  This macro gives access to the individual components of the local data array of an N_Vector.
  The assignment r = NV_Ith_P(v,i) sets r to be the value of the i-th component of the local part of v. The assignment NV_Ith_P(v,i) = r sets the value of the i-th component of the local part of v to be r.
  Here i ranges from 0 to n − 1, where n is the local length.
  Implementation:
#define NV_Ith_P(v,i) ( NV_DATA_P(v)[i] )

8.4.2 NVECTOR_PARALLEL functions

The NVECTOR_PARALLEL module defines parallel implementations of all vector operations listed in Tables 8.1.1, 8.1.2, 8.1.3, and 8.1.4. Their names are obtained from those in these tables by appending the suffix _Parallel (e.g. N_VDestroy_Parallel). The module NVECTOR_PARALLEL provides the following additional user-callable routines:

N_VNew_Parallel
Prototype N_Vector N_VNew_Parallel(MPI_Comm comm, sunindextype local_length, sunindextype global_length);
Description This function creates and allocates memory for a parallel vector.
F2003 Name This function is callable as FN_VNew_Parallel when using the Fortran 2003 interface module.
8.4 The NVECTOR_PARALLEL implementation

N_VNewEmpty_Parallel
Prototype  
N_Vector N_VNewEmpty_Parallel(MPI_Comm comm, sunindextype local_length, sunindextype global_length);
Description  
This function creates a new parallel N_Vector with an empty (NULL) data array.
F2003 Name  
This function is callable as FN_VNewEmpty_Parallel when using the Fortran 2003 interface module.

N_VMake_Parallel
Prototype  
N_Vector N_VMake_Parallel(MPI_Comm comm, sunindextype local_length, sunindextype global_length, realtype *v_data);
Description  
This function creates and allocates memory for a parallel vector with user-provided data array. This function does not allocate memory for v_data itself.
F2003 Name  
This function is callable as FN_VMake_Parallel when using the Fortran 2003 interface module.

N_VCloneVectorArray_Parallel
Prototype  
N_Vector *N_VCloneVectorArray_Parallel(int count, N_Vector w);
Description  
This function creates (by cloning) an array of count parallel vectors.
F2003 Name  
This function is callable as FN_VCloneVectorArray_Parallel when using the Fortran 2003 interface module.

N_VCloneVectorArrayEmpty_Parallel
Prototype  
N_Vector *N_VCloneVectorArrayEmpty_Parallel(int count, N_Vector w);
Description  
This function creates (by cloning) an array of count parallel vectors, each with an empty (NULL) data array.
F2003 Name  
This function is callable as FN_VCloneVectorArrayEmpty_Parallel when using the Fortran 2003 interface module.

N_VDestroyVectorArray_Parallel
Prototype  
void N_VDestroyVectorArray_Parallel(N_Vector *vs, int count);
Description  
This function frees memory allocated for the array of count variables of type N_Vector created with N_VCloneVectorArray_Parallel or with N_VCloneVectorArrayEmpty_Parallel.
F2003 Name  
This function is callable as FN_VDestroyVectorArray_Parallel when using the Fortran 2003 interface module.

N_VGetLocalLength_Parallel
Prototype  
sunindextype N_VGetLocalLength_Parallel(N_Vector v);
Description  
This function returns the local vector length.
F2003 Name  
This function is callable as FN_VGetLocalLength_Parallel when using the Fortran 2003 interface module.
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**N.VPrint_Parallel**

Prototype: 
```c
void N_VPrint_Parallel(N_Vector v);
```
Description: This function prints the local content of a parallel vector to `stdout`
F2003 Name: This function is callable as `FN_VPrint_Parallel` when using the Fortran 2003 interface module.

**N.VPrintFile_Parallel**

Prototype: 
```c
void N_VPrintFile_Parallel(N_Vector v, FILE *outfile);
```
Description: This function prints the local content of a parallel vector to `outfile`
F2003 Name: This function is callable as `FN_VPrintFile_Parallel` when using the Fortran 2003 interface module.

By default all fused and vector array operations are disabled in the `NVECTOR_PARALLEL` module. The following additional user-callable routines are provided to enable or disable fused and vector array operations for a specific vector. To ensure consistency across vectors it is recommended to first create a vector with `N_VNew_Parallel`, enable/disable the desired operations for that vector with the functions below, and create any additional vectors from that vector using `N_VClone` with that vector. This guarantees the new vectors will have the same operations enabled/disabled as cloned vectors inherit the same enable/disable options as the vector they are cloned from while vectors created with `N_VNew_Parallel` will have the default settings for the `NVECTOR_PARALLEL` module.

**N_VEnableFusedOps_Parallel**

Prototype: 
```c
int N_VEnableFusedOps_Parallel(N_Vector v, booleantype tf);
```
Description: This function enables (`SUNTRUE`) or disables (`SUNFALSE`) all fused and vector array operations in the parallel vector. The return value is 0 for success and -1 if the input vector or its ops structure are `NULL`.
F2003 Name: This function is callable as `FN_VEnableFusedOps_Parallel` when using the Fortran 2003 interface module.

**N_VEnableLinearCombination_Parallel**

Prototype: 
```c
int N_VEnableLinearCombination_Parallel(N_Vector v, booleantype tf);
```
Description: This function enables (`SUNTRUE`) or disables (`SUNFALSE`) the linear combination fused operation in the parallel vector. The return value is 0 for success and -1 if the input vector or its ops structure are `NULL`.
F2003 Name: This function is callable as `FN_VEnableLinearCombination_Parallel` when using the Fortran 2003 interface module.

**N_VEnableScaleAddMulti_Parallel**

Prototype: 
```c
int N_VEnableScaleAddMulti_Parallel(N_Vector v, booleantype tf);
```
Description: This function enables (`SUNTRUE`) or disables (`SUNFALSE`) the scale and add a vector to multiple vectors fused operation in the parallel vector. The return value is 0 for success and -1 if the input vector or its ops structure are `NULL`.
F2003 Name: This function is callable as `FN_VEnableScaleAddMulti_Parallel` when using the Fortran 2003 interface module.
8.4 The NVECTOR_PARALLEL implementation

**N_VEnableDotProdMulti_Parallel**

Prototype: `int N_VEnableDotProdMulti_Parallel(N_Vector v, booleantype tf);`

Description: This function enables (`SUNTRUE`) or disables (`SUNFALSE`) the multiple dot products fused operation in the parallel vector. The return value is 0 for success and -1 if the input vector or its `ops` structure are `NULL`.

F2003 Name: This function is callable as `FN_VEnableDotProdMulti_Parallel` when using the Fortran 2003 interface module.

**N_VEnableLinearSumVectorArray_Parallel**

Prototype: `int N_VEnableLinearSumVectorArray_Parallel(N_Vector v, booleantype tf);`

Description: This function enables (`SUNTRUE`) or disables (`SUNFALSE`) the linear sum operation for vector arrays in the parallel vector. The return value is 0 for success and -1 if the input vector or its `ops` structure are `NULL`.

F2003 Name: This function is callable as `FN_VEnableLinearSumVectorArray_Parallel` when using the Fortran 2003 interface module.

**N_VEnableScaleVectorArray_Parallel**

Prototype: `int N_VEnableScaleVectorArray_Parallel(N_Vector v, booleantype tf);`

Description: This function enables (`SUNTRUE`) or disables (`SUNFALSE`) the scale operation for vector arrays in the parallel vector. The return value is 0 for success and -1 if the input vector or its `ops` structure are `NULL`.

F2003 Name: This function is callable as `FN_VEnableScaleVectorArray_Parallel` when using the Fortran 2003 interface module.

**N_VEnableConstVectorArray_Parallel**

Prototype: `int N_VEnableConstVectorArray_Parallel(N_Vector v, booleantype tf);`

Description: This function enables (`SUNTRUE`) or disables (`SUNFALSE`) the const operation for vector arrays in the parallel vector. The return value is 0 for success and -1 if the input vector or its `ops` structure are `NULL`.

F2003 Name: This function is callable as `FN_VEnableConstVectorArray_Parallel` when using the Fortran 2003 interface module.

**N_VEnableWrmsNormVectorArray_Parallel**

Prototype: `int N_VEnableWrmsNormVectorArray_Parallel(N_Vector v, booleantype tf);`

Description: This function enables (`SUNTRUE`) or disables (`SUNFALSE`) the WRMS norm operation for vector arrays in the parallel vector. The return value is 0 for success and -1 if the input vector or its `ops` structure are `NULL`.

F2003 Name: This function is callable as `FN_VEnableWrmsNormVectorArray_Parallel` when using the Fortran 2003 interface module.

**N_VEnableWrmsNormMaskVectorArray_Parallel**

Prototype: `int N_VEnableWrmsNormMaskVectorArray_Parallel(N_Vector v, booleantype tf);`

Description: This function enables (`SUNTRUE`) or disables (`SUNFALSE`) the masked WRMS norm operation for vector arrays in the parallel vector. The return value is 0 for success and -1 if the input vector or its `ops` structure are `NULL`. 
F2003 Name This function is callable as \texttt{FN\_VEnableWrmsNormMaskVectorArray\_Parallel} when using the Fortran 2003 interface module.

\begin{Verbatim}
N\_VEnableScaleAddMultiVectorArray\_Parallel
\end{Verbatim}

Prototype \begin{verbatim}
int N\_VEnableScaleAddMultiVectorArray\_Parallel(N\_Vector v, boolantype tf);
\end{verbatim}

Description This function enables (\texttt{SUNTRUE}) or disables (\texttt{SUNFALSE}) the scale and add a vector array to multiple vector arrays operation in the parallel vector. The return value is 0 for success and -1 if the input vector or its ops structure are NULL.

\begin{Verbatim}
N\_VEnableLinearCombinationVectorArray\_Parallel
\end{Verbatim}

Prototype \begin{verbatim}
int N\_VEnableLinearCombinationVectorArray\_Parallel(N\_Vector v, boolantype tf);
\end{verbatim}

Description This function enables (\texttt{SUNTRUE}) or disables (\texttt{SUNFALSE}) the linear combination operation for vector arrays in the parallel vector. The return value is 0 for success and -1 if the input vector or its ops structure are NULL.

Notes

- When looping over the components of an \texttt{N\_Vector v}, it is more efficient to first obtain the local component array via \texttt{v\_data = NV\_DATA\_P(v)} and then access \texttt{v\_data[i]} within the loop than it is to use \texttt{NV\_Ith\_P(v,i)} within the loop.

- \texttt{N\_VNewEmpty\_Parallel}, \texttt{N\_VMake\_Parallel}, and \texttt{N\_VCloneVectorArray\_Parallel} set the \texttt{own\_data = SUNFALSE}. \texttt{N\_VDestroy\_Parallel} and \texttt{N\_VDestroyVectorArray\_Parallel} will not attempt to free the pointer \texttt{data} for any \texttt{N\_Vector} with \texttt{own\_data set to SUNFALSE}. In such a case, it is the user's responsibility to deallocate the \texttt{data} pointer.

- To maximize efficiency, vector operations in the \texttt{NVECTOR\_PARALLEL} implementation that have more than one \texttt{N\_Vector} argument do not check for consistent internal representation of these vectors. It is the user's responsibility to ensure that such routines are called with \texttt{N\_Vector} arguments that were all created with the same internal representations.

8.4.3 NVECTOR\_PARALLEL Fortran interfaces

For solvers that include a FORTRAN 77 interface module, the \texttt{NVECTOR\_PARALLEL} module also includes a FORTRAN-callable function \texttt{FN\_INITP(COMM, code, NLOCAL, NGLOBAL, IER)}, to initialize this \texttt{NVECTOR\_PARALLEL} module. Here \texttt{COMM} is the MPI communicator, \texttt{code} is an input solver id (1 for \texttt{cvode}, 2 for \texttt{ida}, 3 for \texttt{kinsol}, 4 for \texttt{arkode}); \texttt{NLOCAL} and \texttt{NGLOBAL} are the local and global vector sizes, respectively (declared so as to match C type long int); and \texttt{IER} is an error return flag equal 0 for success and -1 for failure. NOTE: If the header file \texttt{sundials\_config\_h} defines \texttt{SUNDIALS\_MPI\_COMM\_F2C} to be 1 (meaning the MPI implementation used to build SUNDIALS includes the \texttt{MPI\_Comm\_f2c} function), then \texttt{COMM} can be any valid MPI communicator. Otherwise, \texttt{MPI\_COMM\_WORLD} will be used, so just pass an integer value as a placeholder.

8.5 The NVECTOR\_OPENMP implementation

In situations where a user has a multi-core processing unit capable of running multiple parallel threads with shared memory, SUNDIALS provides an implementation of \texttt{NVECTOR} using OpenMP, called \texttt{NVECTOR\_OPENMP}, and an implementation using Pthreads, called \texttt{NVECTOR\_PTHREADS}. Testing has shown that vectors should be of length at least 100,000 before the overhead associated with creating and using the threads is made up by the parallelism in the vector calculations.
The OpenMP NVECTOR implementation provided with SUNDIALS, NVECTOR_OPENMP, defines the content field of N_Vector to be a structure containing the length of the vector, a pointer to the beginning of a contiguous data array, a boolean flag own_data which specifies the ownership of data, and the number of threads. Operations on the vector are threaded using OpenMP.

```
struct _N_VectorContent_OpenMP {
    sunindextype length;
    booleantype own_data;
    realtype *data;
    int num_threads;
};
```

The header file to include when using this module is nvector_openmp.h. The installed module library to link to is libsundials_nvecopenmp.lib where .lib is typically .so for shared libraries and .a for static libraries. The FORTRAN module file to use when using the FORTRAN 2003 interface to this module is fnvector_openmp_mod.mod.

### 8.5.1 NVECTOR_OPENMP accessor macros

The following macros are provided to access the content of an NVECTOR_OPENMP vector. The suffix _OMP in the names denotes the OpenMP version.

- **NV_CONTENT_OMP**
  
  This routine gives access to the contents of the OpenMP vector N_Vector.
  
  The assignment `v_cont = NV_CONTENT_OMP(v)` sets `v_cont` to be a pointer to the OpenMP N_Vector content structure.
  
  Implementation:
  
  ```c
  #define NV_CONTENT_OMP(v) ( (N_VectorContent_OpenMP)(v->content) )
  ```

- **NV_OWN_DATA_OMP, NV_DATA_OMP, NV_LENGTH_OMP, NV_NUM_THREADS_OMP**
  
  These macros give individual access to the parts of the content of a OpenMP N_Vector.
  
  The assignment `v_data = NV_DATA_OMP(v)` sets `v_data` to be a pointer to the first component of the data for the N_Vector `v`. The assignment `NV_DATA_OMP(v) = v_data` sets the component array of `v` to be `v_data` by storing the pointer `v_data`.
  
  The assignment `v_len = NV_LENGTH_OMP(v)` sets `v_len` to be the length of `v`. On the other hand, the call `NV_LENGTH_OMP(v) = len` sets the length of `v` to be `len`.
  
  The assignment `v_num_threads = NV_NUM_THREADS_OMP(v)` sets `v_num_threads` to be the number of threads from `v`. On the other hand, the call `NV_NUM_THREADS_OMP(v) = num_threads` sets the number of threads for `v` to be `num_threads`.
  
  Implementation:
  
  ```c
  #define NV_OWN_DATA_OMP(v) ( NV_CONTENT_OMP(v)->own_data )
  #define NV_DATA_OMP(v) ( NV_CONTENT_OMP(v)->data )
  #define NV_LENGTH_OMP(v) ( NV_CONTENT_OMP(v)->length )
  #define NV_NUM_THREADS_OMP(v) ( NV_CONTENT_OMP(v)->num_threads )
  ```

- **NV_Ith_OMP**
  
  This macro gives access to the individual components of the data array of an N_Vector.
  
  The assignment `r = NV_Ith_OMP(v,i)` sets `r` to be the value of the `i`-th component of `v`. The assignment `NV_Ith_OMP(v,i) = r` sets the value of the `i`-th component of `v` to be `r`.
  
  Here `i` ranges from 0 to `n - 1` for a vector of length `n`.
  
  Implementation:
  
  ```c
  #define NV_Ith_OMP(v,i) ( NV_DATA_OMP(v)[i] )
  ```
8.5.2 NVECTOR_OPENMP functions

The NVECTOR_OPENMP module defines OpenMP implementations of all vector operations listed in Tables 8.1.1, 8.1.2, 8.1.3, and 8.1.4. Their names are obtained from those in these tables by appending the suffix _OpenMP (e.g. N_VDestroy_OpenMP). All the standard vector operations listed in 8.1.1 with the suffix _OpenMP appended are callable via the FORTRAN 2003 interface by prepending an ‘F’ (e.g. FN_VDestroy_OpenMP).

The module NVECTOR_OPENMP provides the following additional user-callable routines:

**N_VNew_OpenMP**
Prototype: N_Vector N_VNew_OpenMP(sunindextype vec_length, int num_threads)
Description: This function creates and allocates memory for a OpenMP N_Vector. Arguments are the vector length and number of threads.
F2003 Name: This function is callable as FN_VNew_OpenMP when using the Fortran 2003 interface module.

**N_VNewEmpty_OpenMP**
Prototype: N_Vector N_VNewEmpty_OpenMP(sunindextype vec_length, int num_threads)
Description: This function creates a new OpenMP N_Vector with an empty (NULL) data array.
F2003 Name: This function is callable as FN_VNewEmpty_OpenMP when using the Fortran 2003 interface module.

**N_VMake_OpenMP**
Prototype: N_Vector N_VMake_OpenMP(sunindextype vec_length, realtype *v_data, int num_threads);
Description: This function creates and allocates memory for a OpenMP vector with user-provided data array. This function does not allocate memory for v_data itself.
F2003 Name: This function is callable as FN_VMake_OpenMP when using the Fortran 2003 interface module.

**N_VCloneVectorArray_OpenMP**
Prototype: N_Vector *N_VCloneVectorArray_OpenMP(int count, N_Vector w)
Description: This function creates (by cloning) an array of count OpenMP vectors.
F2003 Name: This function is callable as FN_VCloneVectorArray_OpenMP when using the Fortran 2003 interface module.

**N_VCloneVectorArrayEmpty_OpenMP**
Prototype: N_Vector *N_VCloneVectorArrayEmpty_OpenMP(int count, N_Vector w)
Description: This function creates (by cloning) an array of count OpenMP vectors, each with an empty (NULL) data array.
F2003 Name: This function is callable as FN_VCloneVectorArrayEmpty_OpenMP when using the Fortran 2003 interface module.
8.5 The NVECTOR_OPENMP implementation

**N_VDestroyVectorArray_OpenMP**

Prototype: `void N_VDestroyVectorArray_OpenMP(N_Vector vs, int count)`

Description: This function frees memory allocated for the array of count variables of type `N_Vector` created with `N_VCloneVectorArray_OpenMP` or with `N_VCloneVectorArrayEmpty_OpenMP`.

F2003 Name: This function is callable as `FN_VDestroyVectorArray_OpenMP` when using the Fortran 2003 interface module.

**N_VPrint_OpenMP**

Prototype: `void N_VPrint_OpenMP(N_Vector v)`

Description: This function prints the content of an OpenMP vector to `stdout`.

F2003 Name: This function is callable as `FN_VPrint_OpenMP` when using the Fortran 2003 interface module.

**N_VPrintFile_OpenMP**

Prototype: `void N_VPrintFile_OpenMP(N_Vector v, FILE *outfile)`

Description: This function prints the content of an OpenMP vector to `outfile`.

F2003 Name: This function is callable as `FN_VPrintFile_OpenMP` when using the Fortran 2003 interface module.

By default all fused and vector array operations are disabled in the `nvector_openmp` module. The following additional user-callable routines are provided to enable or disable fused and vector array operations for a specific vector. To ensure consistency across vectors it is recommended to first create a vector with `N_VNew_OpenMP`, enable/disable the desired operations for that vector with the functions below, and create any additional vectors from that vector using `N_VClone`. This guarantees the new vectors will have the same operations enabled/disabled as cloned vectors inherit the same enable/disable options as the vector they are cloned from while vectors created with `N_VNew_OpenMP` will have the default settings for the `nvector_openmp` module.

**N_VEnableFusedOps_OpenMP**

Prototype: `int N_VEnableFusedOps_OpenMP(N_Vector v, booleantype tf)`

Description: This function enables (`SUNTRUE`) or disables (`SUNFALSE`) all fused and vector array operations in the OpenMP vector. The return value is 0 for success and -1 if the input vector or its `ops` structure are NULL.

F2003 Name: This function is callable as `FN_VEnableFusedOps_OpenMP` when using the Fortran 2003 interface module.

**N_VEnableLinearCombination_OpenMP**

Prototype: `int N_VEnableLinearCombination_OpenMP(N_Vector v, booleantype tf)`

Description: This function enables (`SUNTRUE`) or disables (`SUNFALSE`) the linear combination fused operation in the OpenMP vector. The return value is 0 for success and -1 if the input vector or its `ops` structure are NULL.

F2003 Name: This function is callable as `FN_VEnableLinearCombination_OpenMP` when using the Fortran 2003 interface module.
N_VEnableScaleAddMulti_OpenMP
Prototype int N_VEnableScaleAddMulti_OpenMP(N_Vector v, booleantype tf)
Description This function enables (SUNTRUE) or disables (SUNFALSE) the scale and add a vector to multiple vectors fused operation in the OpenMP vector. The return value is 0 for success and -1 if the input vector or its ops structure are NULL.
F2003 Name This function is callable as FN_VEnableScaleAddMulti_OpenMP when using the Fortran 2003 interface module.

N_VEnableDotProdMulti_OpenMP
Prototype int N_VEnableDotProdMulti_OpenMP(N_Vector v, booleantype tf)
Description This function enables (SUNTRUE) or disables (SUNFALSE) the multiple dot products fused operation in the OpenMP vector. The return value is 0 for success and -1 if the input vector or its ops structure are NULL.
F2003 Name This function is callable as FN_VEnableDotProdMulti_OpenMP when using the Fortran 2003 interface module.

N_VEnableLinearSumVectorArray_OpenMP
Prototype int N_VEnableLinearSumVectorArray_OpenMP(N_Vector v, booleantype tf)
Description This function enables (SUNTRUE) or disables (SUNFALSE) the linear sum operation for vector arrays in the OpenMP vector. The return value is 0 for success and -1 if the input vector or its ops structure are NULL.
F2003 Name This function is callable as FN_VEnableLinearSumVectorArray_OpenMP when using the Fortran 2003 interface module.

N_VEnableScaleVectorArray_OpenMP
Prototype int N_VEnableScaleVectorArray_OpenMP(N_Vector v, booleantype tf)
Description This function enables (SUNTRUE) or disables (SUNFALSE) the scale operation for vector arrays in the OpenMP vector. The return value is 0 for success and -1 if the input vector or its ops structure are NULL.
F2003 Name This function is callable as FN_VEnableScaleVectorArray_OpenMP when using the Fortran 2003 interface module.

N_VEnableConstVectorArray_OpenMP
Prototype int N_VEnableConstVectorArray_OpenMP(N_Vector v, booleantype tf)
Description This function enables (SUNTRUE) or disables (SUNFALSE) the const operation for vector arrays in the OpenMP vector. The return value is 0 for success and -1 if the input vector or its ops structure are NULL.
F2003 Name This function is callable as FN_VEnableConstVectorArray_OpenMP when using the Fortran 2003 interface module.

N_VEnableWrmsNormVectorArray_OpenMP
Prototype int N_VEnableWrmsNormVectorArray_OpenMP(N_Vector v, booleantype tf)
Description This function enables (SUNTRUE) or disables (SUNFALSE) the WRMS norm operation for vector arrays in the OpenMP vector. The return value is 0 for success and -1 if the input vector or its ops structure are NULL.
8.5 The NVECTOR_OPENMP implementation

F2003 Name This function is callable as FN_VEnableWrmsNormVectorArray_OpenMP when using the Fortran 2003 interface module.

**N_VEnableWrmsNormMaskVectorArray_OpenMP**

Prototype  

```c
int N_VEnableWrmsNormMaskVectorArray_OpenMP(N_Vector v, booleantype tf)
```

Description This function enables (SUNTRUE) or disables (SUNFALSE) the masked WRMS norm operation for vector arrays in the OpenMP vector. The return value is 0 for success and -1 if the input vector or its ops structure are NULL.

F2003 Name This function is callable as FN_VEnableWrmsNormMaskVectorArray_OpenMP when using the Fortran 2003 interface module.

**N_VEnableScaleAddMultiVectorArray_OpenMP**

Prototype  

```c
int N_VEnableScaleAddMultiVectorArray_OpenMP(N_Vector v, booleantype tf)
```

Description This function enables (SUNTRUE) or disables (SUNFALSE) the scale and add a vector array to multiple vector arrays operation in the OpenMP vector. The return value is 0 for success and -1 if the input vector or its ops structure are NULL.

**N_VEnableLinearCombinationVectorArray_OpenMP**

Prototype  

```c
int N_VEnableLinearCombinationVectorArray_OpenMP(N_Vector v, booleantype tf)
```

Description This function enables (SUNTRUE) or disables (SUNFALSE) the linear combination operation for vector arrays in the OpenMP vector. The return value is 0 for success and -1 if the input vector or its ops structure are NULL.

Notes

- When looping over the components of an N_Vector v, it is more efficient to first obtain the component array via `v_data = NV_DATA_OMP(v)` and then access `v_data[i]` within the loop than it is to use `NV_Ith_OMP(v,i)` within the loop.

- `N_VNewEmpty_OpenMP`, `N_VMake_OpenMP`, and `N_VCloneVectorArrayEmpty_OpenMP` set the field `own_data = SUNFALSE`. `N_VDestroy_OpenMP` and `N_VDestroyVectorArray_OpenMP` will not attempt to free the pointer `data` for any N_Vector with `own_data` set to SUNFALSE. In such a case, it is the user’s responsibility to deallocate the `data` pointer.

- To maximize efficiency, vector operations in the NVECTOR_OPENMP implementation that have more than one N_Vector argument do not check for consistent internal representation of these vectors. It is the user’s responsibility to ensure that such routines are called with N_Vector arguments that were all created with the same internal representations.

8.5.3 NVECTOR_OPENMP Fortran interfaces

The nvector_openmp module provides a FORTRAN 2003 module as well as FORTRAN 77 style interface functions for use from FORTRAN applications.

FORTRAN 2003 interface module

The nvector_openmp_mod FORTRAN module defines interfaces to most NVECTOR_OPENMP C functions using the intrinsic iso_c_binding module which provides a standardized mechanism for interoperating with C. As noted in the C function descriptions above, the interface functions are named after the corresponding C function, but with a leading ‘F’. For example, the function N_VNew_OpenMP is interfaced as FN_VNew_OpenMP.
The Fortran 2003 nvector_openmp interface module can be accessed with the use statement, i.e. use fnvector_openmp_mod, and linking to the library libsundials_fnvectoropenmp_mod.lib in addition to the C library. For details on where the library and module file fnvector_openmp_mod.mod are installed see Appendix A.

**FORTRAN 77 interface functions**

For solvers that include a Fortran 77 interface module, the nvector_openmp module also includes a Fortran-callable function FNVINITOMP(code, NEQ, NUMTHREADS, IER), to initialize this module. Here code is an input solver id (1 for cvode, 2 for IDA, 3 for KINSOL, 4 for ARKODE); NEQ is the problem size (declared so as to match C type long int); NUMTHREADS is the number of threads; and IER is an error return flag equal 0 for success and -1 for failure.

### 8.6 The NVECTOR_PTHREADS implementation

In situations where a user has a multi-core processing unit capable of running multiple parallel threads with shared memory, SUNDIALS provides an implementation of NVECTOR using OpenMP, called nvector_openmp, and an implementation using Pthreads, called nvector_pthreads. Testing has shown that vectors should be of length at least 100,000 before the overhead associated with creating and using the threads is made up by the parallelism in the vector calculations.

The Pthreads NVECTOR implementation provided with SUNDIALS, denoted nvector_pthreads, defines the content field of N_Vector to be a structure containing the length of the vector, a pointer to the beginning of a contiguous data array, a boolean flag own_data which specifies the ownership of data, and the number of threads. Operations on the vector are threaded using POSIX threads (Pthreads).

```c
struct _N_VectorContent_Pthreads {
    sunindextype length;
    booleantype own_data;
    realtype *data;
    int num_threads;
};
```

The header file to include when using this module is nvector_pthreads.h. The installed module library to link to is libsundials_nvecpthreads.lib where .lib is typically .so for shared libraries and .a for static libraries.

#### 8.6.1 NVECTOR_PTHREADS accessor macros

The following macros are provided to access the content of an NVECTOR_PTHREADS vector. The suffix _PT in the names denotes the Pthreads version.

- **NV_CONTENT_PT**
  
  This routine gives access to the contents of the Pthreads vector N_Vector.
  
  The assignment v_cont = NV_CONTENT_PT(v) sets v_cont to be a pointer to the Pthreads N_Vector content structure.
  
  Implementation:
  
  ```c
  #define NV_CONTENT_PT(v) ( (N_VectorContent_Pthreads)(v->content) )
  ```

- **NV_OWN_DATA_PT, NV_DATA_PT, NV_LENGTH_PT, NV_NUM_THREADS_PT**
  
  These macros give individual access to the parts of the content of a Pthreads N_Vector.
  
  The assignment v_data = NV_DATA_PT(v) sets v_data to be a pointer to the first component of the data for the N_Vector v. The assignment NV_DATA_PT(v) = v_data sets the component array of v to be v_data by storing the pointer v_data.
The assignment \( v_{\text{len}} = \text{NV}_\text{LENGTH}_\text{PT}(v) \) sets \( v_{\text{len}} \) to be the length of \( v \). On the other hand, the call \( \text{NV}_\text{LENGTH}_\text{PT}(v) = \text{len}_v \) sets the length of \( v \) to be \( \text{len}_v \).

The assignment \( v_{\text{num threads}} = \text{NV}_\text{NUM}_\text{THREADS}_\text{PT}(v) \) sets \( v_{\text{num threads}} \) to be the number of threads from \( v \). On the other hand, the call \( \text{NV}_\text{NUM}_\text{THREADS}_\text{PT}(v) = \text{num threads}_v \) sets the number of threads for \( v \) to be \( \text{num threads}_v \).

Implementation:

\[
\begin{align*}
\#define \text{NV}_{\text{OWN DATA}_\text{PT}(v)} & ( \text{NV}_\text{CONTENT}_\text{PT}(v)\rightarrow\text{own data} ) \\
\#define \text{NV}_{\text{DATA}_\text{PT}(v)} & ( \text{NV}_\text{CONTENT}_\text{PT}(v)\rightarrow\text{data} ) \\
\#define \text{NV}_{\text{LENGTH}_\text{PT}(v)} & ( \text{NV}_\text{CONTENT}_\text{PT}(v)\rightarrow\text{length} ) \\
\#define \text{NV}_{\text{NUM THREADS}_\text{PT}(v)} & ( \text{NV}_\text{CONTENT}_\text{PT}(v)\rightarrow\text{num threads} )
\end{align*}
\]

- \( \text{NV}_{\text{Ith}_\text{PT}} \)
  This macro gives access to the individual components of the data array of an NVector.
  The assignment \( r = \text{NV}_{\text{Ith}_\text{PT}}(v,i) \) sets \( r \) to be the value of the \( i \)-th component of \( v \). The assignment \( \text{NV}_{\text{Ith}_\text{PT}}(v,i) = r \) sets the value of the \( i \)-th component of \( v \) to be \( r \).
  Here \( i \) ranges from 0 to \( n - 1 \) for a vector of length \( n \).
  Implementation:
  \[
  \#define \text{NV}_{\text{Ith}_\text{PT}}(v,i) ( \text{NV}_\text{DATA}_\text{PT}(v)[i] )
  \]

### 8.6.2 NVECTOR_PTHREADS functions

The nvector_pthreads module defines Pthreads implementations of all vector operations listed in Tables 8.1.1, 8.1.2, 8.1.3, and 8.1.4. Their names are obtained from those in these tables by appending the suffix _Pthreads (e.g. N_VDestroy_Pthreads). All the standard vector operations listed in 8.1.1 are callable via the FORTRAN 2003 interface by prepending an ‘F’ (e.g. FN_VDestroy_Pthreads). The module nvector_pthreads provides the following additional user-callable routines:

#### N_VNew_Pthreads

Prototype  
\[
\text{N} \_ \text{Vector N}_\text{VNew}_\text{Pthreads}(\text{sunindextype vec_length}, \text{int num_threads})
\]

Description  
This function creates and allocates memory for a Pthreads N_Vector. Arguments are the vector length and number of threads.

F2003 Name  
This function is callable as FN_VNew_Pthreads when using the Fortran 2003 interface module.

#### N_VNewEmpty_Pthreads

Prototype  
\[
\text{N} \_ \text{Vector N}_\text{VNewEmpty}_\text{Pthreads}(\text{sunindextype vec_length}, \text{int num_threads})
\]

Description  
This function creates a new Pthreads N_Vector with an empty (NULL) data array.

F2003 Name  
This function is callable as FN_VNewEmpty_Pthreads when using the Fortran 2003 interface module.

#### N_VMake_Pthreads

Prototype  
\[
\text{N} \_ \text{Vector N}_\text{VMake}_\text{Pthreads}(\text{sunindextype vec_length}, \text{realtype } *v\text{.data}, \text{int num_threads});
\]

Description  
This function creates and allocates memory for a Pthreads vector with user-provided data array. This function does not allocate memory for \( v\text{.data} \) itself.

F2003 Name  
This function is callable as FN_VMake_Pthreads when using the Fortran 2003 interface module.
Description of the NVECTOR module

**N_VCloneVectorArray_Pthreads**

Prototype: `N_Vector *N_VCloneVectorArray_Pthreads(int count, N_Vector w)`

Description: This function creates (by cloning) an array of count Pthreads vectors.

F2003 Name: This function is callable as `FN_VCloneVectorArray_Pthreads` when using the Fortran 2003 interface module.

**N_VCloneVectorArrayEmpty_Pthreads**

Prototype: `N_Vector *N_VCloneVectorArrayEmpty_Pthreads(int count, N_Vector w)`

Description: This function creates (by cloning) an array of count Pthreads vectors, each with an empty (NULL) data array.

F2003 Name: This function is callable as `FN_VCloneVectorArrayEmpty_Pthreads` when using the Fortran 2003 interface module.

**N_VDestroyVectorArray_Pthreads**

Prototype: `void N_VDestroyVectorArray_Pthreads(N_Vector *vs, int count)`

Description: This function frees memory allocated for the array of count variables of type `N_Vector` created with `N_VCloneVectorArray_Pthreads` or with `N_VCloneVectorArrayEmpty_Pthreads`.

F2003 Name: This function is callable as `FN_VDestroyVectorArray_Pthreads` when using the Fortran 2003 interface module.

**N_VPrint_Pthreads**

Prototype: `void N_VPrint_Pthreads(N_Vector v)`

Description: This function prints the content of a Pthreads vector to `stdout`.

F2003 Name: This function is callable as `FN_VPrint_Pthreads` when using the Fortran 2003 interface module.

**N_VPrintFile_Pthreads**

Prototype: `void N_VPrintFile_Pthreads(N_Vector v, FILE *outfile)`

Description: This function prints the content of a Pthreads vector to `outfile`.

F2003 Name: This function is callable as `FN_VPrintFile_Pthreads` when using the Fortran 2003 interface module.

By default all fused and vector array operations are disabled in the `nvector_pthreads` module. The following additional user-callable routines are provided to enable or disable fused and vector array operations for a specific vector. To ensure consistency across vectors it is recommended to first create a vector with `N_VNew_Pthreads`, enable/disable the desired operations for that vector with the functions below, and create any additional vectors from that vector using `N_VClone`. This guarantees the new vectors will have the same operations enabled/disabled as cloned vectors inherit the same enable/disable options as the vector they are cloned from while vectors created with `N_VNew_Pthreads` will have the default settings for the `nvector_pthreads` module.

**N_VEnableFusedOps_Pthreads**

Prototype: `int N_VEnableFusedOps_Pthreads(N_Vector v, booleantype tf)`

Description: This function enables (`SUNTRUE`) or disables (`SUNFALSE`) all fused and vector array operations in the Pthreads vector. The return value is 0 for success and -1 if the input vector or its `ops` structure are NULL.
F2003 Name  This function is callable as \texttt{FN\_VEnableFusedOps\_Pthreads} when using the Fortran 2003 interface module.

\textbf{N\_VEnableLinearCombination\_Pthreads}

Prototype \hspace{2cm} \textbf{int N\_VEnableLinearCombination\_Pthreads(N\_Vector v, booleantype tf)}

Description This function enables (\texttt{SUNTRUE}) or disables (\texttt{SUNFALSE}) the linear combination fused operation in the Pthreads vector. The return value is 0 for success and -1 if the input vector or its \texttt{ops} structure are NULL.

F2003 Name  This function is callable as \texttt{FN\_VEnableLinearCombination\_Pthreads} when using the Fortran 2003 interface module.

\textbf{N\_VEnableScaleAddMulti\_Pthreads}

Prototype \hspace{2cm} \textbf{int N\_VEnableScaleAddMulti\_Pthreads(N\_Vector v, booleantype tf)}

Description This function enables (\texttt{SUNTRUE}) or disables (\texttt{SUNFALSE}) the scale and add a vector to multiple vectors fused operation in the Pthreads vector. The return value is 0 for success and -1 if the input vector or its \texttt{ops} structure are NULL.

F2003 Name  This function is callable as \texttt{FN\_VEnableScaleAddMulti\_Pthreads} when using the Fortran 2003 interface module.

\textbf{N\_VEnableDotProdMulti\_Pthreads}

Prototype \hspace{2cm} \textbf{int N\_VEnableDotProdMulti\_Pthreads(N\_Vector v, booleantype tf)}

Description This function enables (\texttt{SUNTRUE}) or disables (\texttt{SUNFALSE}) the multiple dot products fused operation in the Pthreads vector. The return value is 0 for success and -1 if the input vector or its \texttt{ops} structure are NULL.

F2003 Name  This function is callable as \texttt{FN\_VEnableDotProdMulti\_Pthreads} when using the Fortran 2003 interface module.

\textbf{N\_VEnableLinearSumVectorArray\_Pthreads}

Prototype \hspace{2cm} \textbf{int N\_VEnableLinearSumVectorArray\_Pthreads(N\_Vector v, booleantype tf)}

Description This function enables (\texttt{SUNTRUE}) or disables (\texttt{SUNFALSE}) the linear sum operation for vector arrays in the Pthreads vector. The return value is 0 for success and -1 if the input vector or its \texttt{ops} structure are NULL.

F2003 Name  This function is callable as \texttt{FN\_VEnableLinearSumVectorArray\_Pthreads} when using the Fortran 2003 interface module.

\textbf{N\_VEnableScaleVectorArray\_Pthreads}

Prototype \hspace{2cm} \textbf{int N\_VEnableScaleVectorArray\_Pthreads(N\_Vector v, booleantype tf)}

Description This function enables (\texttt{SUNTRUE}) or disables (\texttt{SUNFALSE}) the scale operation for vector arrays in the Pthreads vector. The return value is 0 for success and -1 if the input vector or its \texttt{ops} structure are NULL.

F2003 Name  This function is callable as \texttt{FN\_VEnableScaleVectorArray\_Pthreads} when using the Fortran 2003 interface module.
**Description of the NVECTOR module**

**N_EnableConstVectorArray_Pthreads**

**Prototype**

```c
int N_EnableConstVectorArray_Pthreads(N_Vector v, booleantype tf)
```

**Description**

This function enables (SUNTRUE) or disables (SUNFALSE) the const operation for vector arrays in the Pthreads vector. The return value is 0 for success and -1 if the input vector or its ops structure are NULL.

**F2003 Name**

This function is callable as FN_EnableConstVectorArray_Pthreads when using the Fortran 2003 interface module.

**N_EnableWrmsNormVectorArray_Pthreads**

**Prototype**

```c
int N_EnableWrmsNormVectorArray_Pthreads(N_Vector v, booleantype tf)
```

**Description**

This function enables (SUNTRUE) or disables (SUNFALSE) the WRMS norm operation for vector arrays in the Pthreads vector. The return value is 0 for success and -1 if the input vector or its ops structure are NULL.

**F2003 Name**

This function is callable as FN_EnableWrmsNormVectorArray_Pthreads when using the Fortran 2003 interface module.

**N_EnableWrmsNormMaskVectorArray_Pthreads**

**Prototype**

```c
int N_EnableWrmsNormMaskVectorArray_Pthreads(N_Vector v, booleantype tf)
```

**Description**

This function enables (SUNTRUE) or disables (SUNFALSE) the masked WRMS norm operation for vector arrays in the Pthreads vector. The return value is 0 for success and -1 if the input vector or its ops structure are NULL.

**F2003 Name**

This function is callable as FN_EnableWrmsNormMaskVectorArray_Pthreads when using the Fortran 2003 interface module.

**N_EnableScaleAddMultiVectorArray_Pthreads**

**Prototype**

```c
int N_EnableScaleAddMultiVectorArray_Pthreads(N_Vector v, booleantype tf)
```

**Description**

This function enables (SUNTRUE) or disables (SUNFALSE) the scale and add a vector array to multiple vector arrays operation in the Pthreads vector. The return value is 0 for success and -1 if the input vector or its ops structure are NULL.

**N_EnableLinearCombinationVectorArray_Pthreads**

**Prototype**

```c
int N_EnableLinearCombinationVectorArray_Pthreads(N_Vector v, booleantype tf)
```

**Description**

This function enables (SUNTRUE) or disables (SUNFALSE) the linear combination operation for vector arrays in the Pthreads vector. The return value is 0 for success and -1 if the input vector or its ops structure are NULL.

**Notes**

- When looping over the components of an N_Vector v, it is more efficient to first obtain the component array via `v->data = NV_DATA_P(v)` and then access `v->data[i]` within the loop than it is to use `NV_Ith_P(v,i)` within the loop.

  ![Warning]

- `N_NewEmpty_Pthreads`, `N_Make_Pthreads`, and `N_CloneVectorArrayEmpty_Pthreads` set the field `own_data = SUNFALSE`. `N_Destroy_Pthreads` and `N_DestroyVectorArray_Pthreads` will not attempt to free the pointer `data` for any N_Vector with `own_data` set to SUNFALSE. In such a case, it is the user's responsibility to deallocate the `data` pointer.
• To maximize efficiency, vector operations in the nvector_pthreads implementation that have more than one N_Vector argument do not check for consistent internal representation of these vectors. It is the user’s responsibility to ensure that such routines are called with N_Vector arguments that were all created with the same internal representations.

8.6.3 NVECTOR_PTHREADS Fortran interfaces

The nvector_pthreads module provides a FORTRAN 2003 module as well as FORTRAN 77 style interface functions for use from FORTRAN applications.

FORTRAN 2003 interface module

The nvector_pthreads_mod FORTRAN module defines interfaces to most nvector_pthreads C functions using the intrinsic iso_c_binding module which provides a standardized mechanism for interoperating with C. As noted in the C function descriptions above, the interface functions are named after the corresponding C function, but with a leading ‘F’. For example, the function N_VNew_Pthreads is interfaced as FN_VNew_Pthreads.

The FORTRAN 2003 nvector_pthreads interface module can be accessed with the use statement, i.e. use fnvector_pthreads_mod, and linking to the library libsunsmill_fnvectorpthreads_mod.lib in addition to the C library. For details on where the library and module file fnvector_pthreads_mod.mod are installed see Appendix A.

FORTRAN 77 interface functions

For solvers that include a FORTRAN interface module, the nvector_pthreads module also includes a FORTRAN-callable function FNVINITPTS(code, NEQ, NUMTHREADS, IER), to initialize this module. Here code is an input solver id (1 for cvode, 2 for IDA, 3 for KINSOL, 4 for ARKODE); NEQ is the problem size (declared so as to match C type long int); NUMTHREADS is the number of threads; and IER is an error return flag equal 0 for success and -1 for failure.

8.7 The NVECTOR_PARHYP implementation

The NVECTOR_PARHYP implementation of the NVECTOR module provided with SUNDIALS is a wrapper around hypre’s ParVector class. Most of the vector kernels simply call hypre vector operations. The implementation defines the content field of N_Vector to be a structure containing the global and local lengths of the vector, a pointer to an object of type HYPRE_ParVector, an MPI communicator, and a boolean flag own_parvector indicating ownership of the hypre parallel vector object x.

struct _N_VectorContent_ParHyp {
    sunindextype local_length;
    sunindextype global_length;
    booleantype own_parvector;
    MPI_Comm comm;
    HYPRE_ParVector x;
};

The header file to include when using this module is nvctor_parhyp.h. The installed module library to link to is libsunsmill_nvecparhyp.lib where .lib is typically .so for shared libraries and .a for static libraries.

Unlike native SUNDIALS vector types, NVECTOR_PARHYP does not provide macros to access its member variables. Note that NVECTOR_PARHYP requires SUNDIALS to be built with MPI support.
8.7.1 NVECTOR_PARHYP functions

The NVECTOR_PARHYP module defines implementations of all vector operations listed in Tables 8.1.1, 8.1.2, 8.1.3, and 8.1.4, except for N_VSetArrayPointer and N_VGetArrayPointer, because accessing raw vector data is handled by low-level hypre functions. As such, this vector is not available for use with SUNDIALS Fortran interfaces. When access to raw vector data is needed, one should extract the hypre vector first, and then use hypre methods to access the data. Usage examples of NVECTOR_PARHYP are provided in the cvAdvDiff_non_ph.c example program for CVODE [34] and the ark_diurnal_kry_ph.c example program for ARKODE [44].

The names of parhyp methods are obtained from those in Tables 8.1.1, 8.1.2, 8.1.3, and 8.1.4 by appending the suffix _ParHyp (e.g. N_VDestroy_ParHyp). The module NVECTOR_PARHYP provides the following additional user-callable routines:

N_VNewEmpty_ParHyp
Prototype N_Vector N_VNewEmpty_ParHyp(MPI_Comm comm, sunindextype local_length, sunindextype global_length)
Description This function creates a new parhyp N_Vector with the pointer to the hypre vector set to NULL.

N_VMake_ParHyp
Prototype N_Vector N_VMake_ParHyp(HYPRE_ParVector x)
Description This function creates an N_Vector wrapper around an existing hypre parallel vector. It does not allocate memory for x itself.

N_VGetVector_ParHyp
Prototype HYPRE_ParVector N_VGetVector_ParHyp(N_Vector v)
Description This function returns the underlying hypre vector.

N_VCloneVectorArray_ParHyp
Prototype N_Vector *N_VCloneVectorArray_ParHyp(int count, N_Vector w)
Description This function creates (by cloning) an array of count parallel vectors.

N_VCloneVectorArrayEmpty_ParHyp
Prototype N_Vector *N_VCloneVectorArrayEmpty_ParHyp(int count, N_Vector w)
Description This function creates (by cloning) an array of count parallel vectors, each with an empty (NULL) data array.

N_VDestroyVectorArray_ParHyp
Prototype void N_VDestroyVectorArray_ParHyp(N_Vector *vs, int count)
Description This function frees memory allocated for the array of count variables of type N_Vector created with N_VCloneVectorArray_ParHyp or with N_VCloneVectorArrayEmpty_ParHyp.

N_VPrint_ParHyp
Prototype void N_VPrint_ParHyp(N_Vector v)
Description This function prints the local content of a parhyp vector to stdout.
8.7 The NVVECTOR_PARHYP implementation

**N_VPrintFile_ParHyp**
Prototype: `void N_VPrintFile_ParHyp(N_Vector v, FILE *outfile)`
Description: This function prints the local content of a parhyp vector to `outfile`.

By default all fused and vector array operations are disabled in the NVVECTOR_PARHYP module. The following additional user-callable routines are provided to enable or disable fused and vector array operations for a specific vector. To ensure consistency across vectors it is recommended to first create a vector with `N_VMake_ParHyp`, enable/disable the desired operations for that vector with the functions below, and create any additional vectors from that vector using `N_VClone`. This guarantees the new vectors will have the same operations enabled/disabled as cloned vectors inherit the same enable/disable options as the vector they are cloned from while vectors created with `N_VMake_ParHyp` will have the default settings for the NVVECTOR_PARHYP module.

**N_VEnableFusedOps_ParHyp**
Prototype: `int N_VEnableFusedOps_ParHyp(N_Vector v, booleantype tf)`
Description: This function enables (`SUNTRUE`) or disables (`SUNFALSE`) all fused and vector array operations in the parhyp vector. The return value is 0 for success and -1 if the input vector or its `ops` structure are NULL.

**N_VEnableLinearCombination_ParHyp**
Prototype: `int N_VEnableLinearCombination_ParHyp(N_Vector v, booleantype tf)`
Description: This function enables (`SUNTRUE`) or disables (`SUNFALSE`) the linear combination fused operation in the parhyp vector. The return value is 0 for success and -1 if the input vector or its `ops` structure are NULL.

**N_VEnableScaleAddMulti_ParHyp**
Prototype: `int N_VEnableScaleAddMulti_ParHyp(N_Vector v, booleantype tf)`
Description: This function enables (`SUNTRUE`) or disables (`SUNFALSE`) the scale and add a vector to multiple vectors fused operation in the parhyp vector. The return value is 0 for success and -1 if the input vector or its `ops` structure are NULL.

**N_VEnableDotProdMulti_ParHyp**
Prototype: `int N_VEnableDotProdMulti_ParHyp(N_Vector v, booleantype tf)`
Description: This function enables (`SUNTRUE`) or disables (`SUNFALSE`) the multiple dot products fused operation in the parhyp vector. The return value is 0 for success and -1 if the input vector or its `ops` structure are NULL.

**N_VEnableLinearSumVectorArray_ParHyp**
Prototype: `int N_VEnableLinearSumVectorArray_ParHyp(N_Vector v, booleantype tf)`
Description: This function enables (`SUNTRUE`) or disables (`SUNFALSE`) the linear sum operation for vector arrays in the parhyp vector. The return value is 0 for success and -1 if the input vector or its `ops` structure are NULL.

**N_VEnableScaleVectorArray_ParHyp**
Prototype: `int N_VEnableScaleVectorArray_ParHyp(N_Vector v, booleantype tf)`

Description

This function enables (SUNTRUE) or disables (SUNFALSE) the scale operation for vector arrays in the parhyp vector. The return value is 0 for success and -1 if the input vector or its ops structure are NULL.

Prototype

int N_VEnableConstVectorArray_ParHyp(N_Vector v, booleantype tf)

Description

This function enables (SUNTRUE) or disables (SUNFALSE) the const operation for vector arrays in the parhyp vector. The return value is 0 for success and -1 if the input vector or its ops structure are NULL.

Prototype

int N_VEnableWrmsNormVectorArray_ParHyp(N_Vector v, booleantype tf)

Description

This function enables (SUNTRUE) or disables (SUNFALSE) the WRMS norm operation for vector arrays in the parhyp vector. The return value is 0 for success and -1 if the input vector or its ops structure are NULL.

Prototype

int N_VEnableWrmsNormMaskVectorArray_ParHyp(N_Vector v, booleantype tf)

Description

This function enables (SUNTRUE) or disables (SUNFALSE) the masked WRMS norm operation for vector arrays in the parhyp vector. The return value is 0 for success and -1 if the input vector or its ops structure are NULL.

Prototype

int N_VEnableScaleAddMultiVectorArray_ParHyp(N_Vector v, booleantype tf)

Description

This function enables (SUNTRUE) or disables (SUNFALSE) the scale and add a vector array to multiple vector arrays operation in the parhyp vector. The return value is 0 for success and -1 if the input vector or its ops structure are NULL.

Prototype

int N_VEnableLinearCombinationVectorArray_ParHyp(N_Vector v, booleantype tf)

Description

This function enables (SUNTRUE) or disables (SUNFALSE) the linear combination operation for vector arrays in the parhyp vector. The return value is 0 for success and -1 if the input vector or its ops structure are NULL.

Notes

- When there is a need to access components of an N_Vector_ParHyp, v, it is recommended to extract the hypre vector via 
  x_vec = N_VGetVector_ParHyp(v) and then access components using appropriate hypre functions.

- N_VNewEmpty_ParHyp, N_VMake_ParHyp, and N_VCloneVectorArrayEmpty_ParHyp set the field own_parvector to SUNFALSE. N_VDestroy_ParHyp and N_VDestroyVectorArray_ParHyp will not attempt to delete an underlying hypre vector for any N_Vector with own_parvector set to SUNFALSE. In such a case, it is the user’s responsibility to delete the underlying vector.
8.8 The NVECTOR_PETSC Implementation

The NVECTOR_PETSC module is an NVECTOR wrapper around the PETSc vector. It defines the content field of a N_Vector to be a structure containing the global and local lengths of the vector, a pointer to the PETSc vector, an MPI communicator, and a boolean flag own_data indicating ownership of the wrapped PETSc vector.

```c
struct _N_VectorContent_Petsc {
    sunindextype local_length;
    sunindextype global_length;
    booleantype own_data;
    Vec *pvec;
    MPI_Comm comm;
};
```

The header file to include when using this module is `nvector_petsc.h`. The installed module library to link to is `libsundials_nvecpetsc.lib` where `.lib` is typically `.so` for shared libraries and `.a` for static libraries.

Unlike native sundials vector types, NVECTOR_PETSC does not provide macros to access its member variables. Note that NVECTOR_PETSC requires SUNDIALS to be built with MPI support.

8.8.1 NVECTOR_PETSC functions

The NVECTOR_PETSC module defines implementations of all vector operations listed in Tables 8.1.1, 8.1.2, 8.1.3, and 8.1.4, except for N_VGetArrayPointer and N_VSetArrayPointer. As such, this vector cannot be used with SUNDIALS Fortran interfaces. When access to raw vector data is needed, it is recommended to extract the PETSc vector first, and then use PETSc methods to access the data. Usage examples of NVECTOR_PETSC are provided in example programs for IDA [32].

The names of vector operations are obtained from those in Tables 8.1.1, 8.1.2, 8.1.3, and 8.1.4 by appending the suffix _Petsc (e.g. N_VDestroy_Petsc). The module NVECTOR_PETSC provides the following additional user-callable routines:

- **N_VNewEmpty_Petsc**
  
  Prototype: N_Vector N_VNewEmpty_Petsc(MPI_Comm comm, sunindextype local_length, sunindextype global_length)
  
  Description: This function creates a new NVECTOR wrapper with the pointer to the wrapped PETSc vector set to (NULL). It is used by the N_VMake_Petsc and N_VClone_Petsc implementations.

- **N_VMake_Petsc**
  
  Prototype: N_Vector N_VMake_Petsc(Vec *pvec)
  
  Description: This function creates and allocates memory for an NVECTOR_PETSC wrapper around a user-provided PETSc vector. It does not allocate memory for the vector pvec itself.

- **N_VGetVector_Petsc**
  
  Prototype: Vec *N_VGetVector_Petsc(N_Vector v)
  
  Description: This function returns a pointer to the underlying PETSc vector.
### N_VCloneVectorArray_Petsc

**Prototype**

```
N_Vector *N_VCloneVectorArray_Petsc(int count, N_Vector w)
```

**Description**

This function creates (by cloning) an array of `count` NVVECTOR_PETSC vectors.

---

### N_VCloneVectorArrayEmpty_Petsc

**Prototype**

```
N_Vector *N_VCloneVectorArrayEmpty_Petsc(int count, N_Vector w)
```

**Description**

This function creates (by cloning) an array of `count` NVVECTOR_PETSC vectors, each with pointers to PETSC vectors set to (NULL).

---

### N_VDestroyVectorArray_Petsc

**Prototype**

```
void N_VDestroyVectorArray_Petsc(N_Vector *vs, int count)
```

**Description**

This function frees memory allocated for the array of `count` variables of type N_Vector created with `N_VCloneVectorArray_Petsc` or with `N_VCloneVectorArrayEmpty_Petsc`.

---

### N_VPrint_Petsc

**Prototype**

```
void N_VPrint_Petsc(N_Vector v)
```

**Description**

This function prints the global content of a wrapped PETSC vector to stdout.

---

### N_VPrintFile_Petsc

**Prototype**

```
void N_VPrintFile_Petsc(N_Vector v, const char fname[])
```

**Description**

This function prints the global content of a wrapped PETSC vector to `fname`.

By default all fused and vector array operations are disabled in the NVECTOR_PETSC module. The following additional user-callable routines are provided to enable or disable fused and vector array operations for a specific vector. To ensure consistency across vectors it is recommended to first create a vector with `N_VMake_Petsc`, enable/disable the desired operations for that vector with the functions below, and create any additional vectors from that vector using `N_VClone`. This guarantees the new vectors will have the same operations enabled/disabled as cloned vectors inherit the same enable/disable options as the vector they are cloned from while vectors created with `N_VMake_Petsc` will have the default settings for the NVECTOR_PETSC module.

---

### N_VEnableFusedOps_Petsc

**Prototype**

```
int N_VEnableFusedOps_Petsc(N_Vector v, booleantype tf)
```

**Description**

This function enables (`SUNTRUE`) or disables (`SUNFALSE`) all fused and vector array operations in the PETSC vector. The return value is 0 for success and -1 if the input vector or its ops structure are NULL.

---

### N_VEnableLinearCombination_Petsc

**Prototype**

```
int N_VEnableLinearCombination_Petsc(N_Vector v, booleantype tf)
```

**Description**

This function enables (`SUNTRUE`) or disables (`SUNFALSE`) the linear combination fused operation in the PETSC vector. The return value is 0 for success and -1 if the input vector or its ops structure are NULL.
### N_VEnableScaleAddMulti_Petsc

**Prototype**

```c
int N_VEnableScaleAddMulti_Petsc(N_Vector v, booleantype tf)
```

**Description**

This function enables (`SUNTRUE`) or disables (`SUNFALSE`) the scale and add a vector to multiple vectors fused operation in the PETSc vector. The return value is 0 for success and -1 if the input vector or its `ops` structure are `NULL`.

### N_VEnableDotProdMulti_Petsc

**Prototype**

```c
int N_VEnableDotProdMulti_Petsc(N_Vector v, booleantype tf)
```

**Description**

This function enables (`SUNTRUE`) or disables (`SUNFALSE`) the multiple dot products fused operation in the PETSc vector. The return value is 0 for success and -1 if the input vector or its `ops` structure are `NULL`.

### N_VEnableLinearSumVectorArray_Petsc

**Prototype**

```c
int N_VEnableLinearSumVectorArray_Petsc(N_Vector v, booleantype tf)
```

**Description**

This function enables (`SUNTRUE`) or disables (`SUNFALSE`) the linear sum operation for vector arrays in the PETSc vector. The return value is 0 for success and -1 if the input vector or its `ops` structure are `NULL`.

### N_VEnableScaleVectorArray_Petsc

**Prototype**

```c
int N_VEnableScaleVectorArray_Petsc(N_Vector v, booleantype tf)
```

**Description**

This function enables (`SUNTRUE`) or disables (`SUNFALSE`) the scale operation for vector arrays in the PETSc vector. The return value is 0 for success and -1 if the input vector or its `ops` structure are `NULL`.

### N_VEnableConstVectorArray_Petsc

**Prototype**

```c
int N_VEnableConstVectorArray_Petsc(N_Vector v, booleantype tf)
```

**Description**

This function enables (`SUNTRUE`) or disables (`SUNFALSE`) the const operation for vector arrays in the PETSc vector. The return value is 0 for success and -1 if the input vector or its `ops` structure are `NULL`.

### N_VEnableWrmsNormVectorArray_Petsc

**Prototype**

```c
int N_VEnableWrmsNormVectorArray_Petsc(N_Vector v, booleantype tf)
```

**Description**

This function enables (`SUNTRUE`) or disables (`SUNFALSE`) the WRMS norm operation for vector arrays in the PETSc vector. The return value is 0 for success and -1 if the input vector or its `ops` structure are `NULL`.

### N_VEnableWrmsNormMaskVectorArray_Petsc

**Prototype**

```c
int N_VEnableWrmsNormMaskVectorArray_Petsc(N_Vector v, booleantype tf)
```

**Description**

This function enables (`SUNTRUE`) or disables (`SUNFALSE`) the masked WRMS norm operation for vector arrays in the PETSc vector. The return value is 0 for success and -1 if the input vector or its `ops` structure are `NULL`.
### Description of the NVECTOR module

#### N_VEnableScaleAddMultiVectorArray_Petsc

**Prototype**

```c
int N_VEnableScaleAddMultiVectorArray_Petsc(N_Vector v, booleantype tf)
```

**Description**

This function enables (SUNTRUE) or disables (SUNFALSE) the scale and add a vector array to multiple vector arrays operation in the PETSc vector. The return value is 0 for success and -1 if the input vector or its ops structure are NULL.

#### N_VEnableLinearCombinationVectorArray_Petsc

**Prototype**

```c
int N_VEnableLinearCombinationVectorArray_Petsc(N_Vector v, booleantype tf)
```

**Description**

This function enables (SUNTRUE) or disables (SUNFALSE) the linear combination operation for vector arrays in the PETSc vector. The return value is 0 for success and -1 if the input vector or its ops structure are NULL.

**Notes**

- When there is a need to access components of an N_Vector_Petsc, v, it is recommended to extract the PETSc vector via `x_vec = N_VGetVector_Petsc(v)` and then access components using appropriate PETSc functions.
- The functions N_VNewEmpty_Petsc, N_VMake_Petsc, and N_VCloneVectorArrayEmpty_Petsc set the field `own_data` to SUNFALSE. N_VDestroy_Petsc and N_VDestroyVectorArray_Petsc will not attempt to free the pointer `pvec` for any N_Vector with `own_data` set to SUNFALSE. In such a case, it is the user’s responsibility to deallocate the `pvec` pointer.
- To maximize efficiency, vector operations in the NVECTOR_PETSC implementation that have more than one N_Vector argument do not check for consistent internal representations of these vectors. It is the user’s responsibility to ensure that such routines are called with N_Vector arguments that were all created with the same internal representations.

### 8.9 The NVECTOR_CUDA implementation

The NVECTOR_CUDA module is an experimental NVECTOR implementation in the CUDA language. The module allows for SUNDIALS vector kernels to run on GPU devices. It is intended for users who are already familiar with CUDA and GPU programming. Building this vector module requires a CUDA compiler and, by extension, a C++ compiler. The class `Vector` in the namespace `suncudavec` manages the vector data layout:

```c
template <class T, class I>
class Vector {
    I size_;    
    I mem_size_;  
    T* h_vec_;  
    T* d_vec_;  
    ThreadPartitioning<T, I>* partStream_;  
    ThreadPartitioning<T, I>* partReduce_;  
    bool ownPartitioning_;  
    bool ownData_;  
    bool ownManaged_;  
    ...  
};
```

The class members are vector size (length), size of the vector data memory block, pointers to vector data on the host and the device, pointers to `ThreadPartitioning` implementations that handle thread
partitioning for streaming and reduction vector kernels, a boolean flag that signals if the vector owns the thread partitioning, a boolean flag that signals if the vector owns the data, and a boolean flag that signals if managed memory is used for the data arrays. The class `Vector` inherits from the empty structure

```c
struct _N_VectorContent_Cuda {}
```

to interface the C++ class with the NVECTOR C code. Due to the rapid progress of CUDA development, we expect that the `suncudavec::Vector` class will change frequently in future SUNDIALS releases. The code is structured so that it can tolerate significant changes in the `suncudavec::Vector` class without requiring changes to the user API.

When instantiated with `N_VNew_Cuda`, the class `Vector` will allocate memory on both the host and the device. Alternatively, a user can provide host and device data arrays by using the `N_VMake_Cuda` constructor. To use CUDA managed memory, the constructors `N_VNewManaged_Cuda` and `N_VMakeManaged_Cuda` are provided. Details on each of these constructors are provided below.

To use the NVECTOR_CUDA module, the header file to include is `nvector_cuda.h`, and the library to link to is `libsundials_nveccuda.lib`. The extension `.lib` is typically `.so` for shared libraries and `.a` for static libraries.

### 8.9.1 NVECTOR_CUDA functions

Unlike other native SUNDIALS vector types, NVECTOR_CUDA does not provide macros to access its member variables. Instead, user should use the accessor functions:

```c
N_VGetHostArrayPointer_Cuda
Prototype realtype *N_VGetHostArrayPointer_Cuda(N_Vector v)
Description This function returns a pointer to the vector data on the host.
```

```c
N_VGetDeviceArrayPointer_Cuda
Prototype realtype *N_VGetDeviceArrayPointer_Cuda(N_Vector v)
Description This function returns a pointer to the vector data on the device.
```

```c
N_VIsManagedMemory_Cuda
Prototype booleantype *N_VIsManagedMemory_Cuda(N_Vector v)
Description This function returns a boolean flag indicating if the vector data is allocated in managed memory or not.
```

The NVECTOR_CUDA module defines implementations of all vector operations listed in Tables 8.1.1, 8.1.2, 8.1.3, and 8.1.4, except for `N_VSetArrayPointer`, and, if using unmanaged memory, `N_VGetArrayPointer`. As such, this vector can only be used with the SUNDIALS Fortran interfaces, and the SUNDIALS direct solvers and preconditioners when using managed memory. The NVECTOR_CUDA module provides separate functions to access data on the host and on the device for the unmanaged memory use case. It also provides methods for copying from the host to the device and vice versa. Usage examples of NVECTOR_CUDA are provided in some example programs for CVODE [34].

The names of vector operations are obtained from those in Tables 8.1.1, 8.1.2, 8.1.3, and 8.1.4 by appending the suffix _Cuda (e.g. `N_VDestroy_Cuda`). The module NVECTOR_CUDA provides the following functions:

```c
N_VNew_Cuda
Prototype N_Vector N_VNew_Cuda(sunindextype length)
Description This function creates and allocates memory for a CUDA N_Vector. The vector data array is allocated on both the host and device.
Description of the NVECTOR module

N_VNewManaged_Cuda
Prototype N_Vector N_VNewManaged_Cuda(sunindextype length)
Description This function creates and allocates memory for a CUDA N_Vector. The vector data array is allocated in managed memory.

N_VNewEmpty_Cuda
Prototype N_Vector N_VNewEmpty_Cuda()
Description This function creates a new NVECTOR wrapper with the pointer to the wrapped CUDA vector set to NULL. It is used by the N_VNew_Cuda, N_VMake_Cuda, and N_VClone_Cuda implementations.

N_VMake_Cuda
Prototype N_Vector N_VMake_Cuda(sunindextype length, realtype *h_data, realtype *dev_data)
Description This function creates an NVECTOR_CUDA with user-supplied vector data arrays h_vdata and d_vdata. This function does not allocate memory for data itself.

N_VMakeManaged_Cuda
Prototype N_Vector N_VMakeManaged_Cuda(sunindextype length, realtype *vdata)
Description This function creates an NVECTOR_CUDA with a user-supplied managed memory data array. This function does not allocate memory for data itself.

N_VMakeWithManagedAllocator_Cuda
Prototype N_Vector N_VMakeWithManagedAllocator_Cuda(sunindextype length, void* (*allocfn)(size_t size), void (*freefn)(void* ptr));
Description This function creates an NVECTOR_CUDA with a user-supplied memory allocator. It requires the user to provide a corresponding free function as well. The memory allocated by the allocator function must behave like CUDA managed memory.

The module NVECTOR_CUDA also provides the following user-callable routines:

N_VSetCudaStream_Cuda
Prototype void N_VSetCudaStream_Cuda(N_Vector v, cudaStream_t *stream)
Description This function sets the CUDA stream that all vector kernels will be launched on. By default an NVECTOR_CUDA uses the default CUDA stream.

Note: All vectors used in a single instance of a SUNDIALS solver must use the same CUDA stream, and the CUDA stream must be set prior to solver initialization. Additionally, if manually instantiating the stream and reduce ThreadPartitioning of a suncudavec::Vector, ensure that they use the same CUDA stream.

N_VCopyToDevice_Cuda
Prototype void N_VCopyToDevice_Cuda(N_Vector v)
Description This function copies host vector data to the device.
8.9 The NV

VECTOR_CUDA implementation

N_VC

opyFromDevice_Cuda
Prototype void N_VC

opyFromDevice_Cuda(N_Vector v)
Description This function copies vector data from the device to the host.

N_VPrint_Cuda
Prototype void N_VPrint_Cuda(N_Vector v)
Description This function prints the content of a CUDA vector to stdout.

N_VPrintFile_Cuda
Prototype void N_VPrintFile_Cuda(N_Vector v, FILE *outfile)
Description This function prints the content of a CUDA vector to outfile.

By default all fused and vector array operations are disabled in the NV

ECTOR_CUDA module. The following additional user-callable routines are provided to enable or disable fused and vector array operations for a specific vector. To ensure consistency across vectors it is recommended to first create a vector with NV

ew_CUDA, enable/disable the desired operations for that vector with the functions below, and create any additional vectors from that vector using NV

lon

. This guarantees the new vectors will have the same operations enabled/disabled as cloned vectors inherit the same enable/disable options as the vector they are cloned from while vectors created with NV

ew_CUDA will have the default settings for the NV

CTOR_CUDA module.

N_VEnableFusedOps_Cuda
Prototype int N_VEnableFusedOps_Cuda(N_Vector v, booleantype tf)
Description This function enables (SUNTRUE) or disables (SUNFALSE) all fused and vector array operations in the CUDA vector. The return value is 0 for success and -1 if the input vector or its ops structure are NULL.

N_VEnableLinearCombination_Cuda
Prototype int N_VEnableLinearCombination_Cuda(N_Vector v, booleantype tf)
Description This function enables (SUNTRUE) or disables (SUNFALSE) the linear combination fused operation in the CUDA vector. The return value is 0 for success and -1 if the input vector or its ops structure are NULL.

N_VEnableScaleAddMulti_Cuda
Prototype int N_VEnableScaleAddMulti_Cuda(N_Vector v, booleantype tf)
Description This function enables (SUNTRUE) or disables (SUNFALSE) the scale and add a vector to multiple vectors fused operation in the CUDA vector. The return value is 0 for success and -1 if the input vector or its ops structure are NULL.

N_VEnableDotProdMulti_Cuda
Prototype int N_VEnableDotProdMulti_Cuda(N_Vector v, booleantype tf)
Description This function enables (SUNTRUE) or disables (SUNFALSE) the multiple dot products fused operation in the CUDA vector. The return value is 0 for success and -1 if the input vector or its ops structure are NULL.
Description of the NVECTOR module

**N_VEnableLinearSumVectorArray_Cuda**

Prototype: `int N_VEnableLinearSumVectorArray_Cuda(N_Vector v, booleantype tf)`

Description: This function enables (SUNTRUE) or disables (SUNFALSE) the linear sum operation for vector arrays in the CUDA vector. The return value is 0 for success and -1 if the input vector or its ops structure are NULL.

**N_VEnableScaleVectorArray_Cuda**

Prototype: `int N_VEnableScaleVectorArray_Cuda(N_Vector v, booleantype tf)`

Description: This function enables (SUNTRUE) or disables (SUNFALSE) the scale operation for vector arrays in the CUDA vector. The return value is 0 for success and -1 if the input vector or its ops structure are NULL.

**N_VEnableConstVectorArray_Cuda**

Prototype: `int N_VEnableConstVectorArray_Cuda(N_Vector v, booleantype tf)`

Description: This function enables (SUNTRUE) or disables (SUNFALSE) the const operation for vector arrays in the CUDA vector. The return value is 0 for success and -1 if the input vector or its ops structure are NULL.

**N_VEnableWrmsNormVectorArray_Cuda**

Prototype: `int N_VEnableWrmsNormVectorArray_Cuda(N_Vector v, booleantype tf)`

Description: This function enables (SUNTRUE) or disables (SUNFALSE) the WRMS norm operation for vector arrays in the CUDA vector. The return value is 0 for success and -1 if the input vector or its ops structure are NULL.

**N_VEnableWrmsNormMaskVectorArray_Cuda**

Prototype: `int N_VEnableWrmsNormMaskVectorArray_Cuda(N_Vector v, booleantype tf)`

Description: This function enables (SUNTRUE) or disables (SUNFALSE) the masked WRMS norm operation for vector arrays in the CUDA vector. The return value is 0 for success and -1 if the input vector or its ops structure are NULL.

**N_VEnableScaleAddMultiVectorArray_Cuda**

Prototype: `int N_VEnableScaleAddMultiVectorArray_Cuda(N_Vector v, booleantype tf)`

Description: This function enables (SUNTRUE) or disables (SUNFALSE) the scale and add a vector array to multiple vector arrays operation in the CUDA vector. The return value is 0 for success and -1 if the input vector or its ops structure are NULL.

**N_VEnableLinearCombinationVectorArray_Cuda**

Prototype: `int N_VEnableLinearCombinationVectorArray_Cuda(N_Vector v, booleantype tf)`

Description: This function enables (SUNTRUE) or disables (SUNFALSE) the linear combination operation for vector arrays in the CUDA vector. The return value is 0 for success and -1 if the input vector or its ops structure are NULL.
Notes

- When there is a need to access components of an N_Vector_Cuda, v, it is recommended to use functions N_VGetDeviceArrayPointer_Cuda or N_VGetHostArrayPointer_Cuda. However, when using managed memory, the function N_VGetArrayPointer may also be used.

- To maximize efficiency, vector operations in the NVECTOR_CUDA implementation that have more than one N_Vector argument do not check for consistent internal representations of these vectors. It is the user’s responsibility to ensure that such routines are called with N_Vector arguments that were all created with the same internal representations.

8.10 The NVECTOR_RAJA implementation

The nvector_raja module is an experimental NVECTOR implementation using the RAJA hardware abstraction layer. In this implementation, RAJA allows for SUNDIALS vector kernels to run on GPU devices. The module is intended for users who are already familiar with RAJA and GPU programming. Building this vector module requires a C++11 compliant compiler and a CUDA software development toolkit. Besides the CUDA backend, RAJA has other backends such as serial, OpenMP, and OpenACC. These backends are not used in this SUNDIALS release. Class Vector in namespace sunrajavec manages the vector data layout:

```cpp
template <class T, class I>
class Vector {
    I size_;  
    I mem_size_;  
    T* h_vec_;  
    T* d_vec_;  
    ...  
};
```

The class members are: vector size (length), size of the vector data memory block, the global vector size (length), a pointer to the vector data on the host, and a pointer to the vector data on the device. The class Vector inherits from an empty structure

```cpp
struct _N_VectorContent_Raja {  
};
```

to interface the C++ class with the NVECTOR C code. When instantiated, the class Vector will allocate memory on both the host and the device. Due to the rapid progress of RAJA development, we expect that the sunrajavec::Vector class will change frequently in future SUNDIALS releases. The code is structured so that it can tolerate significant changes in the sunrajavec::Vector class without requiring changes to the user API.

The header file to include when using this module is nvector_raja.h. The installed module library to link to are libsundials_nveccudaraja.lib. The extension .lib is typically .so for shared libraries and .a for static libraries.

8.10.1 NVECTOR_RAJA functions

Unlike other native SUNDIALS vector types, NVECTOR_RAJA does not provide macros to access its member variables. Instead, user should use the accessor functions:

```cpp
N_VGetHostArrayPointer_Raja
```
Prototype `realtype *N_VGetHostArrayPointer_Raja(N_vector v)`
Description This function returns a pointer to the vector data on the host.
Description of the NVECTOR module

**N_VGetDeviceArrayPointer_Raja**

Prototype: `realtype *N_VGetDeviceArrayPointer_Raja(N_Vector v)`

Description: This function returns a pointer to the vector data on the device.

The NVECTOR_RAJA module defines the implementations of all vector operations listed in Tables 8.1.1, 8.1.2, 8.1.3, and 8.1.4, except for `N_VDotProdMulti`, `N_VWrmsNormVectorArray`, and `N_VWrmsNormMaskVectorArray` as support for arrays of reduction vectors is not yet supported in RAJA. These functions will be added to the NVECTOR_RAJA implementation in the future. Additionally, the vector operations `N_VGetArrayPointer` and `N_VSetArrayPointer` are not implemented by the RAJA vector. As such, this vector cannot be used with the SUNDIALS Fortran interfaces, nor with the SUNDIALS direct solvers and preconditioners. The NVECTOR_RAJA module provides separate functions to access data on the host and on the device. It also provides methods for copying data from the host to the device and vice versa. Usage examples of NVECTOR_RAJA are provided in some example programs for CVODE [34].

The names of vector operations are obtained from those in Tables 8.1.1, 8.1.2, 8.1.3, and 8.1.4 by appending the suffix _Raja (e.g. N_VDestroy_Raja). The module NVECTOR_RAJA provides the following additional user-callable routines:

**N_VNew_Raja**

Prototype: `N_Vector N_VNew_Raja(sunindextype length)`

Description: This function creates and allocates memory for a CUDA N_Vector. The vector data array is allocated on both the host and device.

**N_VNewEmpty_Raja**

Prototype: `N_Vector N_VNewEmpty_Raja()`

Description: This function creates a new NVECTOR wrapper with the pointer to the wrapped RAJA vector set to NULL. It is used by the N_VNew_Raja, N_VMake_Raja, and N_VClone_Raja implementations.

**N_VMake_Raja**

Prototype: `N_Vector N_VMake_Raja(N_VectorContent_Raja c)`

Description: This function creates and allocates memory for an NVECTOR_RAJA wrapper around a user-provided sunrajevec::Vector class. Its only argument is of type N_VectorContent_Raja, which is the pointer to the class.

**N_VCopyToDevice_Raja**

Prototype: `realtype *N_VCopyToDevice_Raja(N_Vector v)`

Description: This function copies host vector data to the device.

**N_VCopyFromDevice_Raja**

Prototype: `realtype *N_VCopyFromDevice_Raja(N_Vector v)`

Description: This function copies vector data from the device to the host.

**N_VPrint_Raja**

Prototype: `void N_VPrint_Raja(N_Vector v)`

Description: This function prints the content of a RAJA vector to stdout.
8.10 The NVECTOR_RAJA implementation

**N_VPrintFile_Raja**

Prototype: `void N_VPrintFile_Raja(N_Vector v, FILE *outfile)`

Description: This function prints the content of a RAJA vector to `outfile`.

By default all fused and vector array operations are disabled in the NVECTOR_RAJA module. The following additional user-callable routines are provided to enable or disable fused and vector array operations for a specific vector. To ensure consistency across vectors it is recommended to first create a vector with `N_VNew_Raja`, enable/disable the desired operations for that vector with the functions below, and create any additional vectors from that vector using `N_VClone`. This guarantees the new vectors will have the same operations enabled/disabled as cloned vectors inherit the same enable/disable options as the vector they are cloned from while vectors created with `N_VNew_Raja` will have the default settings for the NVECTOR_RAJA module.

**N_VEnableFusedOps_Raja**

Prototype: `int N_VEnableFusedOps_Raja(N_Vector v, booleantype tf)`

Description: This function enables (SUNTRUE) or disables (SUNFALSE) all fused and vector array operations in the RAJA vector. The return value is 0 for success and -1 if the input vector or its ops structure are NULL.

**N_VEnableLinearCombination_Raja**

Prototype: `int N_VEnableLinearCombination_Raja(N_Vector v, booleantype tf)`

Description: This function enables (SUNTRUE) or disables (SUNFALSE) the linear combination fused operation in the RAJA vector. The return value is 0 for success and -1 if the input vector or its ops structure are NULL.

**N_VEnableScaleAddMulti_Raja**

Prototype: `int N_VEnableScaleAddMulti_Raja(N_Vector v, booleantype tf)`

Description: This function enables (SUNTRUE) or disables (SUNFALSE) the scale and add a vector to multiple vectors fused operation in the RAJA vector. The return value is 0 for success and -1 if the input vector or its ops structure are NULL.

**N_VEnableLinearSumVectorArray_Raja**

Prototype: `int N_VEnableLinearSumVectorArray_Raja(N_Vector v, booleantype tf)`

Description: This function enables (SUNTRUE) or disables (SUNFALSE) the linear sum operation for vector arrays in the RAJA vector. The return value is 0 for success and -1 if the input vector or its ops structure are NULL.

**N_VEnableScaleVectorArray_Raja**

Prototype: `int N_VEnableScaleVectorArray_Raja(N_Vector v, booleantype tf)`

Description: This function enables (SUNTRUE) or disables (SUNFALSE) the scale operation for vector arrays in the RAJA vector. The return value is 0 for success and -1 if the input vector or its ops structure are NULL.

**N_VEnableConstVectorArray_Raja**

Prototype: `int N_VEnableConstVectorArray_Raja(N_Vector v, booleantype tf)`
Description  This function enables (SUNTRUE) or disables (SUNFALSE) the const operation for vector arrays in the RAJA vector. The return value is 0 for success and -1 if the input vector or its ops structure are NULL.

**N_VEnableScaleAddMultiVectorArray_Raja**

**Prototype**
```c
int N_VEnableScaleAddMultiVectorArray_Raja(N_Vector v, booleantype tf)
```

**Description**  This function enables (SUNTRUE) or disables (SUNFALSE) the scale and add a vector array to multiple vector arrays operation in the RAJA vector. The return value is 0 for success and -1 if the input vector or its ops structure are NULL.

**N_VEnableLinearCombinationVectorArray_Raja**

**Prototype**
```c
int N_VEnableLinearCombinationVectorArray_Raja(N_Vector v, booleantype tf)
```

**Description**  This function enables (SUNTRUE) or disables (SUNFALSE) the linear combination operation for vector arrays in the RAJA vector. The return value is 0 for success and -1 if the input vector or its ops structure are NULL.

**Notes**
- When there is a need to access components of an N_Vector_Raja, v, it is recommended to use functions N_VGetDeviceArrayPointer_Raja or N_VGetHostArrayPointer_Raja.
- To maximize efficiency, vector operations in the NVVECTOR_RAJA implementation that have more than one N_Vector argument do not check for consistent internal representations of these vectors. It is the user’s responsibility to ensure that such routines are called with N_Vector arguments that were all created with the same internal representations.

### 8.11 The NVVECTOR_OPENMPDEV implementation

In situations where a user has access to a device such as a GPU for offloading computation, SUNDIALS provides an NVVECTOR implementation using OpenMP device offloading, called NVVECTOR_OPENMPDEV.

The NVVECTOR_OPENMPDEV implementation defines the content field of the N_Vector to be a structure containing the length of the vector, a pointer to the beginning of a contiguous data array on the host, a pointer to the beginning of a contiguous data array on the device, and a boolean flag own_data which specifies the ownership of host and device data arrays.

```c
struct _N_VectorContent_OpenMPDEV {
    sunindextype length;
    booleantype own_data;
    realtype *host_data;
    realtype *dev_data;
};
```

The header file to include when using this module is nvvector_openmpdev.h. The installed module library to link to is libsundials_nvvecopenmpdev.lib where .lib is typically .so for shared libraries and .a for static libraries.

### 8.11.1 NVVECTOR_OPENMPDEV accessor macros

The following macros are provided to access the content of an NVVECTOR_OPENMPDEV vector.
8.11 The NVector_OPENMPDEV implementation

- **NV_CONTENT_OPENMPDEV**

  This routine gives access to the contents of the NVector_OPENMPDEV vector N_Vector.

  The assignment \( v_{\text{cont}} = \text{NV_CONTENT_OPENMPDEV}(v) \) sets \( v_{\text{cont}} \) to be a pointer to the NVECTOR_OPENMPDEV N_Vector content structure.

  Implementation:

  ```c
  #define NV_CONTENT_OPENMPDEV(v) ( (N_VectorContent_OpenMPDEV)(v->content) )
  ```

- **NV_OWN_DATA_OPENMPDEV, NV_DATA_HOST_OPENMPDEV, NV_DATA_DEV_OPENMPDEV, NV_LENGTH_OPENMPDEV**

  These macros give individual access to the parts of the content of an NVECTOR_OPENMPDEV N_Vector.

  The assignment \( v_{\text{data}} = \text{NV_DATA_HOST_OPENMPDEV}(v) \) sets \( v_{\text{data}} \) to be a pointer to the first component of the data on the host for the N_Vector \( v \). The assignment \( \text{NV_DATA_HOST_OPENMPDEV}(v) = v_{\text{data}} \) sets the host component array of \( v \) to be \( v_{\text{data}} \) by storing the pointer \( v_{\text{data}} \).

  The assignment \( v_{\text{dev\_data}} = \text{NV_DATA_DEV_OPENMPDEV}(v) \) sets \( v_{\text{dev\_data}} \) to be a pointer to the first component of the data on the device for the N_Vector \( v \). The assignment \( \text{NV_DATA_DEV_OPENMPDEV}(v) = v_{\text{dev\_data}} \) sets the device component array of \( v \) to be \( v_{\text{dev\_data}} \) by storing the pointer \( v_{\text{dev\_data}} \).

  The assignment \( v_{\text{len}} = \text{NV_LENGTH_OPENMPDEV}(v) \) sets \( v_{\text{len}} \) to be the length of \( v \). On the other hand, the call \( \text{NV_LENGTH_OPENMPDEV}(v) = \text{len}_v \) sets the length of \( v \) to be \( \text{len}_v \).

  Implementation:

  ```c
  #define NV_OWN_DATA_OPENMPDEV(v) ( NV_CONTENT_OPENMPDEV(v)->own_data )
  #define NV_DATA_HOST_OPENMPDEV(v) ( NV_CONTENT_OPENMPDEV(v)->host_data )
  #define NV_DATA_DEV_OPENMPDEV(v) ( NV_CONTENT_OPENMPDEV(v)->dev_data )
  #define NV_LENGTH_OPENMPDEV(v) ( NV_CONTENT_OPENMPDEV(v)->length )
  ```

8.11.2 NVECTOR_OPENMPDEV functions

The NVECTOR_OPENMPDEV module defines OpenMP device offloading implementations of all vector operations listed in Tables 8.1.1, 8.1.2, 8.1.3, and 8.1.4, except for NVGetArrayPointer and NVSetArrayPointer. As such, this vector cannot be used with the SUNDIALS Fortran interfaces, nor with the SUNDIALS direct solvers and preconditioners. It also provides methods for copying from the host to the device and vice versa.

The names of vector operations are obtained from those in Tables 8.1.1, 8.1.2, 8.1.3, and 8.1.4 by appending the suffix _OPENMPDEV (e.g. NVDestroy_OPENMPDEV). The module NVVECTOR_OPENMPDEV provides the following additional user-callable routines:

```
N_VNew_OpenMPDEV
Prototype    N_Vector N_VNew_OpenMPDEV(sunindextype vec_length)
Description   This function creates and allocates memory for an NVECTOR_OPENMPDEV N_Vector.
```

```
N_VNewEmpty_OpenMPDEV
Prototype N_Vector N_VNewEmpty_OpenMPDEV(sunindextype vec_length)
Description This function creates a new NVECTOR_OPENMPDEV N_Vector with an empty (NULL) host and device data arrays.
```
Description of the NVECTOR module

**N_VMake_OpenMPDEV**
Prototype

```c
N_Vector N_VMake_OpenMPDEV(sunindextype vec_length, realtype *h_vdata, 
                           realtype *d_vdata)
```

Description
This function creates an NVECTOR_OPENMPDEV vector with user-supplied vector data arrays `h_vdata` and `d_vdata`. This function does not allocate memory for data itself.

**N_VCloneVectorArray_OpenMPDEV**
Prototype

```c
N_Vector *N_VCloneVectorArray_OpenMPDEV(int count, N_Vector w)
```

Description
This function creates (by cloning) an array of count NVECTOR_OPENMPDEV vectors.

**N_VCloneVectorArrayEmpty_OpenMPDEV**
Prototype

```c
N_Vector *N_VCloneVectorArrayEmpty_OpenMPDEV(int count, N_Vector w)
```

Description
This function creates (by cloning) an array of count NVECTOR_OPENMPDEV vectors, each with an empty (NULL) data array.

**N_VDestroyVectorArray_OpenMPDEV**
Prototype

```c
void N_VDestroyVectorArray_OpenMPDEV(N_Vector *vs, int count)
```

Description
This function frees memory allocated for the array of count variables of type `N_Vector` created with `N_VCloneVectorArray_OpenMPDEV` or with `N_VCloneVectorArrayEmpty_OpenMPDEV`.

**N_VGetHostArrayPointer_OpenMPDEV**
Prototype

```c
realtype *N_VGetHostArrayPointer_OpenMPDEV(N_Vector v)
```

Description
This function returns a pointer to the host data array.

**N_VGetDeviceArrayPointer_OpenMPDEV**
Prototype

```c
realtype *N_VGetDeviceArrayPointer_OpenMPDEV(N_Vector v)
```

Description
This function returns a pointer to the device data array.

**N_VPrint_OpenMPDEV**
Prototype

```c
void N_VPrint_OpenMPDEV(N_Vector v)
```

Description
This function prints the content of an NVECTOR_OPENMPDEV vector to `stdout`.

**N_VPrintFile_OpenMPDEV**
Prototype

```c
void N_VPrintFile_OpenMPDEV(N_Vector v, FILE *outfile)
```

Description
This function prints the content of an NVECTOR_OPENMPDEV vector to `outfile`.

**N_VCopyToDevice_OpenMPDEV**
Prototype

```c
void N_VCopyToDevice_OpenMPDEV(N_Vector v)
```

Description
This function copies the content of an NVECTOR_OPENMPDEV vector’s host data array to the device data array.
N_VCopyFromDevice_OpenMPDEV

Prototype    void N_VCopyFromDevice_OpenMPDEV(N_Vector v)
Description   This function copies the content of an NVECTOR_OPENMPDEV vector’s device data array to the host data array.

By default all fused and vector array operations are disabled in the NVECTOR_OPENMPDEV module. The following additional user-callable routines are provided to enable or disable fused and vector array operations for a specific vector. To ensure consistency across vectors it is recommended to first create a vector with N_VNew_OpenMPDEV, enable/disable the desired operations for that vector with the functions below, and create any additional vectors from that vector using N_VClone. This guarantees the new vectors will have the same operations enabled/disabled as cloned vectors inherit the same enable/disable options as the vector they are cloned from while vectors created with N_VNew_OpenMPDEV will have the default settings for the NVECTOR_OPENMPDEV module.

N_VEnableFusedOps_OpenMPDEV

Prototype    int N_VEnableFusedOps_OpenMPDEV(N_Vector v, booleantype tf)
Description   This function enables (SUNTRUE) or disables (SUNFALSE) all fused and vector array operations in the NVECTOR_OPENMPDEV vector. The return value is 0 for success and -1 if the input vector or its ops structure are NULL.

N_VEnableLinearCombination_OpenMPDEV

Prototype    int N_VEnableLinearCombination_OpenMPDEV(N_Vector v, booleantype tf)
Description   This function enables (SUNTRUE) or disables (SUNFALSE) the linear combination fused operation in the NVECTOR_OPENMPDEV vector. The return value is 0 for success and -1 if the input vector or its ops structure are NULL.

N_VEnableScaleAddMulti_OpenMPDEV

Prototype    int N_VEnableScaleAddMulti_OpenMPDEV(N_Vector v, booleantype tf)
Description   This function enables (SUNTRUE) or disables (SUNFALSE) the scale and add a vector to multiple vectors fused operation in the NVECTOR_OPENMPDEV vector. The return value is 0 for success and -1 if the input vector or its ops structure are NULL.

N_VEnableDotProdMulti_OpenMPDEV

Prototype    int N_VEnableDotProdMulti_OpenMPDEV(N_Vector v, booleantype tf)
Description   This function enables (SUNTRUE) or disables (SUNFALSE) the multiple dot products fused operation in the NVECTOR_OPENMPDEV vector. The return value is 0 for success and -1 if the input vector or its ops structure are NULL.

N_VEnableLinearSumVectorArray_OpenMPDEV

Prototype    int N_VEnableLinearSumVectorArray_OpenMPDEV(N_Vector v, booleantype tf)
Description   This function enables (SUNTRUE) or disables (SUNFALSE) the linear sum operation for vector arrays in the NVECTOR_OPENMPDEV vector. The return value is 0 for success and -1 if the input vector or its ops structure are NULL.
Description of the NVECTOR module

N_VEnableScaleVectorArray_OpenMPDEV
Prototype  int N_VEnableScaleVectorArray_OpenMPDEV(N_Vector v, booleantype tf)
Description  This function enables (SUNTRUE) or disables (SUNFALSE) the scale operation for vector arrays in the NVECTOR_OPENMPDEV vector. The return value is 0 for success and -1 if the input vector or its ops structure are NULL.

N_VEnableConstVectorArray_OpenMPDEV
Prototype  int N_VEnableConstVectorArray_OpenMPDEV(N_Vector v, booleantype tf)
Description  This function enables (SUNTRUE) or disables (SUNFALSE) the const operation for vector arrays in the NVECTOR_OPENMPDEV vector. The return value is 0 for success and -1 if the input vector or its ops structure are NULL.

N_VEnableWrmsNormVectorArray_OpenMPDEV
Prototype  int N_VEnableWrmsNormVectorArray_OpenMPDEV(N_Vector v, booleantype tf)
Description  This function enables (SUNTRUE) or disables (SUNFALSE) the WRMS norm operation for vector arrays in the NVECTOR_OPENMPDEV vector. The return value is 0 for success and -1 if the input vector or its ops structure are NULL.

N_VEnableWrmsNormMaskVectorArray_OpenMPDEV
Prototype  int N_VEnableWrmsNormMaskVectorArray_OpenMPDEV(N_Vector v, booleantype tf)
Description  This function enables (SUNTRUE) or disables (SUNFALSE) the masked WRMS norm operation for vector arrays in the NVECTOR_OPENMPDEV vector. The return value is 0 for success and -1 if the input vector or its ops structure are NULL.

N_VEnableScaleAddMultiVectorArray_OpenMPDEV
Prototype  int N_VEnableScaleAddMultiVectorArray_OpenMPDEV(N_Vector v, booleantype tf)
Description  This function enables (SUNTRUE) or disables (SUNFALSE) the scale and add a vector array to multiple vector arrays operation in the NVECTOR_OPENMPDEV vector. The return value is 0 for success and -1 if the input vector or its ops structure are NULL.

N_VEnableLinearCombinationVectorArray_OpenMPDEV
Prototype  int N_VEnableLinearCombinationVectorArray_OpenMPDEV(N_Vector v, booleantype tf)
Description  This function enables (SUNTRUE) or disables (SUNFALSE) the linear combination operation for vector arrays in the NVECTOR_OPENMPDEV vector. The return value is 0 for success and -1 if the input vector or its ops structure are NULL.

Notes
- When looping over the components of an N_Vector v, it is most efficient to first obtain the component array via h_data = NV_DATA_HOST_OPENMPDEV(v) for the host array or d_data = NV_DATA_DEV_OPENMPDEV(v) for the device array and then access h_data[i] or d_data[i] within the loop.
• When accessing individual components of an N_Vector v on the host remember to first copy the array back from the device with N_VCopyFromDevice_OpenMPDEV(v) to ensure the array is up to date.

• N_VNewEmpty_OpenMPDEV, N_VMake_OpenMPDEV, and N_VCloneVectorArrayEmpty_OpenMPDEV set the field own_data = SUNFALSE. N_Destroy_OpenMPDEV and N_DestroyVectorArray_OpenMPDEV will not attempt to free the pointer data for any N_Vector with own_data set to SUNFALSE. In such a case, it is the user’s responsibility to deallocate the data pointer.

• To maximize efficiency, vector operations in the NVECTOR_OPENMPDEV implementation that have more than one N_Vector argument do not check for consistent internal representation of these vectors. It is the user’s responsibility to ensure that such routines are called with N_Vector arguments that were all created with the same internal representations.

8.12 The NVECTOR_TRILINOS implementation

The nvector_trilinos module is an NVECTOR wrapper around the Trilinos Tpetra vector. The interface to Tpetra is implemented in the Sundials::TpetraVectorInterface class. This class simply stores a reference counting pointer to a Tpetra vector and inherits from an empty structure

```c
struct _N_VectorContent_Trilinos {
};
```

to interface the C++ class with the nvector C code. A pointer to an instance of this class is kept in the content field of the N_Vector object, to ensure that the Tpetra vector is not deleted for as long as the N_Vector object exists.

The Tpetra vector type in the Sundials::TpetraVectorInterface class is defined as:

```c
typedef Tpetra::Vector<realtype, sunindextype, sunindextype> vector_type;
```

The Tpetra vector will use the sundials-specified realtype as its scalar type, and it will use sunindextype as the global and the local ordinal types. This type definition will use Tpetra’s default node type. Available Kokkos node types in Trilinos 12.14 release are serial (single thread), OpenMP, Pthread, and CUDA. The default node type is selected when building the Kokkos package. For example, the Tpetra vector will use a CUDA node if Tpetra was built with CUDA support and the CUDA node was selected as the default when Tpetra was built.

The header file to include when using this module is nvector_trilinos.h. The installed module library to link to is libsundials_nvectrilinos.lib where .lib is typically .so for shared libraries and .a for static libraries.

8.12.1 NVECTOR_TRILINOS functions

The nvector_trilinos module defines implementations of all vector operations listed in Tables 8.1.1, 8.1.4, and 8.1.4, except for N_VGetArrayPointer and N_VSetArrayPointer. As such, this vector cannot be used with SUNDIALS Fortran interfaces, nor with the SUNDIALS direct solvers and preconditioners. When access to raw vector data is needed, it is recommended to extract the Trilinos Tpetra vector first, and then use Tpetra vector methods to access the data. Usage examples of nvector_trilinos are provided in example programs for IDA [32].

The names of vector operations are obtained from those in Tables 8.1.1, 8.1.4, and 8.1.4 by appending the suffix _Trilinos (e.g. N_VDestroy_Trilinos). Vector operations call existing Tpetra::Vector methods when available. Vector operations specific to SUNDIALS are implemented as standalone functions in the namespace Sundials::TpetraVector, located in the file SundialsTpetraVectorKernels.hpp. The module NVECTOR_TRILINOS provides the following additional user-callable functions:

• N_VGetVector_Trilinos

This C++ function takes an N_Vector as the argument and returns a reference counting pointer to the underlying Tpetra vector. This is a standalone function defined in the global namespace.
Teuchos::RCP<vector_type> N_VGetVector_Trilinos(N_Vector v);

- \texttt{N\_VMake\_Trilinos}
  
  This C++ function creates and allocates memory for an \texttt{N\textunderscore VECTOR\textunderscore TRILINOS} wrapper around a user-provided Tpetra vector. This is a standalone function defined in the global namespace.

\texttt{N\_Vector N\_VMake\_Trilinos(Teuchos::RCP<vector\_type> v)};

\textbf{Notes}

- The template parameter \texttt{vector\_type} should be set as:
  
  \begin{verbatim}
  typedef Sundials::TpetraVectorInterface::vector_type vector_type
  \end{verbatim}
  
  This will ensure that data types used in Tpetra vector match those in \texttt{SUNDIALS}.

- When there is a need to access components of an \texttt{N\_Vector\textunderscore Trilinos}, \texttt{v}, it is recommended to extract the Trilinos vector object via \texttt{x\_vec = N\_VGetVector\_Trilinos(v)} and then access components using the appropriate Trilinos functions.

- The functions \texttt{N\_VDestroy\_Trilinos} and \texttt{N\_VDestroyVectorArray\_Trilinos} only delete the \texttt{N\_Vector} wrapper. The underlying Tpetra vector object will exist for as long as there is at least one reference to it.

\section*{8.13 The NVECTOR\_MANYVECTOR implementation}

The \texttt{NVECTOR\_MANYVECTOR} implementation of the \texttt{NVECTOR} module provided with \texttt{SUNDIALS} is designed to facilitate problems with an inherent data partitioning for the solution vector within a computational node. These data partitions are entirely user-defined, through construction of distinct \texttt{NVECTOR} modules for each component, that are then combined together to form the \texttt{NVECTOR\_MANYVECTOR}. We envision two generic use cases for this implementation:

\textbf{A. Heterogeneous computational architectures:} for users who wish to partition data on a node between different computing resources, they may create architecture-specific subvectors for each partition. For example, a user could create one serial component based on \texttt{NVECTOR\_SERIAL}, another component for GPU accelerators based on \texttt{NVECTOR\_CUDA}, and another threaded component based on \texttt{NVECTOR\_OPENMP}.

\textbf{B. Structure of arrays (SOA) data layouts:} for users who wish to create separate subvectors for each solution component, e.g., in a Navier-Stokes simulation they could have separate subvectors for density, velocities and pressure, which are combined together into a single \texttt{NVECTOR\_MANYVECTOR} for the overall “solution”.

We note that the above use cases are not mutually exclusive, and the \texttt{NVECTOR\_MANYVECTOR} implementation should support arbitrary combinations of these cases.

The \texttt{NVECTOR\_MANYVECTOR} implementation is designed to work with any \texttt{NVECTOR} subvectors that implement the minimum \texttt{required} set of operations. Additionally, \texttt{NVECTOR\_MANYVECTOR} sets no limit on the number of subvectors that may be attached (aside from the limitations of using \texttt{sunindex\_type} for indexing, and standard per-node memory limitations). However, while this ostensibly supports subvectors with one entry each (i.e., one subvector for each solution entry), we anticipate that this extreme situation will hinder performance due to non-stride-one memory accesses and increased function call overhead. We therefore recommend a relatively coarse partitioning of the problem, although actual performance will likely be problem-dependent.

As a final note, in the coming years we plan to introduce additional algebraic solvers and time integration modules that will leverage the problem partitioning enabled by \texttt{NVECTOR\_MANYVECTOR}. However, even at present we anticipate that users will be able to leverage such data partitioning in their problem-defining ODE right-hand side, DAE residual, or nonlinear solver residual functions.
8.13 The NVECTOR_MANYVECTOR implementation

8.13.1 NVECTOR_MANYVECTOR structure

The NVECTOR_MANYVECTOR implementation defines the content field of N_Vector to be a structure containing the number of subvectors comprising the ManyVector, the global length of the ManyVector (including all subvectors), a pointer to the beginning of the array of subvectors, and a boolean flag own_data indicating ownership of the subvectors that populate subvec_array.

```c
struct _N_VectorContent_ManyVector {
    sunindextype num_subvectors; /* number of vectors attached */
    sunindextype global_length; /* overall manyvector length */
    N_Vector* subvec_array; /* pointer to N_Vector array */
    booleantype own_data; /* flag indicating data ownership */
};
```

The header file to include when using this module is nvector_manyvector.h. The installed module library to link against is libsundials_nvecmanyvector.lib where .lib is typically .so for shared libraries and .a for static libraries.

8.13.2 NVECTOR_MANYVECTOR functions

The NVECTOR_MANYVECTOR module implements all vector operations listed in Tables 8.1.1, 8.1.2, 8.1.3, and 8.1.4, except for N_VGetArrayPointer, N_VSetArrayPointer, N_VScaleAddMultiVectorArray, and N_VLinearCombinationVectorArray. As such, this vector cannot be used with the SUNDIALS Fortran-77 interfaces, nor with the SUNDIALS direct solvers and preconditioners. Instead, the NVECTOR_MANYVECTOR module provides functions to access subvectors, whose data may in turn be accessed according to their NVECTOR implementations.

The names of vector operations are obtained from those in Tables 8.1.1, 8.1.2, 8.1.3, and 8.1.4 by appending the suffix _ManyVector (e.g. N_VDestroy_ManyVector). The module NVECTOR_MANYVECTOR provides the following additional user-callable routines:

**N_VNew_ManyVector**

**Prototype**

```c
N_Vector N_VNew_ManyVector(sunindextype num_subvectors,
                           N_Vector *vec_array);
```

**Description**

This function creates a ManyVector from a set of existing NVECTOR objects. This routine will copy all N_Vector pointers from the input vec_array, so the user may modify/free that pointer array after calling this function. However, this routine does not allocate any new subvectors, so the underlying NVECTOR objects themselves should not be destroyed before the ManyVector that contains them.

Upon successful completion, the new ManyVector is returned; otherwise this routine returns NULL (e.g., a memory allocation failure occurred).

Users of the Fortran 2003 interface to this function will first need to use the generic N_Vector utility functions N_VNewVectorArray, and N_VSetVecAtIndexVectorArray to create the N_Vector* argument. This is further explained in Chapter 7.1.3.5, and the functions are documented in Chapter 8.1.5.

**F2003 Name**

This function is callable as FN_VNew_ManyVector when using the Fortran 2003 interface module.

**N_VGetSubvector_ManyVector**

**Prototype**

```c
N_Vector N_VGetSubvector_ManyVector(N_Vector v, sunindextype vec_num);
```

**Description**

This function returns the vec_num subvector from the NVECTOR array.

**F2003 Name**

This function is callable as FN_VGetSubvector_ManyVector when using the Fortran 2003 interface module.
**N_VGetSubvectorArrayPointer_ManyVector**

**Prototype**

```c
realtype *N_VGetSubvectorArrayPointer_ManyVector(N_Vector v, sunindextype vec_num);
```

**Description**

This function returns the data array pointer for the vec_num subvector from the NVECTOR array.

If the input vec_num is invalid, or if the subvector does not support the N_VGetArrayPointer operation, then NULL is returned.

**F2003 Name**

This function is callable as `FN_VGetSubvectorArrayPointer_ManyVector` when using the Fortran 2003 interface module.

---

**N_VSetSubvectorArrayPointer_ManyVector**

**Prototype**

```c
int N_VSetSubvectorArrayPointer_ManyVector(realtype *v_data, N_Vector v, sunindextype vec_num);
```

**Description**

This function sets the data array pointer for the vec_num subvector from the NVECTOR array.

If the input vec_num is invalid, or if the subvector does not support the N_VSetArrayPointer operation, then this routine returns -1; otherwise it returns 0.

**F2003 Name**

This function is callable as `FN_VSetSubvectorArrayPointer_ManyVector` when using the Fortran 2003 interface module.

---

**N_VGetNumSubvectors_ManyVector**

**Prototype**

```c
sunindextype N_VGetNumSubvectors_ManyVector(N_Vector v);
```

**Description**

This function returns the overall number of subvectors in the ManyVector object.

**F2003 Name**

This function is callable as `FN_VGetNumSubvectors_ManyVector` when using the Fortran 2003 interface module.

By default all fused and vector array operations are disabled in the NVECTOR_MANYVECTOR module, except for `N_VWrmsNormVectorArray` and `N_VWrmsNormMaskVectorArray`, that are enabled by default. The following additional user-callable routines are provided to enable or disable fused and vector array operations for a specific vector. To ensure consistency across vectors it is recommended to first create a vector with `N_VNew_ManyVector`, enable/disable the desired operations for that vector with the functions below, and create any additional vectors from that vector using `N_VClone`. This guarantees that the new vectors will have the same operations enabled/disabled, since cloned vectors inherit those configuration options from the vector they are cloned from, while vectors created with `N_VNew_ManyVector` will have the default settings for the NVECTOR_MANYVECTOR module. We note that these routines **do not** call the corresponding routines on subvectors, so those should be set up as desired **before** attaching them to the ManyVector in `N_VNew_ManyVector`.

---

**N_VEnableFusedOps_ManyVector**

**Prototype**

```c
int N_VEnableFusedOps_ManyVector(N_Vector v, booleantype tf);
```

**Description**

This function enables (SUNTRUE) or disables (SUNFALSE) all fused and vector array operations in the ManyVector. The return value is 0 for success and -1 if the input vector or its ops structure are NULL.

**F2003 Name**

This function is callable as `FN_VEnableFusedOps_ManyVector` when using the Fortran 2003 interface module.
8.13 The NVECTOR_MANYVECTOR implementation

N_VEnableLinearCombination_ManyVector
Prototype int N_VEnableLinearCombination_ManyVector(N_Vector v, booleantype tf);
Description This function enables (SUNTRUE) or disables (SUNFALSE) the linear combination fused operation in the ManyVector. The return value is 0 for success and -1 if the input vector or its ops structure are NULL.
F2003 Name This function is callable as FN_VEnableLinearCombination_ManyVector when using the Fortran 2003 interface module.

N_VEnableScaleAddMulti_ManyVector
Prototype int N_VEnableScaleAddMulti_ManyVector(N_Vector v, booleantype tf);
Description This function enables (SUNTRUE) or disables (SUNFALSE) the scale and add a vector to multiple vectors fused operation in the ManyVector. The return value is 0 for success and -1 if the input vector or its ops structure are NULL.
F2003 Name This function is callable as FN_VEnableScaleAddMulti_ManyVector when using the Fortran 2003 interface module.

N_VEnableDotProdMulti_ManyVector
Prototype int N_VEnableDotProdMulti_ManyVector(N_Vector v, booleantype tf);
Description This function enables (SUNTRUE) or disables (SUNFALSE) the multiple dot products fused operation in the ManyVector. The return value is 0 for success and -1 if the input vector or its ops structure are NULL.
F2003 Name This function is callable as FN_VEnableDotProdMulti_ManyVector when using the Fortran 2003 interface module.

N_VEnableLinearSumVectorArray_ManyVector
Prototype int N_VEnableLinearSumVectorArray_ManyVector(N_Vector v, booleantype tf);
Description This function enables (SUNTRUE) or disables (SUNFALSE) the linear sum operation for vector arrays in the ManyVector. The return value is 0 for success and -1 if the input vector or its ops structure are NULL.
F2003 Name This function is callable as FN_VEnableLinearSumVectorArray_ManyVector when using the Fortran 2003 interface module.

N_VEnableScaleVectorArray_ManyVector
Prototype int N_VEnableScaleVectorArray_ManyVector(N_Vector v, booleantype tf);
Description This function enables (SUNTRUE) or disables (SUNFALSE) the scale operation for vector arrays in the ManyVector. The return value is 0 for success and -1 if the input vector or its ops structure are NULL.
F2003 Name This function is callable as FN_VEnableScaleVectorArray_ManyVector when using the Fortran 2003 interface module.

N_VEnableConstVectorArray_ManyVector
Prototype int N_VEnableConstVectorArray_ManyVector(N_Vector v, booleantype tf);
Description This function enables (SUNTRUE) or disables (SUNFALSE) the const operation for vector arrays in the ManyVector. The return value is 0 for success and -1 if the input vector or its ops structure are NULL.
Description of the NVECTOR module

F2003 Name This function is callable as `FN_VEnableConstVectorArray_ManyVector` when using the Fortran 2003 interface module.

```c
N_VEnableWrmsNormVectorArray_ManyVector
```

Prototype `int N_VEnableWrmsNormVectorArray_ManyVector(N_Vector v, booleantype tf);`

Description This function enables (SUNTRUE) or disables (SUNFALSE) the WRMS norm operation for vector arrays in the ManyVector. The return value is 0 for success and -1 if the input vector or its ops structure are NULL.

F2003 Name This function is callable as `FN_VEnableWrmsNormVectorArray_ManyVector` when using the Fortran 2003 interface module.

```c
N_VEnableWrmsNormMaskVectorArray_ManyVector
```

Prototype `int N_VEnableWrmsNormMaskVectorArray_ManyVector(N_Vector v, booleantype tf);`

Description This function enables (SUNTRUE) or disables (SUNFALSE) the masked WRMS norm operation for vector arrays in the ManyVector. The return value is 0 for success and -1 if the input vector or its ops structure are NULL.

F2003 Name This function is callable as `FN_VEnableWrmsNormMaskVectorArray_ManyVector` when using the Fortran 2003 interface module.

Notes

⚠️

- `N_New_ManyVector` sets the field `own_data = SUNFALSE`. `N_Destroy_ManyVector` will not attempt to call `N_Destroy` on any subvectors contained in the subvector array for any `N_Vector` with `own_data` set to SUNFALSE. In such a case, it is the user’s responsibility to deallocate the subvectors.

⚠️

- To maximize efficiency, arithmetic vector operations in the `NVECTOR_MANYVECTOR` implementation that have more than one `N_Vector` argument do not check for consistent internal representation of these vectors. It is the user’s responsibility to ensure that such routines are called with `N_Vector` arguments that were all created with the same subvector representations.

8.14 The NVECTOR_MPIMANYVECTOR implementation

The `NVECTOR_MPIMANYVECTOR` implementation of the `NVECTOR` module provided with SUNDIALS is designed to facilitate problems with an inherent data partitioning for the solution vector, and when using distributed-memory parallel architectures. As such, the MPIManyVector implementation supports all use cases allowed by the MPI-unaware ManyVector implementation, as well as partitioning data between nodes in a parallel environment. These data partitions are entirely user-defined, through construction of distinct `NVECTOR` modules for each component, that are then combined together to form the `NVECTOR_MPIMANYVECTOR`. We envision three generic use cases for this implementation:

A. Heterogeneous computational architectures (single-node or multi-node): for users who wish to partition data on a node between different computing resources, they may create architecture-specific subvectors for each partition. For example, a user could create one MPI-parallel component based on `NVECTOR_PARALLEL`, another single-node component for GPU accelerators based on `NVECTOR_CUDA`, and another threaded single-node component based on `NVECTOR_OPENMP`.

B. Process-based multiphysics decompositions (multi-node): for users who wish to combine separate simulations together, e.g., where one subvector resides on one subset of MPI processes, while another subvector resides on a different subset of MPI processes, and where the user has created a MPI intercommunicator to connect these distinct process sets together.
8.14 The NVECTOR_MPIMANYVECTOR implementation

C. Structure of arrays (SOA) data layouts (single-node or multi-node): for users who wish to create separate subvectors for each solution component, e.g., in a Navier-Stokes simulation they could have separate subvectors for density, velocities and pressure, which are combined together into a single NVECTOR_MPIMANYVECTOR for the overall “solution”.

We note that the above use cases are not mutually exclusive, and the NVECTOR_MPIMANYVECTOR implementation should support arbitrary combinations of these cases.

The NVECTOR_MPIMANYVECTOR implementation is designed to work with any NVECTOR subvectors that implement the minimum required set of operations, however significant performance benefits may be obtained when subvectors additionally implement the optional local reduction operations listed in Table 8.1.4.

Additionally, NVECTOR_MPIMANYVECTOR sets no limit on the number of subvectors that may be attached (aside from the limitations of using sunindextype for indexing, and standard per-node memory limitations). However, while this ostensibly supports subvectors with one entry each (i.e., one subvector for each solution entry), we anticipate that this extreme situation will hinder performance due to non-stride-one memory accesses and increased function call overhead. We therefore recommend a relatively coarse partitioning of the problem, although actual performance will likely be problem-dependent.

As a final note, in the coming years we plan to introduce additional algebraic solvers and time integration modules that will leverage the problem partitioning enabled by NVECTOR_MPIMANYVECTOR. However, even at present we anticipate that users will be able to leverage such data partitioning in their problem-defining ODE right-hand side, DAE residual, or nonlinear solver residual functions.

8.14.1 NVECTOR_MPIMANYVECTOR structure

The NVECTOR_MPIMANYVECTOR implementation defines the content field of N_Vector to be a structure containing the MPI communicator (or MPI_COMM_NULL if running on a single-node), the number of subvectors comprising the MPIManyVector, the global length of the MPIManyVector (including all subvectors on all MPI tasks), a pointer to the beginning of the array of subvectors, and a boolean flag own_data indicating ownership of the subvectors that populate subvec_array.

```c
struct _N_VectorContent_MPIManyVector {
    MPI_Comm comm; /* overall MPI communicator */
    sunindextype num_subvectors; /* number of vectors attached */
    sunindextype global_length; /* overall mpimanyvector length */
    N_Vector* subvec_array; /* pointer to N_Vector array */
    booleantype own_data; /* flag indicating data ownership */
};
```

The header file to include when using this module is nvector_mpimanyvector.h. The installed module library to link against is libsundials_nvecmpimanyvector.lib where .lib is typically .so for shared libraries and .a for static libraries.

**Note:** If SUNDIALS is configured with MPI disabled, then the MPIManyVector library will not be built. Furthermore, any user codes that include nvector_mpimanyvector.h must be compiled using an MPI-aware compiler (whether the specific user code utilizes MPI or not). We note that the NVECTOR_MPIMANYVECTOR implementation is designed for ManyVector use cases in an MPI-unaware environment.

8.14.2 NVECTOR_MPIMANYVECTOR functions

The NVECTOR_MPIMANYVECTOR module implements all vector operations listed in Tables 8.1.1, 8.1.2, 8.1.3, and 8.1.4, except for N_VGetArrayPointer, N_VSetArrayPointer, N_VScaleAddMultiVectorArray, and N_VLinearCombinationVectorArray. As such, this vector cannot be used with the SUNDIALS Fortran-77 interfaces, nor with the SUNDIALS direct solvers and preconditioners. Instead, the NVECTOR_MPIMANYVECTOR module provides functions to access subvectors, whose data may in turn be accessed according to their NVECTOR implementations.
The names of vector operations are obtained from those in Tables 8.1.1, 8.1.2, 8.1.3, and 8.1.4 by appending the suffix \texttt{MPIManyVector} (e.g. \texttt{N_VDestroy(MPIManyVector)}. The module \texttt{NVECTOR\_MPI\_MANY\_VECTOR} provides the following additional user-callable routines:

\textbf{N\_VNew\_MPI\_Many\_Vector}

\textbf{Prototype} \hspace{1cm} \texttt{N\_Vector N\_VNew\_MPI\_Many\_Vector(sunindextype num\_subvectors, N\_Vector \*vec\_array);} \\
\textbf{Description} \hspace{1cm} This function creates an MPIManyVector from a set of existing \texttt{NVECTOR} objects, under the requirement that all MPI-aware subvectors use the same MPI communicator (this is checked internally). If none of the subvectors are MPI-aware, then this may equivalently be used to describe data partitioning within a single node. We note that this routine is designed to support use cases A and C above.

This routine will copy all \texttt{N\_Vector} pointers from the input \texttt{vec\_array}, so the user may modify/free that pointer array after calling this function. However, this routine does \textit{not} allocate any new subvectors, so the underlying \texttt{NVECTOR} objects themselves should not be destroyed before the MPIManyVector that contains them.

Upon successful completion, the new MPIManyVector is returned; otherwise this routine returns \texttt{NULL} (e.g., if two MPI-aware subvectors use different MPI communicators).

Users of the Fortran 2003 interface to this function will first need to use the generic \texttt{N\_Vector} utility functions \texttt{N\_VNew\_Vector\_Array}, and \texttt{N\_VSet\_Vec\_At\_Index\_Vector\_Array} to create the \texttt{N\_Vector\*} argument. This is further explained in Chapter 7.1.3.5, and the functions are documented in Chapter 8.1.5.

\textbf{F2003 Name} \hspace{1cm} This function is callable as \texttt{FN\_VNew\_MPI\_Many\_Vector} when using the Fortran 2003 interface module.

\textbf{N\_VMake\_MPI\_Many\_Vector}

\textbf{Prototype} \hspace{1cm} \texttt{N\_Vector N\_VMake\_MPI\_Many\_Vector(MPI\_Comm comm, sunindextype num\_subvectors, N\_Vector \*vec\_array);} \\
\textbf{Description} \hspace{1cm} This function creates an MPIManyVector from a set of existing \texttt{NVECTOR} objects, and a user-created MPI communicator that “connects” these subvectors. Any MPI-aware subvectors may use different MPI communicators than the input \texttt{comm}. We note that this routine is designed to support any combination of the use cases above.

The input \texttt{comm} should be this user-created MPI communicator. This routine will internally call \texttt{MPI\_Comm\_dup} to create a copy of the input \texttt{comm}, so the user-supplied \texttt{comm} argument need not be retained after the call to \texttt{N\_VMake\_MPI\_Many\_Vector}.

If all subvectors are MPI-unaware, then the input \texttt{comm} argument should be \texttt{MPI\_COMM\_NULL}, although in this case, it would be simpler to call \texttt{N\_VNew\_MPI\_Many\_Vector} instead, or to just use the \texttt{NVECTOR\_MANY\_VECTOR} module.

This routine will copy all \texttt{N\_Vector} pointers from the input \texttt{vec\_array}, so the user may modify/free that pointer array after calling this function. However, this routine does \textit{not} allocate any new subvectors, so the underlying \texttt{NVECTOR} objects themselves should not be destroyed before the MPIManyVector that contains them.

Upon successful completion, the new MPIManyVector is returned; otherwise this routine returns \texttt{NULL} (e.g., if the input \texttt{vec\_array} is \texttt{NULL}).

\textbf{F2003 Name} \hspace{1cm} This function is callable as \texttt{FN\_VMake\_MPI\_Many\_Vector} when using the Fortran 2003 interface module.
8.14 The NVECTOR_MPIMANYVECTOR implementation

N_SendSubvector_MPIManyVector
Prototype N_Vecor N_SendSubvector_MPIManyVector(N_Vecor v, sunindextype vec_num);
Description This function returns the vec_num subvector from the NVECTOR array.
F2003 Name This function is callable as FN_SendSubvector_MPIManyVector when using the Fortran 2003 interface module.

N_SendSubvectorArrayPointer_MPIManyVector
Prototype retaype *N_SendSubvectorArrayPointer_MPIManyVector(N_Vecor v, sunindextype vec_num);
Description This function returns the data array pointer for the vec_num subvector from the NVECTOR array.
If the input vec_num is invalid, or if the subvector does not support the N_VGetArrayPointer operation, then NULL is returned.
F2003 Name This function is callable as FN_SendSubvectorArrayPointer_MPIManyVector when using the Fortran 2003 interface module.

N_SendSubvectorArrayPointer_MPIManyVector
Prototype int N_SendSubvectorArrayPointer_MPIManyVector(retaype *v_data, N_Vecor v, sunindextype vec_num);
Description This function sets the data array pointer for the vec_num subvector from the NVECTOR array.
If the input vec_num is invalid, or if the subvector does not support the N_VSetArrayPointer operation, then this routine returns -1; otherwise it returns 0.
F2003 Name This function is callable as FN_SendSubvectorArrayPointer_MPIManyVector when using the Fortran 2003 interface module.

N_VGetNumSubvectors_MPIManyVector
Prototype sunindextype N_VGetNumSubvectors_MPIManyVector(N_Vecor v);
Description This function returns the overall number of subvectors in the MPIManyVector object.
F2003 Name This function is callable as FN_VGetNumSubvectors_MPIManyVector when using the Fortran 2003 interface module.

By default all fused and vector array operations are disabled in the NVECTOR_MPIMANYVECTOR module, except for N_VWrmsNormVectorArray and N_VWrmsNormMaskVectorArray, that are enabled by default. The following additional user-callable routines are provided to enable or disable fused and vector array operations for a specific vector. To ensure consistency across vectors it is recommended to first create a vector with N_VNew_MPIManyVector or N_VMake_MPIManyVector, enable/disable the desired operations for that vector with the functions below, and create any additional vectors from that vector using N_VClone. This guarantees that the new vectors will have the same operations enabled/disabled, since cloned vectors inherit those configuration options from the vector they are cloned from, while vectors created with N_VNew_MPIManyVector and N_VMake_MPIManyVector will have the default settings for the NVECTOR_MPIMANYVECTOR module. We note that these routines do not call the corresponding routines on subvectors, so those should be set up as desired before attaching them to the MPIManyVector in N_VNew_MPIManyVector or N_VMake_MPIManyVector.
### Description of the NVECTOR module

#### N_VEnableFusedOps_MPIManyVector

**Prototype**

```c
int N_VEnableFusedOps_MPIManyVector(N_Vector v, booleantype tf);
```

**Description**

This function enables (SUNTRUE) or disables (SUNFALSE) all fused and vector array operations in the MPIManyVector. The return value is 0 for success and -1 if the input vector or its ops structure are NULL.

**F2003 Name**

This function is callable as `FN_VEnableFusedOps_MPIManyVector` when using the Fortran 2003 interface module.

#### N_VEnableLinearCombination_MPIManyVector

**Prototype**

```c
int N_VEnableLinearCombination_MPIManyVector(N_Vector v, booleantype tf);
```

**Description**

This function enables (SUNTRUE) or disables (SUNFALSE) the linear combination fused operation in the MPIManyVector. The return value is 0 for success and -1 if the input vector or its ops structure are NULL.

**F2003 Name**

This function is callable as `FN_VEnableLinearCombination_MPIManyVector` when using the Fortran 2003 interface module.

#### N_VEnableScaleAddMulti_MPIManyVector

**Prototype**

```c
int N_VEnableScaleAddMulti_MPIManyVector(N_Vector v, booleantype tf);
```

**Description**

This function enables (SUNTRUE) or disables (SUNFALSE) the scale and add a vector to multiple vectors fused operation in the MPIManyVector. The return value is 0 for success and -1 if the input vector or its ops structure are NULL.

**F2003 Name**

This function is callable as `FN_VEnableScaleAddMulti_MPIManyVector` when using the Fortran 2003 interface module.

#### N_VEnableDotProdMulti_MPIManyVector

**Prototype**

```c
int N_VEnableDotProdMulti_MPIManyVector(N_Vector v, booleantype tf);
```

**Description**

This function enables (SUNTRUE) or disables (SUNFALSE) the multiple dot products fused operation in the MPIManyVector. The return value is 0 for success and -1 if the input vector or its ops structure are NULL.

**F2003 Name**

This function is callable as `FN_VEnableDotProdMulti_MPIManyVector` when using the Fortran 2003 interface module.

#### N_VEnableLinearSumVectorArray_MPIManyVector

**Prototype**

```c
int N_VEnableLinearSumVectorArray_MPIManyVector(N_Vector v, booleantype tf);
```

**Description**

This function enables (SUNTRUE) or disables (SUNFALSE) the linear sum operation for vector arrays in the MPIManyVector. The return value is 0 for success and -1 if the input vector or its ops structure are NULL.

**F2003 Name**

This function is callable as `FN_VEnableLinearSumVectorArray_MPIManyVector` when using the Fortran 2003 interface module.

#### N_VEnableScaleVectorArray_MPIManyVector

**Prototype**

```c
int N_VEnableScaleVectorArray_MPIManyVector(N_Vector v, booleantype tf);
```

**Description**

This function enables (SUNTRUE) or disables (SUNFALSE) the scale operation for vector arrays in the MPIManyVector. The return value is 0 for success and -1 if the input vector or its ops structure are NULL.
8.15 The NVECTOR_MPIPLUSX implementation

F2003 Name This function is callable as FNV_EnableScaleVectorArray_MPIManyVector when using the Fortran 2003 interface module.

Prototype  int N_VEnableConstVectorArray_MPIManyVector(N_Vector v, booleantype tf);

Description  This function enables (SUNTRUE) or disables (SUNFALSE) the const operation for vector arrays in the MPIManyVector. The return value is 0 for success and -1 if the input vector or its ops structure are NULL.

F2003 Name This function is callable as FNV_EnableConstVectorArray_MPIManyVector when using the Fortran 2003 interface module.

Prototype  int N_VEnableWrmsNormVectorArray_MPIManyVector(N_Vector v, booleantype tf);

Description  This function enables (SUNTRUE) or disables (SUNFALSE) the WRMS norm operation for vector arrays in the MPIManyVector. The return value is 0 for success and -1 if the input vector or its ops structure are NULL.

F2003 Name This function is callable as FNV_EnableWrmsNormVectorArray_MPIManyVector when using the Fortran 2003 interface module.

Prototype  int N_VEnableWrmsNormMaskVectorArray_MPIManyVector(N_Vector v, booleantype tf);

Description  This function enables (SUNTRUE) or disables (SUNFALSE) the masked WRMS norm operation for vector arrays in the MPIManyVector. The return value is 0 for success and -1 if the input vector or its ops structure are NULL.

F2003 Name This function is callable as FNV_EnableWrmsNormMaskVectorArray_MPIManyVector when using the Fortran 2003 interface module.

Notes

- N_New_MPIManyVector and N_Make_MPIManyVector set the field own_data = SUNFALSE. N_Destroy_MPIManyVector will not attempt to call N_Destroy on any subvectors contained in the subvector array for any N_Vector with own_data set to SUNFALSE. In such a case, it is the user’s responsibility to deallocate the subvectors.

- To maximize efficiency, arithmetic vector operations in the NVECTOR_MPI MANYVECTOR implementation that have more than one N_Vector argument do not check for consistent internal representation of these vectors. It is the user’s responsibility to ensure that such routines are called with N_Vector arguments that were all created with the same subvector representations.

8.15 The NVECTOR_MPIPLUSX implementation

The NVECTOR_MPIPLUSX implementation of the NVECTOR module provided with SUNDIALS is designed to facilitate the MPI+X paradigm, where X is some form of on-node (local) parallelism (e.g. OpenMP, CUDA). This paradigm is becoming increasingly popular with the rise of heterogeneous computing architectures.

The NVECTOR_MPIPLUSX implementation is designed to work with any NVECTOR that implements the minimum required set of operations. However, it is not recommended to use the NVECTOR_PARALLEL, NVECTOR_PARHYP, NVECTOR_PETSC, or NVECTOR_TRILINOS implementations underneath the NVECTOR_MPIPLUSX module since they already provide MPI capabilities.
8.15.1 NVECTOR_MPIPLUSX structure

The NVECTOR_MPIPLUSX implementation is a thin wrapper around the NVECTOR_MPMANYVECTOR. Accordingly, it adopts the same content structure as defined in Section 8.14.1.

The header file to include when using this module is nvector_mpiplusx.h. The installed module library to link against is lib sundials_nvecmpiplusx. The installed module library to link against is lib sundials_nvecmpiplusx.lib where .lib is typically .so for shared libraries and .a for static libraries.

**Note:** If SUNDIALS is configured with MPI disabled, then the mpiplusx library will not be built. Furthermore, any user codes that include nvector_mpiplusx.h must be compiled using an MPI-aware compiler.

8.15.2 NVECTOR_MPIPLUSX functions

The NVECTOR_MPIPLUSX module adopts all vector operations listed in Tables 8.1.1, 8.1.2, 8.1.3, and 8.1.4, from the NVECTOR_MPMANYVECTOR (see section 8.14.2) except for N_VGetArrayPointer and N_VSetArrayPointer; the module provides its own implementation of these functions that call the local vector implementations. Therefore, the NVECTOR_MPIPLUSX module implements all of the operations listed in the referenced sections except for N_VScaleAddMultiVectorArray, and N_VLinearCombinationVectorArray.

Accordingly, it’s compatibility with the SUNDIALS Fortran-77 interface, and with the SUNDIALS direct solvers and preconditioners depends on the local vector implementation.

The module NVECTOR_MPIPLUSX provides the following additional user-callable routines:

**N_VMake_MPIPlusX**

Prototype:  
N_Vector N_VMake_MPIPlusX(MPI_Comm comm,  
N_Vector *local_vector);

Description: This function creates an MPIPlusX vector from an existing local (i.e. on-node) NVECTOR object, and a user-created MPI communicator.

The input comm should be this user-created MPI communicator. This routine will internally call MPI_Comm_dup to create a copy of the input comm, so the user-supplied comm argument need not be retained after the call to N_VMake_MPIPlusX.

This routine will copy the N_Vector pointer to the input local_vector, so the underlying local nvector object should not be destroyed before the mpiplusx that contains it.

Upon successful completion, the new MPIPlusX is returned; otherwise this routine returns NULL (e.g., if the input local_vector is NULL).

F2003 Name: This function is callable as FN_VMake_MPIPlusX when using the Fortran 2003 interface module.

**N_VGetLocalVector_MPIPlusX**

Prototype:  
N_Vector N_VGetLocalVector_MPIPlusX(N_Vector v);

Description: This function returns the local vector underneath the the MPIPlusX NVECTOR.

F2003 Name: This function is callable as FN_VGetLocalVector_MPIPlusX when using the Fortran 2003 interface module.

**N_VGetArrayPointer_MPIPlusX**

Prototype:  
realtype* N_VGetArrayPointer_MPIPlusX(N_Vector v);

Description: This function returns the data array pointer for the local vector if the local vector implements the N_VGetArrayPointer operation; otherwise it returns NULL.

F2003 Name: This function is callable as FN_VGetArrayPointer_MPIPlusX when using the Fortran 2003 interface module.
8.16 NVECTOR Examples

**N_VSetArrayPointer_MPIPlusX**

**Prototype**

```c
void N_VSetArrayPointer_MPIPlusX(realtype *data, N_Vector v);
```

**Description**

This function sets the data array pointer for the local vector if the local vector implements the `N_VSetArrayPointer` operation.

**F2003 Name**

This function is callable as `FN_VSetArrayPointer_MPIPlusX` when using the Fortran 2003 interface module.

The `N_VECTOR_MPIPLUSX` module does not implement any fused or vector array operations. Instead users should enable/disable fused operations on the local vector.

**Notes**

- `N_VMake_MPIPlusX` sets the field `own_data = SUNFALSE`. and `N_VDestroy_MPIPlusX` will not call `N_VDestroy` on the local vector. In this case, it is the user’s responsibility to deallocate the local vector.
- To maximize efficiency, arithmetic vector operations in the `NVECTOR_MPIPLUSX` implementation that have more than one `N_Vector` argument do not check for consistent internal representation of these vectors. It is the user’s responsibility to ensure that such routines are called with `N_Vector` arguments that were all created with the same local vector representations.

### 8.16 NVECTOR Examples

There are `NVector` examples that may be installed for the implementations provided with **SUNDIALS**. Each implementation makes use of the functions in `test_nvector.c`. These example functions show simple usage of the `NVector` family of functions. The input to the examples are the vector length, number of threads (if threaded implementation), and a print timing flag.

The following is a list of the example functions in `test_nvector.c`:

- **Test_N_VClone**: Creates clone of vector and checks validity of clone.
- **Test_N_VCloneEmpty**: Creates clone of empty vector and checks validity of clone.
- **Test_N_VCloneVectorArray**: Creates clone of vector array and checks validity of cloned array.
- **Test_N_VCloneVectorArray**: Creates clone of empty vector array and checks validity of cloned array.
- **Test_N_VGetArrayPointer**: Get array pointer.
- **Test_N_VSetArrayPointer**: Allocate new vector, set pointer to new vector array, and check values.
- **Test_N_VGetLength**: Compares self-reported length to calculated length.
- **Test_N_VGetCommunicator**: Compares self-reported communicator to the one used in constructor; or for MPI-unaware vectors it ensures that NULL is reported.
- **Test_N_VLinearSum** Case 1a: Test \( y = x + y \)
- **Test_N_VLinearSum** Case 1b: Test \( y = -x + y \)
- **Test_N_VLinearSum** Case 1c: Test \( y = ax + y \)
- **Test_N_VLinearSum** Case 2a: Test \( x = x + y \)
- **Test_N_VLinearSum** Case 2b: Test \( x = x - y \)
- **Test_N_VLinearSum** Case 2c: Test \( x = x + by \)
Description of the NVECTOR module

- Test_N_VLinearSum Case 3: Test $z = x + y$
- Test_N_VLinearSum Case 4a: Test $z = x - y$
- Test_N_VLinearSum Case 4b: Test $z = -x + y$
- Test_N_VLinearSum Case 5a: Test $z = x + by$
- Test_N_VLinearSum Case 5b: Test $z = ax + y$
- Test_N_VLinearSum Case 6a: Test $z = -x + by$
- Test_N_VLinearSum Case 6b: Test $z = ax - y$
- Test_N_VLinearSum Case 7: Test $z = a(x + y)$
- Test_N_VLinearSum Case 8: Test $z = a(x - y)$
- Test_N_VLinearSum Case 9: Test $z = ax + by$
- Test_N_VConst: Fill vector with constant and check result.
- Test_N_VProd: Test vector multiply: $z = x \times y$
- Test_N_VDiv: Test vector division: $z = x / y$
- Test_N_VScale: Case 1: scale: $x = cx$
- Test_N_VScale: Case 2: copy: $z = x$
- Test_N_VScale: Case 3: negate: $z = -x$
- Test_N_VScale: Case 4: combination: $z = cx$
- Test_N_VAbs: Create absolute value of vector.
- Test_N_VAddConst: add constant vector: $z = c + x$
- Test_N_VDotProd: Calculate dot product of two vectors.
- Test_N_VMaxNorm: Create vector with known values, find and validate the max norm.
- Test_N_VWrmsNorm: Create vector of known values, find and validate the weighted root mean square.
- Test_N_VWrmsNormMask: Create vector of known values, find and validate the weighted root mean square using all elements except one.
- Test_N_VMin: Create vector, find and validate the min.
- Test_N_VL2Norm: Create vector, find and validate the weighted Euclidean L2 norm.
- Test_N_VL1Norm: Create vector, find and validate the L1 norm.
- Test_N_VCompare: Compare vector with constant returning and validating comparison vector.
- Test_N_VInvTest: Test $z[i] = 1 / x[i]$
- Test_N_VConstrMask: Test mask of vector $x$ with vector $c$.
- Test_N_VMinQuotient: Fill two vectors with known values. Calculate and validate minimum quotient.
- Test_N_VLinearCombination Case 1a: Test $x = a x$
• Test_N_VLinearCombination Case 1b: Test $z = a \times$

• Test_N_VLinearCombination Case 2a: Test $x = a \times b \times$

• Test_N_VLinearCombination Case 2b: Test $z = a \times b \times$

• Test_N_VLinearCombination Case 3a: Test $x = x + a \times b \times$

• Test_N_VLinearCombination Case 3b: Test $x = a \times b \times c \times$

• Test_N_VLinearCombination Case 3c: Test $w = a \times b \times c \times$

• Test_N_VScaleAddMulti Case 1a: $y = a \times + y$

• Test_N_VScaleAddMulti Case 1b: $z = a \times + y$

• Test_N_VScaleAddMulti Case 2a: $Y[i] = c[i] \times + Y[i]$, $i = 1, 2, 3$

• Test_N_VScaleAddMulti Case 2b: $Z[i] = c[i] \times + Y[i]$, $i = 1, 2, 3$

• Test_N_VDotProdMulti Case 1: Calculate the dot product of two vectors

• Test_N_VDotProdMulti Case 2: Calculate the dot product of one vector with three other vectors in a vector array.

• Test_N_VLinearSumVectorArray Case 1: $z = a \times + b \times$

• Test_N_VLinearSumVectorArray Case 2a: $Z[i] = a \times X[i] + b \times Y[i]$

• Test_N_VLinearSumVectorArray Case 2b: $X[i] = a \times X[i] + b \times Y[i]$

• Test_N_VLinearSumVectorArray Case 2c: $Y[i] = a \times X[i] + b \times Y[i]$

• Test_N_VScaleVectorArray Case 1a: $y = c \times$

• Test_N_VScaleVectorArray Case 1b: $z = c \times$

• Test_N_VScaleVectorArray Case 2a: $Y[i] = c[i] \times Y[i]$

• Test_N_VScaleVectorArray Case 2b: $Z[i] = c[i] \times Y[i]$

• Test_N_VScaleVectorArray Case 1a: $z = c$

• Test_N_VScaleVectorArray Case 1b: $Z[i] = c$

• Test_N_VWrmsNormVectorArray Case 1a: Create a vector of know values, find and validate the weighted root mean square norm.

• Test_N_VWrmsNormVectorArray Case 1b: Create a vector array of three vectors of know values, find and validate the weighted root mean square norm of each.

• Test_N_VWrmsNormMaskVectorArray Case 1a: Create a vector of know values, find and validate the weighted root mean square norm using all elements except one.

• Test_N_VWrmsNormMaskVectorArray Case 1b: Create a vector array of three vectors of know values, find and validate the weighted root mean square norm of each using all elements except one.

• Test_N_VScaleAddMultiVectorArray Case 1a: $y = a \times + y$

• Test_N_VScaleAddMultiVectorArray Case 1b: $z = a \times + y$

• Test_N_VScaleAddMultiVectorArray Case 2a: $Y[j][0] = a[j] \times X[0] + Y[j][0]$
- Test_N_VScaleAddMultiVectorArray Case 2b: $Z[j][0] = a[j] \times X[0] + Y[j][0]$
- Test_N_VScaleAddMultiVectorArray Case 3a: $Y[0][i] = a[0] \times X[i] + Y[0][i]$
- Test_N_VScaleAddMultiVectorArray Case 3b: $Z[0][i] = a[0] \times X[i] + Y[0][i]$
- Test_N_VScaleAddMultiVectorArray Case 4a: $Y[j][i] = a[j] \times X[i] + Y[j][i]$
- Test_N_VScaleAddMultiVectorArray Case 4b: $Z[j][i] = a[j] \times X[i] + Y[j][i]$
- Test_N_VLinearCombinationVectorArray Case 1a: $x = a \times x$
- Test_N_VLinearCombinationVectorArray Case 1b: $z = a \times x$
- Test_N_VLinearCombinationVectorArray Case 2a: $x = a \times x + b \times y$
- Test_N_VLinearCombinationVectorArray Case 2b: $z = a \times x + b \times y$
- Test_N_VLinearCombinationVectorArray Case 3a: $x = a \times x + b \times y + c \times z$
- Test_N_VLinearCombinationVectorArray Case 3b: $w = a \times x + b \times y + c \times z$
- Test_N_VLinearCombinationVectorArray Case 4a: $X[0][i] = c[0] \times X[0][i]$
- Test_N_VLinearCombinationVectorArray Case 4b: $Z[i] = c[0] \times X[0][i]$
- Test_N_VLinearCombinationVectorArray Case 5a: $X[0][i] = c[0] \times X[0][i] + c[1] \times X[1][i]$
- Test_N_VLinearCombinationVectorArray Case 5b: $Z[i] = c[0] \times X[0][i] + c[1] \times X[1][i]$
- Test_N_VLinearCombinationVectorArray Case 6a: $X[0][i] = X[0][i] + c[1] \times X[1][i] + c[2] \times X[2][i]$
- Test_N_VLinearCombinationVectorArray Case 6b: $X[0][i] = c[0] \times X[0][i] + c[1] \times X[1][i] + c[2] \times X[2][i]$
- Test_N_VLinearCombinationVectorArray Case 6c: $Z[i] = c[0] \times X[0][i] + c[1] \times X[1][i] + c[2] \times X[2][i]$
- Test_N_VDotProdLocal: Calculate MPI task-local portion of the dot product of two vectors.
- Test_N_VMaxNormLocal: Create vector with known values, find and validate the MPI task-local portion of the max norm.
- Test_N_VMinLocal: Create vector, find and validate the MPI task-local min.
- Test_N_VL1NormLocal: Create vector, find and validate the MPI task-local portion of the L1 norm.
- Test_N_VWSqrSumLocal: Create vector of known values, find and validate the MPI task-local portion of the weighted squared sum of two vectors.
- Test_N_VWSqrSumMaskLocal: Create vector of known values, find and validate the MPI task-local portion of the weighted squared sum of two vectors, using all elements except one.
- Test_N_VInvTestLocal: Test the MPI task-local portion of $z[i] = 1 / x[i]$
- Test_N_VConstrMaskLocal: Test the MPI task-local portion of the mask of vector $x$ with vector $c$.
- Test_N_VMinQuotientLocal: Fill two vectors with known values. Calculate and validate the MPI task-local minimum quotient.
Table 8.2: List of vector functions usage by IDAS code modules

<table>
<thead>
<tr>
<th>Function</th>
<th>IDAS</th>
<th>IDALS</th>
<th>IDABDPR</th>
<th>IDAA</th>
</tr>
</thead>
<tbody>
<tr>
<td>N_VGetVectorID</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N_VGetLength</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N_VClone</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>N_VCloneEmpty</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N_VDestroy</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>N_VCloneVectorArray</td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N_VDestroyVectorArray</td>
<td>✓</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>N_VSpace</td>
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<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>N_VGetArrayPointer</td>
<td>1</td>
<td>✓</td>
<td></td>
<td></td>
</tr>
<tr>
<td>N_VSetArrayPointer</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N_VLinearSum</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>N_VConst</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>N_VProd</td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N_VDiv</td>
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<td></td>
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<td></td>
</tr>
<tr>
<td>N_VScale</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>N_VAbs</td>
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<td></td>
</tr>
<tr>
<td>N_VAbs</td>
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<td></td>
</tr>
<tr>
<td>N_VInv</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>N_VAddConst</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>N_VMaxNorm</td>
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<td></td>
</tr>
<tr>
<td>N_VWRmsNorm</td>
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<td>✓</td>
<td></td>
<td></td>
</tr>
<tr>
<td>N_VMin</td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N_VMinQuotient</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N_VConstrMask</td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N_VWRmsNormMask</td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N_VCompare</td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N_VLinearCombination</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N_VScaleAddMulti</td>
<td>✓</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>N_VDotProdMulti</td>
<td>3</td>
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<td></td>
</tr>
<tr>
<td>N_VLinearSumVectorArray</td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N_VScaleVectorArray</td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N_VConstVectorArray</td>
<td>✓</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>N_VWRmsNormVectorArray</td>
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<tr>
<td>N_VWRmsNormMaskVectorArray</td>
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<td></td>
</tr>
<tr>
<td>N_VScaleAddMultiVectorArray</td>
<td>✓</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>N_VLinearCombinationVectorArray</td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Chapter 9

Description of the SUNMatrix module

For problems that involve direct methods for solving linear systems, the SUNDIALS solvers not only operate on generic vectors, but also on generic matrices (of type SUNMatrix), through a set of operations defined by the particular SUNMATRIX implementation. Users can provide their own specific implementation of the SUNMATRIX module, particularly in cases where they provide their own NVECTOR and/or linear solver modules, and require matrices that are compatible with those implementations. Alternately, we provide three SUNMATRIX implementations: dense, banded, and sparse. The generic operations are described below, and descriptions of the implementations provided with SUNDIALS follow.

9.1 The SUNMatrix API

The SUNMATRIX API can be grouped into two sets of functions: the core matrix operations, and utility functions. Section 9.1.1 lists the core operations, while Section 9.1.2 lists the utility functions.

9.1.1 SUNMatrix core functions

The generic SUNMatrix object defines the following set of core operations:

- **SUNMatGetID**
  
  Call: `id = SUNMatGetID(A);`
  
  Description: Returns the type identifier for the matrix `A`. It is used to determine the matrix implementation type (e.g. dense, banded, sparse,...) from the abstract SUNMatrix interface. This is used to assess compatibility with SUNDIALS-provided linear solver implementations.
  
  Arguments: `A` (SUNMatrix) a SUNMATRIX object
  
  Return value: A SUNMATRIX_ID, possible values are given in the Table 9.2.
  
  F2003 Name: FSUNMatGetID

- **SUNMatClone**
  
  Call: `B = SUNMatClone(A);`
  
  Description: Creates a new SUNMatrix of the same type as an existing matrix `A` and sets the ops field. It does not copy the matrix, but rather allocates storage for the new matrix.
  
  Arguments: `A` (SUNMatrix) a SUNMATRIX object
Return value SUNMatrix
F2003 Name FSUNMatClone
F2003 Call type(SUNMatrix), pointer :: B
B => FSUNMatClone(A)

SUNMatDestroy
Call SUNMatDestroy(A);
Description Destroys A and frees memory allocated for its internal data.
Arguments A (SUNMatrix) a SUNMATRIX object
Return value None
F2003 Name FSUNMatDestroy

SUNMatSpace
Call ier = SUNMatSpace(A, &lrw, &liw);
Description Returns the storage requirements for the matrix A. lrw is a long int containing the
number of realtype words and liw is a long int containing the number of integer words.
Arguments A (SUNMatrix) a SUNMATRIX object
lrw (sunindextype*) the number of realtype words
liw (sunindextype*) the number of integer words
Return value None
Notes This function is advisory only, for use in determining a user’s total space requirements;
it could be a dummy function in a user-supplied SUNMATRIX module if that information
is not of interest.
F2003 Name FSUNMatSpace
F2003 Call integer(c_long) :: lrw(1), liw(1)
ier = FSUNMatSpace(A, lrw, liw)

SUNMatZero
Call ier = SUNMatZero(A);
Description Performs the operation $A_{ij} = 0$ for all entries of the matrix A.
Arguments A (SUNMatrix) a SUNMATRIX object
Return value A SUNMATRIX return code of type int denoting success/failure
F2003 Name FSUNMatZero

SUNMatCopy
Call ier = SUNMatCopy(A,B);
Description Performs the operation $B_{ij} = A_{i,j}$ for all entries of the matrices A and B.
Arguments A (SUNMatrix) a SUNMATRIX object
B (SUNMatrix) a SUNMATRIX object
Return value A SUNMATRIX return code of type int denoting success/failure
F2003 Name FSUNMatCopy
9.1 The SUNMatrix API

**SUNMatScaleAdd**

Call: `ier = SUNMatScaleAdd(c, A, B);`

Description: Performs the operation \( A = cA + B \).

Arguments:
- `c` (realtype) constant that scales \( A \)
- `A` (SUNMatrix) a SUNMATRIX object
- `B` (SUNMatrix) a SUNMATRIX object

Return value: A SUNMATRIX return code of type `int` denoting success/failure

F2003 Name: FSUNMatScaleAdd

**SUNMatScaleAddI**

Call: `ier = SUNMatScaleAddI(c, A);`

Description: Performs the operation \( A = cA + I \).

Arguments:
- `c` (realtype) constant that scales \( A \)
- `A` (SUNMatrix) a SUNMATRIX object

Return value: A SUNMATRIX return code of type `int` denoting success/failure

F2003 Name: FSUNMatScaleAddI

**SUNMatMatvecSetup**

Call: `ier = SUNMatMatvecSetup(A);`

Description: Performs any setup necessary to perform a matrix-vector product. It is useful for SUNMatrix implementations which need to prepare the matrix itself, or communication structures before performing the matrix-vector product.

Arguments:
- `A` (SUNMatrix) a SUNMATRIX object

Return value: A SUNMATRIX return code of type `int` denoting success/failure

F2003 Name: FSUNMatMatvecSetup

**SUNMatMatvec**

Call: `ier = SUNMatMatvec(A, x, y);`

Description: Performs the matrix-vector product operation, \( y = Ax \). It should only be called with vectors \( x \) and \( y \) that are compatible with the matrix \( A \) – both in storage type and dimensions.

Arguments:
- `A` (SUNMatrix) a SUNMATRIX object
- `x` (N_Vector) a NVECTOR object
- `y` (N_Vector) an output NVECTOR object

Return value: A SUNMATRIX return code of type `int` denoting success/failure

F2003 Name: FSUNMatMatvec

9.1.2 SUNMatrix utility functions

To aid in the creation of custom SUNMATRIX modules the generic SUNMATRIX module provides two utility functions `SUNMatNewEmpty` and `SUNMatVCopyOps`. 
Description of the SUNMatrix module

**SUNMatNewEmpty**
Call

```
A = SUNMatNewEmpty();
```

**Description**
The function `SUNMatNewEmpty` allocates a new generic SUNMatrix object and initializes its content pointer and the function pointers in the operations structure to NULL.

**Arguments**
None

**Return value**
This function returns a SUNMatrix object. If an error occurs when allocating the object, then this routine will return NULL.

F2003 Name FSUNMatNewEmpty

**SUNMatFreeEmpty**
Call

```
SUNMatFreeEmpty(A);
```

**Description**
This routine frees the generic SUNMatrix object, under the assumption that any implementation-specific data that was allocated within the underlying content structure has already been freed. It will additionally test whether the ops pointer is NULL, and, if it is not, it will free it as well.

**Arguments**
A (SUNMatrix) a SUNMatrix object

**Return value**
None

F2003 Name FSUNMatFreeEmpty

**SUNMatCopyOps**
Call

```
retval = SUNMatCopyOps(A, B);
```

**Description**
The function `SUNMatCopyOps` copies the function pointers in the ops structure of A into the ops structure of B.

**Arguments**
A (SUNMatrix) the matrix to copy operations from
B (SUNMatrix) the matrix to copy operations to

**Return value**
This returns 0 if successful and a non-zero value if either of the inputs are NULL or the ops structure of either input is NULL.

F2003 Name FSUNMatCopyOps

### 9.1.3 SUNMatrix return codes

The functions provided to SUNMATRIX modules within the SUNDIALS-provided SUNMATRIX implementations utilize a common set of return codes, shown in Table 9.1. These adhere to a common pattern: 0 indicates success, and a negative value indicates a failure. The actual values of each return code are primarily to provide additional information to the user in case of a failure.

<table>
<thead>
<tr>
<th>Name</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>SUNMAT_SUCCESS</td>
<td>0</td>
<td>successful call or converged solve</td>
</tr>
</tbody>
</table>

*continued on next page*
Table 9.2: Identifiers associated with matrix kernels supplied with Sundials.

<table>
<thead>
<tr>
<th>Matrix ID</th>
<th>Matrix type</th>
<th>ID Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>SUNMATRIX_DENSE</td>
<td>Dense $M \times N$ matrix</td>
<td>0</td>
</tr>
<tr>
<td>SUNMATRIX_BAND</td>
<td>Band $M \times M$ matrix</td>
<td>1</td>
</tr>
<tr>
<td>SUNMATRIX_SPARSE</td>
<td>Sparse (CSR or CSC) $M \times N$ matrix</td>
<td>2</td>
</tr>
<tr>
<td>SUNMATRIX_SLUUNRLOC</td>
<td>Adapter for the SuperLU_DIST SuperMatrix</td>
<td>3</td>
</tr>
<tr>
<td>SUNMATRIX_CUSTOM</td>
<td>User-provided custom matrix</td>
<td>4</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Name</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>SUNMAT_Ill_INPUT</td>
<td>-701</td>
<td>an illegal input has been provided to the function</td>
</tr>
<tr>
<td>SUNMAT_MEM_FAIL</td>
<td>-702</td>
<td>failed memory access or allocation</td>
</tr>
<tr>
<td>SUNMAT_OPERATION_FAIL</td>
<td>-703</td>
<td>a SUNMatrix operation returned nonzero</td>
</tr>
<tr>
<td>SUNMAT_MATVEC_SETUP_REQUIRED</td>
<td>-704</td>
<td>the SUNMatMatvecSetup routine needs to be called before calling SUNMatMatvec</td>
</tr>
</tbody>
</table>
struct _generic_SUNMatrix {
    void *content;
    struct _generic_SUNMatrix_Ops *ops;
};

The _generic_SUNMatrix_Ops structure is essentially a list of pointers to the various actual matrix operations, and is defined as

struct _generic_SUNMatrix_Ops {
    SUNMatrix_ID (*getid)(SUNMatrix);
    SUNMatrix (*clone)(SUNMatrix);
    void (*destroy)(SUNMatrix);
    int (*zero)(SUNMatrix);
    int (*copy)(SUNMatrix, SUNMatrix);
    int (*scaleadd)(realtype, SUNMatrix, SUNMatrix);
    int (*scaleaddi)(realtype, SUNMatrix);
    int (*matvecsetup)(SUNMatrix);
    int (*matvec)(SUNMatrix, N_Vector, N_Vector);
    int (*space)(SUNMatrix, long int*, long int*);
};

The generic SUNMATRIX module defines and implements the matrix operations acting on SUNMatrix objects. These routines are nothing but wrappers for the matrix operations defined by a particular SUNMATRIX implementation, which are accessed through the ops field of the SUNMatrix structure. To illustrate this point we show below the implementation of a typical matrix operation from the generic SUNMATRIX module, namely SUNMatZero, which sets all values of a matrix A to zero, returning a flag denoting a successful/failed operation:

```c
int SUNMatZero(SUNMatrix A) {
    return((int) A->ops->zero(A));
}
```

Section 9.1.1 contains a complete list of all matrix operations defined by the generic SUNMATRIX module.

The Fortran 2003 interface provides a bind(C) derived-type for the _generic_SUNMatrix and the _generic_SUNMatrix_Ops structures. Their definition is given below.

```fortran
type, bind(C), public :: SUNMatrix
    type(C_PTR), public :: content
    type(C_PTR), public :: ops
end type SUNMatrix

type, bind(C), public :: SUNMatrix_Ops
    type(C_FUNPTR), public :: getid
    type(C_FUNPTR), public :: clone
    type(C_FUNPTR), public :: destroy
    type(C_FUNPTR), public :: zero
    type(C_FUNPTR), public :: copy
    type(C_FUNPTR), public :: scaleadd
    type(C_FUNPTR), public :: scaleaddi
    type(C_FUNPTR), public :: matvecsetup
    type(C_FUNPTR), public :: matvec
    type(C_FUNPTR), public :: space
end type SUNMatrix_Ops
```
9.1.7 Implementing a custom SUNMatrix

A particular implementation of the SUNMATRIX module must:

- Specify the content field of the SUNMatrix object.
- Define and implement a minimal subset of the matrix operations. See the documentation for each SUNDIALS solver to determine which SUNMATRIX operations they require.
  
  Note that the names of these routines should be unique to that implementation in order to permit using more than one SUNMATRIX module (each with different SUNMatrix internal data representations) in the same code.
- Define and implement user-callable constructor and destructor routines to create and free a SUNMatrix with the new content field and with ops pointing to the new matrix operations.
- Optionally, define and implement additional user-callable routines acting on the newly defined SUNMatrix (e.g., a routine to print the content for debugging purposes).
- Optionally, provide accessor macros or functions as needed for that particular implementation to access different parts of the content field of the newly defined SUNMatrix.

It is recommended that a user-supplied SUNMATRIX implementation use the SUNMATRIX_CUSTOM identifier.

To aid in the creation of custom SUNMATRIX modules the generic SUNMATRIX module provides two utility functions SUNMatNewEmpty and SUNMatVCopyOps. When used in custom SUNMATRIX constructors and clone routines these functions will ease the introduction of any new optional matrix operations to the SUNMATRIX API by ensuring only required operations need to be set and all operations are copied when cloning a matrix. These functions are described in Section 9.1.2.

9.2 SUNMatrix functions used by IDAS

In Table 9.4, we list the matrix functions in the SUNMATRIX module used within the IDAS package. The table also shows, for each function, which of the code modules uses the function. The main IDAS integrator does not call any SUNMATRIX functions directly, so the table columns are specific to the IDALS interface and the IDABBDPRE preconditioner module. We further note that the IDALS interface only utilizes these routines when supplied with a matrix-based linear solver, i.e., the SUNMATRIX object passed to IDASSetLinearSolver was not NULL.

At this point, we should emphasize that the IDAS user does not need to know anything about the usage of matrix functions by the IDAS code modules in order to use IDAS. The information is presented as an implementation detail for the interested reader.

<table>
<thead>
<tr>
<th>Function</th>
<th>IDALS</th>
<th>IDABBDPRE</th>
</tr>
</thead>
<tbody>
<tr>
<td>SUNMatGetID</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>SUNMatDestroy</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>SUNMatZero</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>SUNMatSpace</td>
<td></td>
<td>†</td>
</tr>
</tbody>
</table>

The matrix functions listed in Section 9.1.1 with a † symbol are optionally used, in that these are only called if they are implemented in the SUNMATRIX module that is being used (i.e. their function pointers are non-NULL). The matrix functions listed in Section 9.1.1 that are not used by IDAS
are: SUNMatCopy, SUNMatClone, SUNMatScaleAdd, SUNMatScaleAddI and SUNMatMatvec. Therefore
a user-supplied SUNMATRIX module for IDAS could omit these functions.

9.3 The SUNMatrix_Dense implementation

The dense implementation of the SUNMATRIX module provided with SUNDIALS, SUNMATRIX_DENSE,
defines the content field of SUNMatrix to be the following structure:

```c
struct _SUNMatrixContent_Dense {
    sunindextype M;
    sunindextype N;
    realtype *data;
    sunindextype ldata;
    realtype **cols;
};
```

These entries of the content field contain the following information:

- **M** - number of rows
- **N** - number of columns
- **data** - pointer to a contiguous block of realtype variables. The elements of the dense matrix are
  stored columnwise, i.e. the \((i,j)\)-th element of a dense SUNMATRIX \(A\) (with \(0 \leq i < M\) and \(0 \leq j < N\)) may be accessed via \(\text{data}[j*M+i]\).
- **ldata** - length of the data array (= \(M\cdot N\)).
- **cols** - array of pointers. \(\text{cols}[j]\) points to the first element of the \(j\)-th column of the matrix in the
  array data. The \((i,j)\)-th element of a dense SUNMATRIX \(A\) (with \(0 \leq i < M\) and \(0 \leq j < N\)) may be accessed via \(\text{cols}[j][i]\).

The header file to include when using this module is `sunmatrix/sunmatrix_dense.h`. The SUNMA-
TRIX_DENSE module is accessible from all SUNDIALS solvers without linking to the
libsundials_sunmatrixdense module library.

9.3.1 SUNMatrix_Dense accessor macros

The following macros are provided to access the content of a SUNMATRIX_DENSE matrix. The prefix
SM, in the names denotes that these macros are for SUNMatrix implementations, and the suffix _D
denotes that these are specific to the dense version.

- **SM_CONTENT_D**
  This macro gives access to the contents of the dense SUNMatrix.

  The assignment \(A_{\text{cont}} = \text{SM_CONTENT_D}(A)\) sets \(A_{\text{cont}}\) to be a pointer to the dense SUNMatrix
  content structure.

  Implementation:
  ```c
  #define SM_CONTENT_D(A) ( (SUNMatrixContent_Dense)(A->content) )
  ```

- **SM_ROWS_D, SM_COLUMNS_D, and SM_LDATA_D**
  These macros give individual access to various lengths relevant to the content of a dense
  SUNMatrix.

  These may be used either to retrieve or to set these values. For example, the assignment \(A_{\text{rows}}
  = \text{SM_ROWS_D}(A)\) sets \(A_{\text{rows}}\) to be the number of rows in the matrix \(A\). Similarly, the assignment
  \(\text{SM_COLUMNS_D}(A) = A_{\text{cols}}\) sets the number of columns in \(A\) to equal \(A_{\text{cols}}\).

  Implementation:
  ```c
  #define SM_ROWS_D(A) ( SM_CONTENT_D(A)->M )
  ```
9.3 The SUNMatrix_Dense implementation

#define SM_COLUMNS_D(A) ( SM_CONTENT_D(A)->N )
#define SM_LDATA_D(A) ( SM_CONTENT_D(A)->ldata )

• SM_DATA_D and SM_COLS_D

These macros give access to the data and cols pointers for the matrix entries.

The assignment A_data = SM_DATA_D(A) sets A_data to be a pointer to the first component of the data array for the dense SUNMatrix A. The assignment SM_DATA_D(A) = A_data sets the data array of A to be A_data by storing the pointer A_data.

Similarly, the assignment A_cols = SM_COLS_D(A) sets A_cols to be a pointer to the array of column pointers for the dense SUNMatrix A. The assignment SM_COLS_D(A) = A_cols sets the column pointer array of A to be A_cols by storing the pointer A_cols.

Implementation:
#define SM_DATA_D(A) ( SM_CONTENT_D(A)->data )
#define SM_COLS_D(A) ( SM_CONTENT_D(A)->cols )

• SM_COLUMN_D and SM_ELEMENT_D

These macros give access to the individual columns and entries of the data array of a dense SUNMatrix.

The assignment col_j = SM_COLUMN_D(A,j) sets col_j to be a pointer to the first entry of the j-th column of the M × N dense matrix A (with 0 ≤ j < N). The type of the expression SM_COLUMN_D(A,j) is realtype *. The pointer returned by the call SM_COLUMN_D(A,j) can be treated as an array which is indexed from 0 to M − 1.

The assignments SM_ELEMENT_D(A,i,j) = a_{ij} and a_{ij} = SM_ELEMENT_D(A,i,j) reference the (i,j)-th element of the M × N dense matrix A (with 0 ≤ i < M and 0 ≤ j < N).

Implementation:
#define SM_COLUMN_D(A,j) ( (SM_CONTENT_D(A)->cols)[j] )
#define SM_ELEMENT_D(A,i,j) ( (SM_CONTENT_D(A)->cols)[j][i] )

9.3.2 SUNMatrix_Dense functions

The SUNMATRIX_DENSE module defines dense implementations of all matrix operations listed in Section 9.1.1. Their names are obtained from those in Section 9.1.1 by appending the suffix _Dense (e.g. SUNMatCopy_Dense). All the standard matrix operations listed in Section 9.1.1 with the suffix _Dense appended are callable via the FORTRAN 2003 interface by prepending an ‘F’ (e.g. FSUNMatCopy_Dense).

The module SUNMATRIX_DENSE provides the following additional user-callable routines:

SUNDenseMatrix
Prototype SUNMatrix SUNDenseMatrix(sunindextype M, sunindextype N)
Description This constructor function creates and allocates memory for a dense SUNMatrix. Its arguments are the number of rows, M, and columns, N, for the dense matrix.
F2003 Name This function is callable as FSUNDenseMatrix when using the Fortran 2003 interface module.

SUNDenseMatrix_Print
Prototype void SUNDenseMatrix_Print(SUNMatrix A, FILE* outfile)
Description This function prints the content of a dense SUNMatrix to the output stream specified by outfile. Note: stdout or stderr may be used as arguments for outfile to print directly to standard output or standard error, respectively.
Description of the SUNMatrix module

**SUNDenseMatrix_Rows**
Prototype: `sunindextype SUNDenseMatrix_Rows(SUNMatrix A)`
Description: This function returns the number of rows in the dense SUNMatrix.
F2003 Name: This function is callable as `FSUNDenseMatrix_Rows` when using the Fortran 2003 interface module.

**SUNDenseMatrix_Columns**
Prototype: `sunindextype SUNDenseMatrix_Columns(SUNMatrix A)`
Description: This function returns the number of columns in the dense SUNMatrix.
F2003 Name: This function is callable as `FSUNDenseMatrix_Columns` when using the Fortran 2003 interface module.

**SUNDenseMatrix_LData**
Prototype: `sunindextype SUNDenseMatrix_LData(SUNMatrix A)`
Description: This function returns the length of the data array for the dense SUNMatrix.
F2003 Name: This function is callable as `FSUNDenseMatrix_LData` when using the Fortran 2003 interface module.

**SUNDenseMatrix_Data**
Prototype: `realtype* SUNDenseMatrix_Data(SUNMatrix A)`
Description: This function returns a pointer to the data array for the dense SUNMatrix.
F2003 Name: This function is callable as `FSUNDenseMatrix_Data` when using the Fortran 2003 interface module.

**SUNDenseMatrix_Cols**
Prototype: `realtype** SUNDenseMatrix_Cols(SUNMatrix A)`
Description: This function returns a pointer to the cols array for the dense SUNMatrix.

**SUNDenseMatrix_Column**
Prototype: `realtype* SUNDenseMatrix_Column(SUNMatrix A, sunindextype j)`
Description: This function returns a pointer to the first entry of the jth column of the dense SUNMatrix. The resulting pointer should be indexed over the range 0 to M – 1.
F2003 Name: This function is callable as `FSUNDenseMatrix_Column` when using the Fortran 2003 interface module.

Notes
- When looping over the components of a dense SUNMatrix A, the most efficient approaches are to:
  - First obtain the component array via `A_data = SM_DATA_D(A)` or `A_data = SUNDenseMatrix_Data(A)` and then access `A_data[i]` within the loop.
  - First obtain the array of column pointers via `A_cols = SM_COLS_D(A)` or `A_cols = SUNDenseMatrix_Cols(A)`, and then access `A_cols[j][i]` within the loop.
  - Within a loop over the columns, access the column pointer via `A_colj = SUNDenseMatrix_Column(A, j)`, and then to access the entries within that column using `A_colj[i]` within the loop.
9.4 The SUNMatrix_Band implementation

All three of these are more efficient than using \texttt{SM\_ELEMENT\_D(A, i, j)} within a double loop.

- Within the \texttt{SUNMatMatvec\_Dense} routine, internal consistency checks are performed to ensure that the matrix is called with consistent \texttt{nvector} implementations. These are currently limited to: \texttt{nvector\_serial}, \texttt{nvector\_openmp}, and \texttt{nvector\_pthread}. As additional compatible vector implementations are added to SUNDIALS, these will be included within this compatibility check.

### 9.3.3 SUNMatrix\_Dense Fortran interfaces

The \texttt{sunmatrix\_dense} module provides a FORTRAN 2003 module as well as FORTRAN 77 style interface functions for use from FORTRAN applications.

**FORTRAN 2003 interface module**

The \texttt{fsunmatrix\_dense\_mod} FORTRAN module defines interfaces to most \texttt{sunmatrix\_dense} C functions using the intrinsic \texttt{iso\_c\_binding} module which provides a standardized mechanism for inter-operating with C. As noted in the C function descriptions above, the interface functions are named after the corresponding C function, but with a leading ‘F’. For example, the function \texttt{SUNDenseMatrix} is interfaced as \texttt{FSUNDenseMatrix}.

The FORTRAN 2003 SUNMATRIX\_DENSE interface module can be accessed with the \texttt{use} statement, i.e. \texttt{use fsunmatrix\_dense\_mod}, and linking to the library \texttt{libsundials\_fsunmatrixdense\_mod.lib} in addition to the C library. For details on where the library and module file \texttt{fsunmatrix\_dense\_mod.mod} are installed see Appendix A. We note that the module is accessible from the FORTRAN 2003 SUNDIALS integrators without separately linking to the \texttt{libsundials\_fsunmatrixdense\_mod} library.

**FORTRAN 77 interface functions**

For solvers that include a FORTRAN interface module, the \texttt{sunmatrix\_dense} module also includes the FORTRAN-callable function \texttt{FSUNDenseMatInit(code, M, N, ier)} to initialize this SUNMATRIX\_DENSE module for a given SUNDIALS solver. Here \texttt{code} is an integer input solver id (1 for \texttt{cvode}, 2 for \texttt{ida}, 3 for \texttt{kinsol}, 4 for \texttt{arkode}); \texttt{M} and \texttt{N} are the corresponding dense matrix construction arguments (declared to match C type \texttt{long\ int}); and \texttt{ier} is an error return flag equal to 0 for success and -1 for failure. Both \texttt{code} and \texttt{ier} are declared to match C type \texttt{int}. Additionally, when using \texttt{arkode} with a non-identity mass matrix, the FORTRAN-callable function \texttt{FSUNDenseMassMatInit(M, N, ier)} initializes this SUNMATRIX\_DENSE module for storing the mass matrix.

### 9.4 The SUNMatrix\_Band implementation

The banded implementation of the SUNMATRIX module provided with SUNDIALS, SUNMATRIX\_BAND, defines the \texttt{content} field of SUNMatrix to be the following structure:

```
struct _SUNMatrixContent_Band {
    sunindextype M;
    sunindextype N;
    sunindextype mu;
    sunindextype ml;
    sunindextype s_mu;
    sunindextype ldim;
    realtype *data;
    sunindextype ldata;
    realtype **cols;
};
```

A diagram of the underlying data representation in a banded matrix is shown in Figure 9.1. A more complete description of the parts of this \texttt{content} field is given below:
M - number of rows
N - number of columns (N = M)
mu - upper half-bandwidth, 0 ≤ mu < N
ml - lower half-bandwidth, 0 ≤ ml < N
s.mu - storage upper bandwidth, mu ≤ s.mu < N. The LU decomposition routines in the associated SUNLINSOL_BAND and SUNLINSOL_LAPACKBAND modules write the LU factors into the storage for A. The upper triangular factor U, however, may have an upper bandwidth as big as min(N-1,mu+ml) because of partial pivoting. The s.mu field holds the upper half-bandwidth allocated for A.
ldim - leading dimension (ldim ≥ s.mu+ml+1)
data - pointer to a contiguous block of realtype variables. The elements of the banded matrix are stored columnwise (i.e. columns are stored one on top of the other in memory). Only elements within the specified half-bandwidths are stored. data is a pointer to ldata contiguous locations which hold the elements within the band of A.
ldata - length of the data array (= ldim·N)
cols - array of pointers. cols[j] is a pointer to the uppermost element within the band in the j-th column. This pointer may be treated as an array indexed from s.mu−mu (to access the uppermost element within the band in the j-th column) to s.mu+ml (to access the lowest element within the band in the j-th column). Indices from 0 to s.mu−mu−1 give access to extra storage elements required by the LU decomposition function. Finally, cols[j][i−j+s.mu] is the (i,j)-th element with j−mu ≤ i ≤ j+ml.
The header file to include when using this module is sunmatrix/sunmatrix_band.h. The SUNMATRIX_BAND module is accessible from all SUNDIALS solvers without linking to the libsundials_sunmatrixband module library.

9.4.1 SUNMatrix_Band accessor macros
The following macros are provided to access the content of a SUNMATRIX_BAND matrix. The prefix SM_ in the names denotes that these macros are for SUNMatrix implementations, and the suffix _B denotes that these are specific to the banded version.

- SM_CONTENT_B
  This routine gives access to the contents of the banded SUNMatrix.
  The assignment A_cont = SM_CONTENT_B(A) sets A_cont to be a pointer to the banded SUNMatrix content structure.
  Implementation:
  #define SM_CONTENT_B(A) ( (SUNMatrixContent_Band)(A->content) )

  These macros give individual access to various lengths relevant to the content of a banded SUNMatrix.
  These may be used either to retrieve or to set these values. For example, the assignment A_rows = SM_ROWS_B(A) sets A_rows to be the number of rows in the matrix A. Similarly, the assignment SM_COLUMNS_B(A) = A_cols sets the number of columns in A to equal A_cols.
  Implementation:
  #define SM_ROWS_B(A) ( SM_CONTENT_B(A)->M )
  #define SM_COLUMNS_B(A) ( SM_CONTENT_B(A)->N )
  #define SM_UBAND_B(A) ( SM_CONTENT_B(A)->mu )
  #define SM_LBAND_B(A) ( SM_CONTENT_B(A)->ml )
9.4 The SUNMatrix_Band implementation

Figure 9.1: Diagram of the storage for the SUNMATRIX_BAND module. Here $A$ is an $N \times N$ band matrix with upper and lower half-bandwidths $\mu$ and $m_l$, respectively. The rows and columns of $A$ are numbered from 0 to $N - 1$ and the $(i, j)$-th element of $A$ is denoted $A(i, j)$. The greyed out areas of the underlying component storage are used by the associated SUNLINSOL_BAND linear solver.
#define SM_SUBAND_B(A)  (SM_CONTENT_B(A)->s_mu)
#define SM_LDIM_B(A)  (SM_CONTENT_B(A)->ldim)
#define SM_LDATA_B(A)  (SM_CONTENT_B(A)->ldata)

- **SM_DATA_B** and **SM_COLS_B**

These macros give access to the data and cols pointers for the matrix entries.

The assignment \( A_{\text{data}} = \text{SM\_DATA\_B}(A) \) sets \( A_{\text{data}} \) to be a pointer to the first component of the data array for the banded SUNMatrix \( A \). The assignment \( \text{SM\_DATA\_B}(A) = A_{\text{data}} \) sets the data array of \( A \) to be \( A_{\text{data}} \) by storing the pointer \( A_{\text{data}} \).

Similarly, the assignment \( A_{\text{cols}} = \text{SM\_COLS\_B}(A) \) sets \( A_{\text{cols}} \) to be a pointer to the array of column pointers for the banded SUNMatrix \( A \). The assignment \( \text{SM\_COLS\_B}(A) = A_{\text{cols}} \) sets the column pointer array of \( A \) to be \( A_{\text{cols}} \) by storing the pointer \( A_{\text{cols}} \).

Implementation:

- **SM\_COLUMN\_B**, **SM\_COLUMN\_ELEMENT\_B**, and **SM\_ELEMENT\_B**

These macros give access to the individual columns and entries of the data array of a banded SUNMatrix.

The assignments \( \text{SM\_ELEMENT\_B}(A,i,j) = a_{ij} \) and \( a_{ij} = \text{SM\_ELEMENT\_B}(A,i,j) \) reference the \((i,j)\)-th element of the \( N \times N \) band matrix \( A \), where \( 0 \leq i, j \leq N - 1 \). The location \((i,j)\) should further satisfy \( j - \mu \leq i \leq j + ml \).

The assignment \( \text{col}_j = \text{SM\_COLUMN\_B}(A,j) \) sets \( \text{col}_j \) to be a pointer to the diagonal element of the \( j \)-th column of the \( N \times N \) band matrix \( A \), \( 0 \leq j \leq N - 1 \). The type of the expression \( \text{SM\_COLUMN\_B}(A,j) \) is \( \text{realtype} * \). The pointer returned by the call \( \text{SM\_COLUMN\_B}(A,j) \) can be treated as an array which is indexed from \(-\mu\) to \( ml \).

The assignments \( \text{SM\_COLUMN\_ELEMENT\_B} (\text{col}_j,i,j) = a_{ij} \) and \( a_{ij} = \text{SM\_COLUMN\_ELEMENT\_B} (\text{col}_j,i,j) \) reference the \((i,j)\)-th entry of the band matrix \( A \) when used in conjunction with \( \text{SM\_COLUMN\_B} \) to reference the \( j \)-th column through \( \text{col}_j \). The index \((i,j)\) should satisfy \( j - \mu \leq i \leq j + ml \).

Implementation:

- **SUNMatrix\_Band** functions

The SUNMATRIX\_BAND module defines banded implementations of all matrix operations listed in Section 9.1.1. Their names are obtained from those in Section 9.1.1 by appending the suffix \_Band\ (e.g. SUNMatCopy\_Band). All the standard matrix operations listed in Section 9.1.1 with the suffix \_Band appended are callable via the FORTRAN 2003 interface by prepending an ‘F’ (e.g. FSUNMatCopy\_Band).

The module SUNMATRIX\_BAND provides the following additional user-callable routines:
Description This constructor function creates and allocates memory for a banded SUNMatrix. Its arguments are the matrix size, \( N \), and the upper and lower half-bandwidths of the matrix, \( \mu \) and \( \ml \). The stored upper bandwidth is set to \( \mu + \ml \) to accommodate subsequent factorization in the SUNLINSOL_BAND and SUNLINSOL_LAPACKBAND modules.

F2003 Name This function is callable as FSUNBandMatrix when using the Fortran 2003 interface module.

**SUNBandMatrixStorage**

Prototype `SUNMatrix SUNBandMatrixStorage(sunindextype N, sunindextype mu, sunindextype ml, sunindextype smu)`

Description This constructor function creates and allocates memory for a banded SUNMatrix. Its arguments are the matrix size, \( N \), the upper and lower half-bandwidths of the matrix, \( \mu \) and \( \ml \), and the stored upper bandwidth, \( \smu \). When creating a band SUNMatrix, this value should be

- at least \( \min(N-1,\mu+\ml) \) if the matrix will be used by the SUNLINSOL_BAND module;
- exactly equal to \( \mu + \ml \) if the matrix will be used by the SUNLINSOL_LAPACKBAND module;
- at least \( \mu \) if used in some other manner.

*Note: it is strongly recommended that users call the default constructor, SUNBandMatrix, in all standard use cases. This advanced constructor is used internally within SUNDIALS solvers, and is provided to users who require banded matrices for non-default purposes.*

**SUNBandMatrix_Print**

Prototype `void SUNBandMatrix_Print(SUNMatrix A, FILE* outfile)`

Description This function prints the content of a banded SUNMatrix to the output stream specified by `outfile`. Note: `stdout` or `stderr` may be used as arguments for `outfile` to print directly to standard output or standard error, respectively.

**SUNBandMatrix_Rows**

Prototype `sunindextype SUNBandMatrix_Rows(SUNMatrix A)`

Description This function returns the number of rows in the banded SUNMatrix.

F2003 Name This function is callable as FSUNBandMatrix_Rows when using the Fortran 2003 interface module.

**SUNBandMatrix_Columns**

Prototype `sunindextype SUNBandMatrix_Columns(SUNMatrix A)`

Description This function returns the number of columns in the banded SUNMatrix.

F2003 Name This function is callable as FSUNBandMatrix_Columns when using the Fortran 2003 interface module.

**SUNBandMatrix_LowerBandwidth**

Prototype `sunindextype SUNBandMatrix_LowerBandwidth(SUNMatrix A)`

Description This function returns the lower half-bandwidth of the banded SUNMatrix.

F2003 Name This function is callable as FSUNBandMatrix_LowerBandwidth when using the Fortran 2003 interface module.
Description of the SUNMatrix module

**SUNBandMatrix_UpperBandwidth**

Protoype: `sunindextype SUNBandMatrix_UpperBandwidth(SUNMatrix A)`

Description: This function returns the upper half-bandwidth of the banded SUNMatrix.

F2003 Name: This function is callable as `FSUNBandMatrix_UpperBandwidth` when using the Fortran 2003 interface module.

**SUNBandMatrix_StoredUpperBandwidth**

Protoype: `sunindextype SUNBandMatrix_StoredUpperBandwidth(SUNMatrix A)`

Description: This function returns the stored upper half-bandwidth of the banded SUNMatrix.

F2003 Name: This function is callable as `FSUNBandMatrix_StoredUpperBandwidth` when using the Fortran 2003 interface module.

**SUNBandMatrix_LDim**

Protoype: `sunindextype SUNBandMatrix_LDim(SUNMatrix A)`

Description: This function returns the length of the leading dimension of the banded SUNMatrix.

F2003 Name: This function is callable as `FSUNBandMatrix_LDim` when using the Fortran 2003 interface module.

**SUNBandMatrix_Data**

Protoype: `realtype* SUNBandMatrix_Data(SUNMatrix A)`

Description: This function returns a pointer to the data array for the banded SUNMatrix.

F2003 Name: This function is callable as `FSUNBandMatrix_Data` when using the Fortran 2003 interface module.

**SUNBandMatrix_Cols**

Protoype: `realtype** SUNBandMatrix_Cols(SUNMatrix A)`

Description: This function returns a pointer to the cols array for the banded SUNMatrix.

**SUNBandMatrix_Column**

Protoype: `realtype* SUNBandMatrix_Column(SUNMatrix A, sunindextype j)`

Description: This function returns a pointer to the diagonal entry of the j-th column of the banded SUNMatrix. The resulting pointer should be indexed over the range \(-\mu_u\) to \(\mu_l\).

F2003 Name: This function is callable as `FSUNBandMatrix_Column` when using the Fortran 2003 interface module.

Notes:

- When looping over the components of a banded SUNMatrix \(A\), the most efficient approaches are to:
  - First obtain the component array via \(A\_data = \text{SM\_DATA\_B}(A)\) or \(A\_data = \text{SUNBandMatrix\_Data}(A)\) and then access \(A\_data[i]\) within the loop.
  - First obtain the array of column pointers via \(A\_cols = \text{SM\_COLS\_B}(A)\) or \(A\_cols = \text{SUNBandMatrix\_Cols}(A)\), and then access \(A\_cols[j][i]\) within the loop.
  - Within a loop over the columns, access the column pointer via \(A\_colj = \text{SUNBandMatrix\_Column}(A, j)\) and then to access the entries within that column using \(\text{SM\_COLUMN\_ELEMENT\_B}(A\_colj, i, j)\).
All three of these are more efficient than using `SM_ELEMENT_B(A,i,j)` within a double loop.

- Within the `SUNMatMatvec_Band` routine, internal consistency checks are performed to ensure that the matrix is called with consistent `nvector` implementations. These are currently limited to: `nvector_serial`, `nvector_openmp`, and `nvector_pthreads`. As additional compatible vector implementations are added to `sundials`, these will be included within this compatibility check.

### 9.4.3 SUNMatrix_Band Fortran interfaces

The `sunmatrix_band` module provides a FORTRAN 2003 module as well as FORTRAN 77 style interface functions for use from FORTRAN applications.

#### FORTRAN 2003 interface module

The `fsunmatrix_band_mod` FORTRAN module defines interfaces to most `sunmatrix_band` C functions using the intrinsic `iso_c_binding` module which provides a standardized mechanism for interoperating with C. As noted in the C function descriptions above, the interface functions are named after the corresponding C function, but with a leading ‘F’. For example, the function `SUNBandMatrix` is interfaced as `FSUNBandMatrix`.

The FORTRAN 2003 SUNMATRIX_BAND interface module can be accessed with the `use` statement, i.e. `use fsunmatrix_band_mod`, and linking to the library `libsundials_fsunmatrixband_mod.lib` in addition to the C library. For details on where the library and module file `fsunmatrix_band_mod.mod` are installed see Appendix A. We note that the module is accessible from the FORTRAN 2003 SUNDIALS integrators *without* separately linking to the `libsundials_fsunmatrixband_mod.lib` library.

#### FORTRAN 77 interface functions

For solvers that include a FORTRAN interface module, the SUNMATRIX_BAND module also includes the FORTRAN-callable function `FSUNBandMatInit(code, N, mu, ml, ier)` to initialize this SUNMATRIX_BAND module for a given SUNDIALS solver. Here `code` is an integer input solver id (1 for `cvode`, 2 for `ida`, 3 for `kinsol`, 4 for `arkode`); `N`, `mu`, and `ml` are the corresponding band matrix construction arguments (declared to match C type `long int`); and `ier` is an error return flag equal to 0 for success and -1 for failure. Both `code` and `ier` are declared to match C type `int`. Additionally, when using `arkode` with a non-identity mass matrix, the FORTRAN-callable function `FSUNBandMassMatInit(N, mu, ml, ier)` initializes this SUNMATRIX_BAND module for storing the mass matrix.

### 9.5 The SUNMatrix_Sparse implementation

The sparse implementation of the SUNMATRIX module provided with SUNDIALS, SUNMATRIX_SPARSE, is designed to work with either `compressed-sparse-column` (CSC) or `compressed-sparse-row` (CSR) sparse matrix formats. To this end, it defines the `content` field of SUNMatrix to be the following structure:

```c
struct _SUNMatrixContent_Sparse {
    sunindextype M;
    sunindextype N;
    sunindextype NNZ;
    sunindextype NP;
    realtype *data;
    int sparsetype;
    sunindextype *indexvals;
    sunindextype *indexptrs;
    /* CSC indices */
    sunindextype **rowvals;
```
sunindextype **colptrs;
/* CSR indices */
sunindextype **colvals;
sunindextype **rowptrs;
};

A diagram of the underlying data representation for a CSC matrix is shown in Figure 9.2 (the CSR format is similar). A more complete description of the parts of this content field is given below:

- **M** - number of rows
- **N** - number of columns
- **NNZ** - maximum number of nonzero entries in the matrix (allocated length of **data** and **indexvals** arrays)
- **NP** - number of index pointers (e.g. number of column pointers for CSC matrix). For CSC matrices **NP** = **N**, and for CSR matrices **NP** = **M**. This value is set automatically based on the input for **sparsetype**.
- **data** - pointer to a contiguous block of **realttype** variables (of length **NNZ**), containing the values of the nonzero entries in the matrix
- **sparsetype** - type of the sparse matrix (**CSC_MAT** or **CSR_MAT**)
- **indexvals** - pointer to a contiguous block of **int** variables (of length **NNZ**), containing the row indices (if CSC) or column indices (if CSR) of each nonzero matrix entry held in **data**
- **indexptrs** - pointer to a contiguous block of **int** variables (of length **NP+1**). For CSC matrices each entry provides the index of the first column entry into the **data** and **indexvals** arrays, e.g. if **indexptr[3]** = 7, then the first nonzero entry in the fourth column of the matrix is located in **data[7]**, and is located in row **indexvals[7]** of the matrix. The last entry contains the total number of nonzero values in the matrix and hence points one past the end of the active data in the **data** and **indexvals** arrays. For CSR matrices, each entry provides the index of the first row entry into the **data** and **indexvals** arrays.

The following pointers are added to the **SlsMat** type for user convenience, to provide a more intuitive interface to the CSC and CSR sparse matrix data structures. They are set automatically when creating a sparse **SUNMATRIX**, based on the sparse matrix storage type.

- **rowvals** - pointer to **indexvals** when **sparsetype** is **CSC_MAT**, otherwise set to **NULL**.
- **colptrs** - pointer to **indexptrs** when **sparsetype** is **CSC_MAT**, otherwise set to **NULL**.
- **colvals** - pointer to **indexvals** when **sparsetype** is **CSR_MAT**, otherwise set to **NULL**.
- **rowptrs** - pointer to **indexptrs** when **sparsetype** is **CSR_MAT**, otherwise set to **NULL**.

For example, the $5 \times 4$ CSC matrix

$$
\begin{bmatrix}
0 & 3 & 1 & 0 \\
3 & 0 & 0 & 2 \\
0 & 7 & 0 & 0 \\
1 & 0 & 0 & 9 \\
0 & 0 & 0 & 5
\end{bmatrix}
$$

could be stored in this structure as either

- **M** = 5;
- **N** = 4;
- **NNZ** = 8;
- **NP** = **N**;
- **data** = \{3.0, 1.0, 3.0, 7.0, 1.0, 2.0, 9.0, 5.0\};
- **sparsetype** = **CSC_MAT**;
- **indexvals** = \{1, 3, 0, 2, 0, 1, 3, 4\};
- **indexptrs** = \{0, 2, 4, 5, 8\};

or
9.5 The SUNMatrix Sparse implementation

\begin{verbatim}
M = 5;
N = 4;
NNZ = 10;
NP = N;
data = {3.0, 1.0, 3.0, 7.0, 1.0, 2.0, 9.0, 5.0, *, *};
sparsetype = CSC_MAT;
indexvals = {1, 3, 0, 2, 0, 1, 3, 4, *, *};
indexptrs = {0, 2, 4, 5, 8};
\end{verbatim}
where the first has no unused space, and the second has additional storage (the entries marked with * may contain any values). Note in both cases that the final value in \texttt{indexptrs} is 8, indicating the total number of nonzero entries in the matrix.

Similarly, in CSR format, the same matrix could be stored as

\begin{verbatim}
M = 5;
N = 4;
NNZ = 8;
NP = N;
data = {3.0, 1.0, 3.0, 2.0, 7.0, 1.0, 9.0, 5.0};
sparsetype = CSR_MAT;
indexvals = {1, 2, 0, 3, 1, 0, 3, 3};
indexptrs = {0, 2, 4, 5, 7, 8};
\end{verbatim}

The header file to include when using this module is \texttt{sunmatrix/sunmatrix_sparse.h}. The \texttt{sunmatrix_sparse} module is accessible from all \texttt{sundials} solvers \textit{without} linking to the \texttt{libsundials} \texttt{sunmatrixsparse} module library.

9.5.1 SUNMatrix_Sparse accessor macros

The following macros are provided to access the content of a \texttt{sunmatrix_sparse} matrix. The prefix \texttt{SM} in the names denotes that these macros are for \texttt{SUNMatrix} implementations, and the suffix \texttt{S} denotes that these are specific to the \texttt{sparse} version.

- \texttt{SM\_CONTENT\_S}

  This routine gives access to the contents of the sparse SUNMatrix.

  The assignment \texttt{A\_cont = SM\_CONTENT\_S(A)} sets \texttt{A\_cont} to be a pointer to the sparse SUNMatrix content structure.

  Implementation:
  \begin{verbatim}
  #define SM\_CONTENT\_S(A) ( (SUNMatrixContent_Sparse)(A->content) )
  \end{verbatim}

- \texttt{SM\_ROWS\_S}, \texttt{SM\_COLUMNS\_S}, \texttt{SM\_NNZ\_S}, \texttt{SM\_NP\_S}, and \texttt{SM\_SPARSETYPE\_S}

  These macros give individual access to various lengths relevant to the content of a sparse SUNMatrix.

  These may be used either to retrieve or to set these values. For example, the assignment \texttt{A\_rows = SM\_ROWS\_S(A)} sets \texttt{A\_rows} to be the number of rows in the matrix \texttt{A}. Similarly, the assignment \texttt{SM\_COLUMNS\_S(A) = A\_cols} sets the number of columns in \texttt{A} to equal \texttt{A\_cols}.

  Implementation:
  \begin{verbatim}
  #define SM\_ROWS\_S(A) ( SM\_CONTENT\_S(A)->M )
  #define SM\_COLUMNS\_S(A) ( SM\_CONTENT\_S(A)->N )
  #define SM\_NNZ\_S(A) ( SM\_CONTENT\_S(A)->NNZ )
  #define SM\_NP\_S(A) ( SM\_CONTENT\_S(A)->NP )
  #define SM\_SPARSETYPE\_S(A) ( SM\_CONTENT\_S(A)->sparsetype )
  \end{verbatim}

Figure 9.2: Diagram of the storage for a compressed-sparse-column matrix. Here $A$ is an $M \times N$ sparse matrix with storage for up to $NNZ$ nonzero entries (the allocated length of both $\text{data}$ and $\text{indexvals}$). The entries in $\text{indexvals}$ may assume values from 0 to $M - 1$, corresponding to the row index (zero-based) of each nonzero value. The entries in $\text{data}$ contain the values of the nonzero entries, with the row $i$, column $j$ entry of $A$ (again, zero-based) denoted as $A(i,j)$. The $\text{indexptrs}$ array contains $N + 1$ entries; the first $N$ denote the starting index of each column within the $\text{indexvals}$ and $\text{data}$ arrays, while the final entry points one past the final nonzero entry. Here, although $NNZ$ values are allocated, only $nz$ are actually filled in; the greyed-out portions of $\text{data}$ and $\text{indexvals}$ indicate extra allocated space.
9.5 The SUNMatrix Sparse implementation

- SM_DATA_S, SM_INDEXVALS_S, and SM_INDEXPTRS_S

These macros give access to the data and index arrays for the matrix entries.

The assignment $A_{\text{data}} = \text{SM\_DATA\_S}(A)$ sets $A_{\text{data}}$ to be a pointer to the first component of the data array for the sparse SUNMatrix $A$. The assignment $\text{SM\_DATA\_S}(A) = A_{\text{data}}$ sets the data array of $A$ to be $A_{\text{data}}$ by storing the pointer $A_{\text{data}}$.

Similarly, the assignment $A_{\text{indexvals}} = \text{SM\_INDEXVALS\_S}(A)$ sets $A_{\text{indexvals}}$ to be a pointer to the array of index values (i.e. row indices for a CSC matrix, or column indices for a CSR matrix) for the sparse SUNMatrix $A$. The assignment $\text{A\_INDEXPTRS\_S}(A) = A_{\text{indexptrs}}$ sets $A_{\text{indexptrs}}$ to be a pointer to the array of index pointers (i.e. the starting indices in the data/indexvals arrays for each row or column in CSR or CSC formats, respectively).

Implementation:

```c
#define SM_DATA_S(A) ( SM_CONTENT_S(A)->data )
#define SM_INDEXVALS_S(A) ( SM_CONTENT_S(A)->indexvals )
#define SM_INDEXPTRS_S(A) ( SM_CONTENT_S(A)->indexptrs )
```

9.5.2 SUNMatrix Sparse functions

The SUNMATRIX_SPARSE module defines sparse implementations of all matrix operations listed in Section 9.1.1. Their names are obtained from those in Section 9.1.1 by appending the suffix _Sparse (e.g. SUNMatCopy_Sparse). All the standard matrix operations listed in Section 9.1.1 with the suffix _Sparse appended are callable via the FORTRAN 2003 interface by prepending an ‘F’ (e.g. FSUNMatCopy_Sparse).

The module SUNMATRIX_SPARSE provides the following additional user-callable routines:

**SUNSparseMatrix**

Prototype: `SUNMatrix SUNSparseMatrix(sunindextype M, sunindextype N, sunindextype NNZ, int sparsetype)`

Description: This function creates and allocates memory for a sparse SUNMatrix. Its arguments are the number of rows and columns of the matrix, $M$ and $N$, the maximum number of nonzeros to be stored in the matrix, $NNZ$, and a flag $\text{sparsetype}$ indicating whether to use CSR or CSC format (valid arguments are CSR\_MAT or CSC\_MAT).

F2003 Name: This function is callable as FSUNSparseMatrix when using the Fortran 2003 interface module.

**SUNSparseFromDenseMatrix**

Prototype: `SUNMatrix SUNSparseFromDenseMatrix(SUNMatrix A, realtype droptol, int sparsetype);`

Description: This function creates a new sparse matrix from an existing dense matrix by copying all values with magnitude larger than $\text{droptol}$ into the sparse matrix structure.

Requirements:

- $A$ must have type SUNMATRIX\_DENSE;
- $\text{droptol}$ must be non-negative;
- $\text{sparsetype}$ must be either CSC\_MAT or CSR\_MAT.

The function returns NULL if any requirements are violated, or if the matrix storage request cannot be satisfied.

F2003 Name: This function is callable as FSUNSparseFromDenseMatrix when using the Fortran 2003 interface module.
Description of the SUNMatrix module

**SUNSparseFromBandMatrix**

Prototype: `SUNMatrix SUNSparseFromBandMatrix(SUNMatrix A, realtype droptol, int sparsetype);`

Description: This function creates a new sparse matrix from an existing band matrix by copying all values with magnitude larger than `droptol` into the sparse matrix structure.

Requirements:
- `A` must have type `SUNMATRIX_BAND`;
- `droptol` must be non-negative;
- `sparsetype` must be either `CSC_MAT` or `CSR_MAT`.

The function returns NULL if any requirements are violated, or if the matrix storage request cannot be satisfied.

F2003 Name: This function is callable as `FSUNSparseFromBandMatrix` when using the Fortran 2003 interface module.

**SUNSparseMatrix_Realloc**

Prototype: `int SUNSparseMatrix_Realloc(SUNMatrix A)`

Description: This function reallocates internal storage arrays in a sparse matrix so that the resulting sparse matrix has no wasted space (i.e. the space allocated for nonzero entries equals the actual number of nonzeros, `indexptrs[NP]`). Returns 0 on success and 1 on failure (e.g. if the input matrix is not sparse).

F2003 Name: This function is callable as `FSUNSparseMatrix_Realloc` when using the Fortran 2003 interface module.

**SUNSparseMatrix_Reallocate**

Prototype: `int SUNSparseMatrix_Reallocate(SUNMatrix A, sunindextype NNZ)`

Description: This function reallocates internal storage arrays in a sparse matrix so that the resulting sparse matrix has storage for a specified number of nonzeros. Returns 0 on success and 1 on failure (e.g. if the input matrix is not sparse or if `NNZ` is negative).

F2003 Name: This function is callable as `FSUNSparseMatrix_Reallocate` when using the Fortran 2003 interface module.

**SUNSparseMatrix_Print**

Prototype: `void SUNSparseMatrix_Print(SUNMatrix A, FILE* outfile)`

Description: This function prints the content of a sparse `SUNMatrix` to the output stream specified by `outfile`. Note: `stdout` or `stderr` may be used as arguments for `outfile` to print directly to standard output or standard error, respectively.

**SUNSparseMatrix_Rows**

Prototype: `sunindextype SUNSparseMatrix_Rows(SUNMatrix A)`

Description: This function returns the number of rows in the sparse `SUNMatrix`.

F2003 Name: This function is callable as `FSUNSparseMatrix_Rows` when using the Fortran 2003 interface module.
9.5 The SUNMatrix_Sparse implementation

[SUNSparseMatrix_Columns]
Prototype: sunindextype SUNSparseMatrix_Columns(SUNMatrix A)
Description: This function returns the number of columns in the sparse SUNMatrix.
F2003 Name: This function is callable as FSUNSparseMatrix_Columns when using the Fortran 2003 interface module.

[SUNSparseMatrix_NNZ]
Prototype: sunindextype SUNSparseMatrix_NNZ(SUNMatrix A)
Description: This function returns the number of entries allocated for nonzero storage for the sparse matrix SUNMatrix.
F2003 Name: This function is callable as FSUNSparseMatrix_NNZ when using the Fortran 2003 interface module.

[SUNSparseMatrix_NP]
Prototype: sunindextype SUNSparseMatrix_NP(SUNMatrix A)
Description: This function returns the number of columns/rows for the sparse SUNMatrix, depending on whether the matrix uses CSC/CSR format, respectively. The indexptrs array has NP+1 entries.
F2003 Name: This function is callable as FSUNSparseMatrix_NP when using the Fortran 2003 interface module.

[SUNSparseMatrix_SparseType]
Prototype: int SUNSparseMatrix_SparseType(SUNMatrix A)
Description: This function returns the storage type (CSR_MAT or CSC_MAT) for the sparse SUNMatrix.
F2003 Name: This function is callable as FSUNSparseMatrix_SparseType when using the Fortran 2003 interface module.

[SUNSparseMatrix_Data]
Prototype: realtype* SUNSparseMatrix_Data(SUNMatrix A)
Description: This function returns a pointer to the data array for the sparse SUNMatrix.
F2003 Name: This function is callable as FSUNSparseMatrix_Data when using the Fortran 2003 interface module.

[SUNSparseMatrix_IndexValues]
Prototype: sunindextype* SUNSparseMatrix_IndexValues(SUNMatrix A)
Description: This function returns a pointer to index value array for the sparse SUNMatrix: for CSR format this is the column index for each nonzero entry, for CSC format this is the row index for each nonzero entry.
F2003 Name: This function is callable as FSUNSparseMatrix_IndexValues when using the Fortran 2003 interface module.
Description of the SUNMatrix module

**Prototype**

sunindextype* SUNSparseMatrix_IndexPointers(SUNMatrix A)

**Description**

This function returns a pointer to the index pointer array for the sparse SUNMatrix. For CSR format this is the location of the first entry of each row in the data and indexvalues arrays, for CSC format this is the location of the first entry of each column.

**F2003 Name**

This function is callable as FSUNSparseMatrix_IndexPointers when using the Fortran 2003 interface module.

Within the SUNMatMatvec_Sparse routine, internal consistency checks are performed to ensure that the matrix is called with consistent nvector implementations. These are currently limited to: nvector_serial, nvector_openmp, nvector_pthreads, and nvector_CUDA when using managed memory. As additional compatible vector implementations are added to Sundials, these will be included within this compatibility check.

### 9.5.3 SUNMatrix_Sparse Fortran interfaces

The SUNMatrix_sparse module provides a Fortran 2003 module as well as Fortran 77 style interface functions for use from Fortran applications.

#### FORTRAN 2003 interface module

The fsunmatrix_sparse_mod Fortran module defines interfaces to most SUNMATRIX_SPARSE C functions using the intrinsic iso_c_binding module which provides a standardized mechanism for interop-erating with C. As noted in the function descriptions above, the interface functions are named after the corresponding C function, but with a leading ‘F’. For example, the function SUNSparseMatrix is interfaced as FSUNSparseMatrix.

The Fortran 2003 SUNMATRIX_SPARSE interface module can be accessed with the use statement, i.e. use fsunmatrix_sparse_mod, and linking to the library libsundials_fsunmatrixsparse_mod.lib in addition to the C library. For details on where the library and module file fsunmatrix_sparse_mod.mod are installed see Appendix A. We note that the module is accessible from the Fortran 2003 Sundials integrators without separately linking to the libsundials_fsunmatrixsparse_mod library.

#### FORTRAN 77 interface functions

For solvers that include a Fortran interface module, the SUNMATRIX_SPARSE module also includes the Fortran-callable function FSUNSparseMatInit(code, M, N, NNZ, sparsetype, ier) to initialize this SUNMATRIX_SPARSE module for a given Sundials solver. Here code is an integer input for the solver id (1 for CVODE, 2 for IDA, 3 for KINSOL, 4 for ARKODE); M, N and NNZ are the corresponding sparse matrix construction arguments (declared to match C type long int); sparsetype is an integer flag indicating the sparse storage type (0 for CSC, 1 for CSR); and ier is an error return flag equal to 0 for success and -1 for failure. Each of code, sparsetype and ier are declared so as to match C type int. Additionally, when using ARKODE with a non-identity mass matrix, the Fortran-callable function FSUNSparseMassMatInit(M, N, NNZ, sparsetype, ier) initializes this SUNMATRIX_SPARSE module for storing the mass matrix.

### 9.6 The SUNMatrix_SLUNRloc implementation

The SUNMATRIX_SLUNRLOC implementation of the SUNMATRIX module provided with Sundials is an adapter for the SuperMatrix structure provided by the SuperLU_DIST sparse matrix factorization and solver library written by X. Sherry Li [3, 25, 40, 41]. It is designed to be used with the SUNLINSOL_SUPERLU_DIST linear solver discussed in Section 10.10. To this end, it defines the content field of SUNMatrix to be the following structure:
structure _SUNMatrixContent_SLUNRloc {
    bool own_data;
    gridinfo_t *grid;
    sunindextype *row_to_proc;
    pdgsmv_comm_t *gsmv_comm;
    SuperMatrix *A_super;
    SuperMatrix *ACS_super;
};

A more complete description of the this content field is given below:

- **own_data** - a flag which indicates if the SUNMatrix is responsible for freeing A_super
- **grid** - pointer to the SuperLU_DIST structure that stores the 2D process grid
- **row_to_proc** - a mapping between the rows in the matrix and the process it resides on; will be NULL until the SUNMatMatvecSetup routine is called
- **gsmv_comm** - pointer to the SuperLU_DIST structure that stores the communication information needed for matrix-vector multiplication; will be NULL until the SUNMatMatvecSetup routine is called
- **A_super** - pointer to the underlying SuperLU_DIST SuperMatrix with Stype = SLU_NR_loc, Dtype = SLU_D, Mtype = SLU_GE; must have the full diagonal present to be used with SUNMatScaleAddI routine
- **ACS_super** - a column-sorted version of the matrix needed to perform matrix-vector multiplication; will be NULL until the routine SUNMatMatvecSetup routine is called

The header file to include when using this module is sunmatrix/sunmatrix_slunrloc.h. The installed module library to link to is libsundials_sunmatrixslunrloc.lib where.lib is typically .so for shared libraries and .a for static libraries.

### 9.6.1 SUNMatrix_SLUNRloc functions

The module SUNMATRIX_SLUNRLOC provides the following user-callable routines:

- **SUNMatrix_SLUNRloc**

  **Call:** A = SUNMatrix_SLUNRloc(Asuper, grid);

  **Description:** The function SUNMatrix_SLUNRloc creates and allocates memory for a SUNMATRIX_SLUNRLOC object.

  **Arguments:**
  - Asuper (SuperMatrix*) a fully-allocated SuperLU_DIST SuperMatrix that the SUNMatrix will wrap; must have Stype = SLU_NR_loc, Dtype = SLU_D, Mtype = SLU_GE to be compatible
  - grid (gridinfo_t*) the initialized SuperLU_DIST 2D process grid structure

  **Return value:** a SUNMatrix object if Asuper is compatible else NULL

- **SUNMatrix_SLUNRloc_Print**

  **Call:** SUNMatrix_SLUNRloc_Print(A, fp);

  **Description:** The function SUNMatrix_SLUNRloc_Print prints the underlying SuperMatrix content.

  **Arguments:**
  - A (SUNMatrix) the matrix to print
  - fp (FILE) the file pointer used for printing

  **Return value:** void

- **Notes:**
Description of the SUNMatrix module

**SUNMatrix_SLUNRloc_SuperMatrix**

Call: \[ \text{Asuper} = \text{SUNMatrix\_SLUNRloc\_SuperMatrix}(A); \]

Description: The function `SUNMatrix_SLUNRloc_SuperMatrix` provides access to the underlying `SuperLU\_DIST SuperMatrix` of `A`.

Arguments: `A` (SUNMatrix) the matrix to access

Return value: `SuperMatrix*`

Notes:

**SUNMatrix_SLUNRloc_ProcessGrid**

Call: \[ \text{grid} = \text{SUNMatrix\_SLUNRloc\_ProcessGrid}(A); \]

Description: The function `SUNMatrix_SLUNRloc_ProcessGrid` provides access to the `SuperLU\_DIST gridinfo_t` structure associated with `A`.

Arguments: `A` (SUNMatrix) the matrix to access

Return value: `gridinfo_t*`

Notes:

**SUNMatrix_SLUNRloc_OwnData**

Call: \[ \text{does\_own\_data} = \text{SUNMatrix\_SLUNRloc\_OwnData}(A); \]

Description: The function `SUNMatrix_SLUNRloc_OwnData` returns true if the SUNMatrix object is responsible for freeing `A\_super`, otherwise it returns false.

Arguments: `A` (SUNMatrix) the matrix to access

Return value: `boolean` type

Notes:

The SUNMATRIX_SLUNRLOC module defines implementations of all generic SUNMatrix operations listed in Section 9.1.1:

- `SUNMatGetID_SLUNRloc` - returns SUNMATRIX_SLUNRLOC
- `SUNMatClone_SLUNRloc`
- `SUNMatDestroy_SLUNRloc`
- `SUNMatSpace_SLUNRloc` - this only returns information for the storage within the matrix interface, i.e. storage for `row_to_proc`
- `SUNMatZero_SLUNRloc`
- `SUNMatCopy_SLUNRloc`
- `SUNMatScaleAdd_SLUNRloc` - performs \( A = cA + B \), but \( A \) and \( B \) must have the same sparsity pattern
- `SUNMatScaleAddI_SLUNRloc` - performs \( A = cA + I \), but the diagonal of \( A \) must be present
- `SUNMatMatvecSetup_SLUNRloc` - initializes the SuperLU DIST parallel communication structures needed to perform a matrix-vector product; only needs to be called before the first call to `SUNMatMatvec` or if the matrix changed since the last setup
- `SUNMatMatvec_SLUNRloc`

The SUNMATRIX_SLUNRLOC module requires that the complete diagonal, i.e. nonzeros and zeros, is present in order to use the `SUNMatScaleAddI` operation.
Chapter 10

Description of the SUNLinearSolver module

For problems that involve the solution of linear systems of equations, the SUNDIALS packages operate using generic linear solver modules defined through the SUNLINSOL API. This allows SUNDIALS packages to utilize any valid SUNLINSOL implementation that provides a set of required functions. These functions can be divided into three categories. The first are the core linear solver functions. The second group consists of “set” routines to supply the linear solver object with functions provided by the SUNDIALS package, or for modification of solver parameters. The last group consists of “get” routines for retrieving artifacts (statistics, residual vectors, etc.) from the linear solver. All of these functions are defined in the header file sundials/sundials_linear solver.h.

The implementations provided with SUNDIALS work in coordination with the SUNDIALS generic NVVECTOR and SUNMATRIX modules to provide a set of compatible data structures and solvers for the solution of linear systems using direct or iterative (matrix-based or matrix-free) methods. Moreover, advanced users can provide a customized SUNLinearSolver implementation to any SUNDIALS package, particularly in cases where they provide their own NVVECTOR and/or SUNMATRIX modules.

Historically, the SUNDIALS packages have been designed to specifically leverage the use of either direct linear solvers or matrix-free, scaled, preconditioned, iterative linear solvers. However, matrix-based iterative linear solvers are also supported.

The iterative linear solvers packaged with SUNDIALS leverage scaling and preconditioning, as applicable, to balance error between solution components and to accelerate convergence of the linear solver. To this end, instead of solving the linear system \(Ax = b\) directly, these apply the underlying iterative algorithm to the transformed system

\[
\tilde{A}\tilde{x} = \tilde{b}
\]

where

\[
\tilde{A} = S_1 P_1^{-1} A P_2^{-1} S_2^{-1},
\]

\[
\tilde{b} = S_1 P_1^{-1} b,
\]

\[
\tilde{x} = S_2 P_2 x,
\]

and where

- \(P_1\) is the left preconditioner,
- \(P_2\) is the right preconditioner,
- \(S_1\) is a diagonal matrix of scale factors for \(P_1^{-1} b\),
- \(S_2\) is a diagonal matrix of scale factors for \(P_2 x\).
The scaling matrices are chosen so that \( S_1 P^{-1} b \) and \( S_2 P x \) have dimensionless components. If preconditioning is done on the left only \((P_2 = I)\), by a matrix \( P \), then \( S_2 \) must be a scaling for \( x \), while \( S_1 \) is a scaling for \( P^{-1} b \), and so may also be taken as a scaling for \( x \). Similarly, if preconditioning is done on the right only \((P_1 = I \text{ and } P_2 = P)\), then \( S_1 \) must be a scaling for \( b \), while \( S_2 \) is a scaling for \( P x \), and may also be taken as a scaling for \( b \).

SUNDIALS packages request that iterative linear solvers stop based on the 2-norm of the scaled preconditioned residual meeting a prescribed tolerance

\[
\left\| \tilde{b} - \tilde{A} \tilde{x} \right\|_2 < \text{tol}.
\]

When provided an iterative SUNLINSOL implementation that does not support the scaling matrices \( S_1 \) and \( S_2 \), SUNDIALS’ packages will adjust the value of \text{tol} accordingly (see §10.4.2 for more details). In this case, they instead request that iterative linear solvers stop based on the criteria

\[
\left\| P_1^{-1} b - P_1^{-1} A x \right\|_2 < \text{tol}.
\]

We note that the corresponding adjustments to \text{tol} in this case are non-optimal, in that they cannot balance error between specific entries of the solution \( x \), only the aggregate error in the overall solution vector.

We further note that not all of the SUNDIALS-provided iterative linear solvers support the full range of the above options (e.g., separate left/right preconditioning), and that some of the SUNDIALS packages only utilize a subset of these options. Further details on these exceptions are described in the documentation for each SUNLINSOL implementation, or for each SUNDIALS package.

For users interested in providing their own SUNLINSOL module, the following section presents the SUNLINSOL API and its implementation beginning with the definition of SUNLINSOL functions in sections 10.1.1 – 10.1.3. This is followed by the definition of functions supplied to a linear solver implementation in section 10.1.4. A table of linear solver return codes is given in section 10.1.5. The SUNLinearSolver type and the generic SUNLINSOL module are defined in section 10.1.6. The section 10.2 discusses compatibility between the SUNDIALS-provided SUNLINSOL modules and SUNMATRIX modules. Section 10.3 lists the requirements for supplying a custom SUNLINSOL module and discusses some intended use cases. Users wishing to supply their own SUNLINSOL module are encouraged to use the SUNLINSOL implementations provided with SUNDIALS as a template for supplying custom linear solver modules. The SUNLINSOL functions required by this SUNDIALS package as well as other package specific details are given in section 10.4. The remaining sections of this chapter present the SUNLINSOL modules provided with SUNDIALS.

## 10.1 The SUNLinearSolver API

The SUNLINSOL API defines several linear solver operations that enable SUNDIALS packages to utilize any SUNLINSOL implementation that provides the required functions. These functions can be divided into three categories. The first are the core linear solver functions. The second group of functions consists of set routines to supply the linear solver with functions provided by the SUNDIALS time integrators and to modify solver parameters. The final group consists of get routines for retrieving linear solver statistics. All of these functions are defined in the header file `sundials/sundials_linearsolver.h`.

### 10.1.1 SUNLinearSolver core functions

The core linear solver functions consist of two required functions to get the linear solver type (`SUNLinSolGetType`) and solve the linear system \( Ax = b \) (`SUNLinSolSolve`). The remaining functions are for getting the solver ID (`SUNLinSolGetID`), initializing the linear solver object once all solver-specific options have been set (`SUNLinSolInitialize`), setting up the linear solver object to utilize an updated matrix \( A \) (`SUNLinSolSetup`), and for destroying the linear solver object (`SUNLinSolFree`) are optional.
10.1 The SUNLinearSolver API

SUNLinSolGetType

Call \[ \text{type} = \text{SUNLinSolGetType}(LS); \]

Description The required function SUNLinSolGetType returns the type identifier for the linear solver LS. It is used to determine the solver type (direct, iterative, or matrix-iterative) from the abstract SUNLinearSolver interface.

Arguments LS (SUNLinearSolver) a SUNLINSOL object.

Return value The return value type (of type int) will be one of the following:

- SUNLINEARSOLVER_DIRECT – 0, the SUNLINSOL module requires a matrix, and computes an ‘exact’ solution to the linear system defined by that matrix.
- SUNLINEARSOLVER_ITERATIVE – 1, the SUNLINSOL module does not require a matrix (though one may be provided), and computes an inexact solution to the linear system using a matrix-free iterative algorithm. That is it solves the linear system defined by the package-supplied ATimes routine (see SUNLinSolSetATimes below), even if that linear system differs from the one encoded in the matrix object (if one is provided). As the solver computes the solution only inexactly (or may diverge), the linear solver should check for solution convergence/accuracy as appropriate.
- SUNLINEARSOLVER_MATRIX_ITERATIVE – 2, the SUNLINSOL module requires a matrix, and computes an inexact solution to the linear system defined by that matrix using an iterative algorithm. That is it solves the linear system defined by the matrix object even if that linear system differs from that encoded by the package-supplied ATimes routine. As the solver computes the solution only inexactly (or may diverge), the linear solver should check for solution convergence/accuracy as appropriate.

Notes See section 10.3.1 for more information on intended use cases corresponding to the linear solver type.

F2003 Name FSUNLinSolGetType

SUNLinSolGetID

Call \[ \text{id} = \text{SUNLinSolGetID}(LS); \]

Description The optional function SUNLinSolGetID returns the identifier for the linear solver LS.

Arguments LS (SUNLinearSolver) a SUNLINSOL object.

Return value The return value id (of type int) will be a non-negative value defined by the enumeration SUNLinearSolver_ID.

Notes It is recommended that a user-supplied SUNLinearSolver return the SUNLINEARSOLVER_CUSTOM identifier.

F2003 Name FSUNLinSolGetID

SUNLinSolInitialize

Call \[ \text{retval} = \text{SUNLinSolInitialize}(LS); \]

Description The optional function SUNLinSolInitialize performs linear solver initialization (assuming that all solver-specific options have been set).

Arguments LS (SUNLinearSolver) a SUNLINSOL object.

Return value This should return zero for a successful call, and a negative value for a failure, ideally returning one of the generic error codes listed in Table 10.1.

F2003 Name FSUNLinSolInitialize
Description of the SUNLinearSolver module

**SUNLinSolSetup**

Call `retval = SUNLinSolSetup(LS, A);`

Description The *optional* function `SUNLinSolSetup` performs any linear solver setup needed, based on an updated system `sunmatrix` `A`. This may be called frequently (e.g., with a full Newton method) or infrequently (for a modified Newton method), based on the type of integrator and/or nonlinear solver requesting the solves.

Arguments
- `LS` (*SUNLinearSolver*) a SUNLINSOL object.
- `A` (*SUNMatrix*) a SUNMATRIX object.

Return value This should return zero for a successful call, a positive value for a recoverable failure and a negative value for an unrecoverable failure, ideally returning one of the generic error codes listed in Table 10.1.

F2003 Name `FSUNLinSolSetup`

**SUNLinSolSolve**

Call `retval = SUNLinSolSolve(LS, A, x, b, tol);`

Description The *required* function `SUNLinSolSolve` solves a linear system `Ax = b`.

Arguments
- `LS` (*SUNLinearSolver*) a SUNLINSOL object.
- `A` (*SUNMatrix*) a SUNMATRIX object.
- `x` (*N_Vector*) a NVECTOR object containing the initial guess for the solution of the linear system, and the solution to the linear system upon return.
- `b` (*N_Vector*) a NVECTOR object containing the linear system right-hand side.
- `tol` (*realtype*) the desired linear solver tolerance.

Return value This should return zero for a successful call, a positive value for a recoverable failure and a negative value for an unrecoverable failure, ideally returning one of the generic error codes listed in Table 10.1.

Notes
- **Direct solvers**: can ignore the `tol` argument.
- **Matrix-free solvers**: (those that identify as `SUNLINEARSOLVER_ITERATIVE`) can ignore the SUNMATRIX input `A`, and should instead rely on the matrix-vector product function supplied through the routine `SUNLinSolSetATimes`.
- **Iterative solvers**: (those that identify as `SUNLINEARSOLVER_ITERATIVE` or `SUNLINEARSOLVER_MATRIX_ITERATIVE`) should attempt to solve to the specified tolerance `tol` in a weighted 2-norm. If the solver does not support scaling then it should just use a 2-norm.

F2003 Name `FSUNLinSolSolve`

**SUNLinSolFree**

Call `retval = SUNLinSolFree(LS);`

Description The *optional* function `SUNLinSolFree` frees memory allocated by the linear solver.

Arguments
- `LS` (*SUNLinearSolver*) a SUNLINSOL object.

Return value This should return zero for a successful call and a negative value for a failure.

F2003 Name `FSUNLinSolFree`
10.1.2 SUNLinearSolver set functions

The following set functions are used to supply linear solver modules with functions defined by the SUNDIALS packages and to modify solver parameters. Only the routine for setting the matrix-vector product routine is required, and that is only for matrix-free linear solver modules. Otherwise, all other set functions are optional. SUNLINSOL implementations that do not provide the functionality for any optional routine should leave the corresponding function pointer NULL instead of supplying a dummy routine.

**SUNLinSolSetATimes**

Call: \( \text{retval} = \text{SUNLinSolSetATimes}(\text{LS}, \text{A.data}, \text{ATimes}) \);

Description: The function SUNLinSolSetATimes is required for matrix-free linear solvers; otherwise it is optional.

This routine provides an ATimesFn function pointer, as well as a void* pointer to a data structure used by this routine, to a linear solver object. SUNDIALS packages will call this function to set the matrix-vector product function to either a solver-provided difference-quotient via vector operations or a user-supplied solver-specific routine.

Arguments:
- \( \text{LS} \) (SUNLinearSolver) a SUNLINSOL object.
- \( \text{A.data} \) (void*) data structure passed to ATimes.
- \( \text{ATimes} \) (ATimesFn) function pointer implementing the matrix-vector product routine.

Return value: This routine should return zero for a successful call, and a negative value for a failure, ideally returning one of the generic error codes listed in Table 10.1.

F2003 Name: FSUNLinSolSetATimes

**SUNLinSolSetPreconditioner**

Call: \( \text{retval} = \text{SUNLinSolSetPreconditioner}(\text{LS}, \text{Pdata}, \text{Pset}, \text{Psol}) \);

Description: The optional function SUNLinSolSetPreconditioner provides PSetupFn and PSolveFn function pointers that implement the preconditioner solves \( P^{-1}_1 \) and \( P^{-1}_2 \) from equations (10.1)-(10.2). This routine will be called by a SUNDIALS package, which will provide translation between the generic Pset and Psol calls and the package- or user-supplied routines.

Arguments:
- \( \text{LS} \) (SUNLinearSolver) a SUNLINSOL object.
- \( \text{Pdata} \) (void*) data structure passed to both Pset and Psol.
- \( \text{Pset} \) (PSetupFn) function pointer implementing the preconditioner setup.
- \( \text{Psol} \) (PSolveFn) function pointer implementing the preconditioner solve.

Return value: This routine should return zero for a successful call, and a negative value for a failure, ideally returning one of the generic error codes listed in Table 10.1.

F2003 Name: FSUNLinSolSetPreconditioner

**SUNLinSolSetScalingVectors**

Call: \( \text{retval} = \text{SUNLinSolSetScalingVectors}(\text{LS}, \text{s1}, \text{s2}) \);

Description: The optional function SUNLinSolSetScalingVectors provides left/right scaling vectors for the linear system solve. Here, \( s_1 \) and \( s_2 \) are NVVECTOR of positive scale factors containing the diagonal of the matrices \( S_1 \) and \( S_2 \) from equations (10.1)-(10.2), respectively. Neither of these vectors need to be tested for positivity, and a NULL argument for either indicates that the corresponding scaling matrix is the identity.

Arguments:
- \( \text{LS} \) (SUNLinearSolver) a SUNLINSOL object.
- \( \text{s1} \) (NV_VECTOR) diagonal of the matrix \( S_1 \)
s2 (N_Vector) diagonal of the matrix $S_2$

Return value This routine should return zero for a successful call, and a negative value for a failure, ideally returning one of the generic error codes listed in Table 10.1.

F2003 Name FSUNLinSolSetScalingVectors

10.1.3 SUNLinearSolver get functions

The following get functions allow SUNDIALS packages to retrieve results from a linear solve. All routines are optional.

**SUNLinSolNumIters**

Call $\text{its} = \text{SUNLinSolNumIters}(\text{LS})$;

Description The optional function SUNLinSolNumIters should return the number of linear iterations performed in the last ‘solve’ call.

Arguments LS (SUNLinearSolver) a SUNLINSOL object.

Return value int containing the number of iterations

F2003 Name FSUNLinSolNumIters

**SUNLinSolResNorm**

Call $\text{rnorm} = \text{SUNLinSolResNorm}(\text{LS})$;

Description The optional function SUNLinSolResNorm should return the final residual norm from the last ‘solve’ call.

Arguments LS (SUNLinearSolver) a SUNLINSOL object.

Return value realtype containing the final residual norm

F2003 Name FSUNLinSolResNorm

**SUNLinSolResid**

Call $\text{rvec} = \text{SUNLinSolResid}(\text{LS})$;

Description If an iterative method computes the preconditioned initial residual and returns with a successful solve without performing any iterations (i.e., either the initial guess or the preconditioner is sufficiently accurate), then this optional routine may be called by the SUNDIALS package. This routine should return the NVECTOR containing the preconditioned initial residual vector.

Arguments LS (SUNLinearSolver) a SUNLINSOL object.

Return value N_Vector containing the final residual vector

Notes Since N_Vector is actually a pointer, and the results are not modified, this routine should not require additional memory allocation. If the SUNLINSOL object does not retain a vector for this purpose, then this function pointer should be set to NULL in the implementation.

F2003 Name FSUNLinSolResid

**SUNLinSolLastFlag**

Call $\text{lflag} = \text{SUNLinSolLastFlag}(\text{LS})$;

Description The optional function SUNLinSolLastFlag should return the last error flag encountered within the linear solver. This is not called by the SUNDIALS packages directly; it allows the user to investigate linear solver issues after a failed solve.
Arguments  LS (SUNLinearSolver) a SUNLINSOL object.

Return value  sunindextype containing the most recent error flag

F2003 Name  FSUNLinSolLastFlag

SUNLinSolSpace

Call  retval = SUNLinSolSpace(LS, &lrw, &liw);

Description  The optional function SUNLinSolSpace should return the storage requirements for the linear solver LS.

Arguments  LS (SUNLinearSolver) a SUNLINSOL object.

lrw (long int*) the number of realtype words stored by the linear solver.

liw (long int*) the number of integer words stored by the linear solver.

Return value  This should return zero for a successful call, and a negative value for a failure, ideally returning one of the generic error codes listed in Table 10.1.

Notes  This function is advisory only, for use in determining a user’s total space requirements.

F2003 Name  FSUNLinSolSpace

10.1.4 Functions provided by SUNDIALS packages

To interface with the SUNLINSOL modules, the SUNDIALS packages supply a variety of routines for evaluating the matrix-vector product, and setting up and applying the preconditioner. These package-provided routines translate between the user-supplied ODE, DAE, or nonlinear systems and the generic interfaces to the linear systems of equations that result in their solution. The types for functions provided to a SUNLINSOL module are defined in the header file sundials/sundials_iterative.h, and are described below.

ATimesFn

Definition  typedef int (*ATimesFn)(void *A_data, N_Vector v, N_Vector z);

Purpose  These functions compute the action of a matrix on a vector, performing the operation \( z = Av \). Memory for \( z \) should already be allocated prior to calling this function. The vector \( v \) should be left unchanged.

Arguments  A_data is a pointer to client data, the same as that supplied to SUNLinSolSetATimes.

v is the input vector to multiply.

z is the output vector computed.

Return value  This routine should return 0 if successful and a non-zero value if unsuccessful.

PSetupFn

Definition  typedef int (*PSetupFn)(void *P_data)

Purpose  These functions set up any requisite problem data in preparation for calls to the corresponding PSolveFn.

Arguments  P_data is a pointer to client data, the same pointer as that supplied to the routine SUNLinSolSetPreconditioner.

Return value  This routine should return 0 if successful and a non-zero value if unsuccessful.
**PSolveFn**

**Definition**
```
typedef int (*PSolveFn)(void *P_data, N_Vector r, N_Vector z, realtype tol, int lr)
```

**Purpose**
These functions solve the preconditioner equation $Pz = r$ for the vector $z$. Memory for $z$ should already be allocated prior to calling this function. The parameter $P_data$ is a pointer to any information about $P$ which the function needs in order to do its job (set up by the corresponding $PSetupFn$). The parameter $lr$ is input, and indicates whether $P$ is to be taken as the left preconditioner or the right preconditioner: $lr = 1$ for left and $lr = 2$ for right. If preconditioning is on one side only, $lr$ can be ignored. If the preconditioner is iterative, then it should strive to solve the preconditioner equation so that

$$\|Pz - r\|_{\text{wrms}} < tol$$

where the weight vector for the WRMS norm may be accessed from the main package memory structure. The vector $r$ should not be modified by the $PSolveFn$.

**Arguments**
- $P_data$ is a pointer to client data, the same pointer as that supplied to the routine $SUNLinSolSetPreconditioner$.
- $r$ is the right-hand side vector for the preconditioner system.
- $z$ is the solution vector for the preconditioner system.
- $tol$ is the desired tolerance for an iterative preconditioner.
- $lr$ is flag indicating whether the routine should perform left (1) or right (2) preconditioning.

**Return value**
This routine should return 0 if successful and a non-zero value if unsuccessful. On a failure, a negative return value indicates an unrecoverable condition, while a positive value indicates a recoverable one, in which the calling routine may reattempt the solution after updating preconditioner data.

### 10.1.5 SUNLinearSolver return codes

The functions provided to SUNLINSOL modules by each SUNDIALS package, and functions within the SUNDIALS-provided SUNLINSOL implementations utilize a common set of return codes, shown in Table 10.1. These adhere to a common pattern: 0 indicates success, a positive value corresponds to a recoverable failure, and a negative value indicates a non-recoverable failure. Aside from this pattern, the actual values of each error code are primarily to provide additional information to the user in case of a linear solver failure.

Table 10.1: Description of the SUNLinearSolver error codes

<table>
<thead>
<tr>
<th>Name</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>SUNLS_SUCCESS</td>
<td>0</td>
<td>successful call or converged solve</td>
</tr>
<tr>
<td>SUNLS_MEM_NULL</td>
<td>-801</td>
<td>the memory argument to the function is NULL</td>
</tr>
<tr>
<td>SUNLS_ILL_INPUT</td>
<td>-802</td>
<td>an illegal input has been provided to the function</td>
</tr>
<tr>
<td>SUNLS_MEM_FAIL</td>
<td>-803</td>
<td>failed memory access or allocation</td>
</tr>
<tr>
<td>SUNLS_ATIMES_FAIL_UNREC</td>
<td>-804</td>
<td>an unrecoverable failure occurred in the $ATimes$ routine</td>
</tr>
</tbody>
</table>

*continued on next page*
### 10.1 The SUNLinearSolver API

<table>
<thead>
<tr>
<th>Name</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>SUNLS_PSET_FAIL_UNREC</td>
<td>-805</td>
<td>an unrecoverable failure occurred in the Pset routine</td>
</tr>
<tr>
<td>SUNLS_PSOLVE_FAIL_UNREC</td>
<td>-806</td>
<td>an unrecoverable failure occurred in the Psolve routine</td>
</tr>
<tr>
<td>SUNLS_PACKAGE_FAIL_UNREC</td>
<td>-807</td>
<td>an unrecoverable failure occurred in an external linear solver package</td>
</tr>
<tr>
<td>SUNLS_GS_FAIL</td>
<td>-808</td>
<td>a failure occurred during Gram-Schmidt orthogonalization</td>
</tr>
<tr>
<td>(SUNLINSOL_SPGMR/SUNLINSOL_SPFGMR)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SUNLS_QRSOL_FAIL</td>
<td>-809</td>
<td>a singular $R$ matrix was encountered in a QR factorization</td>
</tr>
<tr>
<td>(SUNLINSOL_SPGMR/SUNLINSOL_SPFGMR)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SUNLS_RES_REduced</td>
<td>801</td>
<td>an iterative solver reduced the residual, but did not converge to the desired tolerance</td>
</tr>
<tr>
<td>SUNLS_CONV_FAIL</td>
<td>802</td>
<td>an iterative solver did not converge (and the residual was not reduced)</td>
</tr>
<tr>
<td>SUNLS_ATIMES_FAIL_REC</td>
<td>803</td>
<td>a recoverable failure occurred in the ATimes routine</td>
</tr>
<tr>
<td>SUNLS_PSET_FAIL_REC</td>
<td>804</td>
<td>a recoverable failure occurred in the Pset routine</td>
</tr>
<tr>
<td>SUNLS_PSOLVE_FAIL_REC</td>
<td>805</td>
<td>a recoverable failure occurred in the Psolve routine</td>
</tr>
<tr>
<td>SUNLS_PACKAGE_FAIL_REC</td>
<td>806</td>
<td>a recoverable failure occurred in an external linear solver package</td>
</tr>
<tr>
<td>SUNLS_QRFACT_FAIL</td>
<td>807</td>
<td>a singular matrix was encountered during a QR factorization</td>
</tr>
<tr>
<td>(SUNLINSOL_SPGMR/SUNLINSOL_SPFGMR)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SUNLS_LUFUNCT_FAIL</td>
<td>808</td>
<td>a singular matrix was encountered during a LU factorization</td>
</tr>
<tr>
<td>(SUNLINSOL_DENSE/SUNLINSOL_BAND)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

#### 10.1.6 The generic SUNLinearSolver module

SUNDIALS packages interact with specific SUNLINSOL implementations through the generic SUNLINSOL module on which all other SUNLINSOL implementations are built. The SUNLinearSolver type is a pointer to a structure containing an implementation-dependent content field, and an ops field. The type SUNLinearSolver is defined as

```c
typedef struct _generic_SUNLinearSolver *SUNLinearSolver;

struct _generic_SUNLinearSolver {
    void *content;
    struct _generic_SUNLinearSolver_Ops *ops;
};
```

where the _generic_SUNLinearSolver_Ops structure is a list of pointers to the various actual linear solver operations provided by a specific implementation. The _generic_SUNLinearSolver_Ops structure is defined as

```c
struct _generic_SUNLinearSolver_Ops {
    SUNLinearSolver_Type (*gettype)(SUNLinearSolver);
    SUNLinearSolver_ID (*getid)(SUNLinearSolver);
    int (*setatimes)(SUNLinearSolver, void*, ATimesFn);
    int (*setpreconditioner)(SUNLinearSolver, void*,
                              PSetupFn, PSolveFn);
    int (*setscalingvectors)(SUNLinearSolver, N_Vector, N_Vector);
    int (*initialize)(SUNLinearSolver);
};
```
The generic SUNLINSOL module defines and implements the linear solver operations defined in Sections 10.1.1-10.1.3. These routines are in fact only wrappers to the linear solver operations defined by a particular SUNLINSOL implementation, which are accessed through the ops field of the SUNLinearSolver structure. To illustrate this point we show below the implementation of a typical linear solver operation from the generic SUNLINSOL module, namely SUNLinSolInitialize, which initializes a SUNLINSOL object for use after it has been created and configured, and returns a flag denoting a successful/failed operation:

```c
int SUNLinSolInitialize(SUNLinearSolver S)
{
    return ((int) S->ops->initialize(S));
}
```

The Fortran 2003 interface provides a bind(C) derived-type for the _generic_SUNLinearSolver and the _generic_SUNLinearSolver_Ops structures. Their definition is given below.

```fortran
module SUNLinearSolver_Ops

    type, bind(C), public :: SUNLinearSolver_Ops
    type(C_PTR), public :: gettype
    type(C_PTR), public :: setatimes
    type(C_PTR), public :: setpreconditioner
    type(C_PTR), public :: setscalingvectors
    type(C_PTR), public :: initialize
    type(C_PTR), public :: setup
    type(C_PTR), public :: solve
    type(C_PTR), public :: numiters
    type(C_PTR), public :: resnorm
    type(C_PTR), public :: lastflag
    type(C_PTR), public :: space
    type(C_PTR), public :: resid
    type(C_PTR), public :: free
end module SUNLinearSolver_Ops
```

### 10.2 Compatibility of SUNLinearSolver modules

We note that not all SUNLINSOL types are compatible with all SUNMATRIX and NVECTOR types provided with SUNDIALS. In Table 10.2 we show the matrix-based linear solvers available as SUNLINSOL modules, and the compatible matrix implementations. Recall that Table 1.4 shows the compatibility between all SUNLINSOL modules and vector implementations.
10.3 Implementing a custom SUNLinearSolver module

A particular implementation of the SUNLINSOL module must:

- Specify the content field of the SUNLinearSolver object.

- Define and implement a minimal subset of the linear solver operations. See the section 10.4 to determine which SUNLINSOL operations are required for this SUNDIALS package.

  Note that the names of these routines should be unique to that implementation in order to permit using more than one SUNLINSOL module (each with different SUNLinearSolver internal data representations) in the same code.

- Define and implement user-callable constructor and destructor routines to create and free a SUNLinearSolver with the new content field and with ops pointing to the new linear solver operations.

We note that the function pointers for all unsupported optional routines should be set to NULL in the ops structure. This allows the SUNDIALS package that is using the SUNLINSOL object to know that the associated functionality is not supported.

To aid in the creation of custom SUNLINSOL modules the generic SUNLINSOL module provides the utility functions SUNLinSolNewEmpty and SUNLinSolFreeEmpty. When used in custom SUNLINSOL constructors the function SUNLinSolNewEmpty will ease the introduction of any new optional linear solver operations to the SUNLINSOL API by ensuring only required operations need to be set.

### SUNLinSolNewEmpty

**Call**

```c
LS = SUNLinSolNewEmpty();
```

**Description**
The function SUNLinSolNewEmpty allocates a new generic SUNLINSOL object and initializes its content pointer and the function pointers in the operations structure to NULL.

**Arguments**
None

**Return value**
This function returns a SUNLinearSolver object. If an error occurs when allocating the object, then this routine will return NULL.

**F2003 Name**
FSUNLinSolNewEmpty

---

### SUNLinSolFreeEmpty

**Call**

```c
SUNLinSolFreeEmpty(LS);
```
Description of the SUNLinearSolver module

Description  This routine frees the generic SUNLinSolFreeEmpty object, under the assumption that any implementation-specific data that was allocated within the underlying content structure has already been freed. It will additionally test whether the ops pointer is NULL, and, if it is not, it will free it as well.

Arguments  LS (SUNLinearSolver)

Return value  None

F2003 Name  FSUNLinSolFreeEmpty

Additionally, a SUNLINSOL implementation may do the following:

- Define and implement additional user-callable “set” routines acting on the SUNLinearSolver, e.g., for setting various configuration options to tune the linear solver to a particular problem.
- Provide additional user-callable “get” routines acting on the SUNLinearSolver object, e.g., for returning various solve statistics.

10.3.1 Intended use cases

The SUNLINSOL (and SUNMATRIX) APIs are designed to require a minimal set of routines to ease interfacing with custom or third-party linear solver libraries. External solvers provide similar routines with the necessary functionality and thus will require minimal effort to wrap within custom SUNMATRIX and SUNLINSOL implementations. Sections 9.2 and 10.4 include a list of the required set of routines that compatible SUNMATRIX and SUNLINSOL implementations must provide. As SUNDIALS packages utilize generic SUNLINSOL modules allowing for user-supplied SUNLinearSolver implementations, there exists a wide range of possible linear solver combinations. Some intended use cases for both the SUNDIALS-provided and user-supplied SUNLINSOL modules are discussed in the following sections.

Direct linear solvers

Direct linear solver modules require a matrix and compute an ‘exact’ solution to the linear system defined by the matrix. Multiple matrix formats and associated direct linear solvers are supplied with SUNDIALS through different SUNMATRIX and SUNLINSOL implementations. SUNDIALS packages strive to amortize the high cost of matrix construction by reusing matrix information for multiple nonlinear iterations. As a result, each package’s linear solver interface recomputes Jacobian information as infrequently as possible.

Alternative matrix storage formats and compatible linear solvers that are not currently provided by, or interfaced with, SUNDIALS can leverage this infrastructure with minimal effort. To do so, a user must implement custom SUNMATRIX and SUNLINSOL wrappers for the desired matrix format and/or linear solver following the APIs described in Chapters 9 and 10. This user-supplied SUNLINSOL module must then self-identify as having SUNLINEARSOLVER_DIRECT type.

Matrix-free iterative linear solvers

Matrix-free iterative linear solver modules do not require a matrix and compute an inexact solution to the linear system defined by the package-supplied ATimes routine. SUNDIALS supplies multiple scaled, preconditioned iterative linear solver (spils) SUNLINSOL modules that support scaling to allow users to handle non-dimensionalization (as best as possible) within each SUNDIALS package and retain variables and define equations as desired in their applications. For linear solvers that do not support left/right scaling, the tolerance supplied to the linear solver is adjusted to compensate (see section 10.4.2 for more details); however, this use case may be non-optimal and cannot handle situations where the magnitudes of different solution components or equations vary dramatically within a single problem.

To utilize alternative linear solvers that are not currently provided by, or interfaced with, SUNDIALS a user must implement a custom SUNLINSOL wrapper for the linear solver following the API described in Chapter 10. This user-supplied SUNLINSOL module must then self-identify as having SUNLINEARSOLVER_ITERATIVE type.
Matrix-based iterative linear solvers (reusing $A$)

Matrix-based iterative linear solver modules require a matrix and compute an inexact solution to the linear system defined by the matrix. This matrix will be updated infrequently and reused across multiple solves to amortize cost of matrix construction. As in the direct linear solver case, only wrappers for the matrix and linear solver in SUNMATRIX and SUNLINSOL implementations need to be created to utilize a new linear solver. This user-supplied SUNLINSOL module must then self-identify as having SUNLINEARSOLVER\_MATRIX\_ITERATIVE type.

At present, SUNDIALS has one example problem that uses this approach for wrapping a structured-grid matrix, linear solver, and preconditioner from the hypre library that may be used as a template for other customized implementations (see examples/arkode/CXX\_parhyp/ark\_heat2D\_hypre.cpp).

Matrix-based iterative linear solvers (current $A$)

For users who wish to utilize a matrix-based iterative linear solver module where the matrix is purely for preconditioning and the linear system is defined by the package-supplied ATimes routine, we envision two current possibilities.

The preferred approach is for users to employ one of the SUNDIALS spils SUNLINSOL implementations (SUNLINSOL\_SPGMR, SUNLINSOL\_SPFGMR, SUNLINSOL\_SPBCGS, SUNLINSOL\_SPTQMR, or SUNLINSOL\_PCG) as the outer solver. The creation and storage of the preconditioner matrix, and interfacing with the corresponding linear solver, can be handled through a package’s preconditioner ‘setup’ and ‘solve’ functionality (see §4.5.8.2) without creating SUNMATRIX and SUNLINSOL implementations. This usage mode is recommended primarily because the SUNDIALS-provided spils modules support the scaling as described above.

A second approach supported by the linear solver APIs is as follows. If the SUNLINSOL implementation is matrix-based, self-identifies as having SUNLINEARSOLVER\_ITERATIVE type, and also provides a non-NULL SUNLinSolSetATimes routine, then each SUNDIALS package will call that routine to attach its package-specific matrix-vector product routine to the SUNLINSOL object. The SUNDIALS package will then call the SUNLINSOL-provided SUNLinSolSetup routine (infrequently) to update matrix information, but will provide current matrix-vector products to the SUNLINSOL implementation through the package-supplied ATimesFn routine.

### 10.4 IDAS SUNLinearSolver interface

Table 10.3 below lists the SUNLINSOL module linear solver functions used within the IDALS interface. As with the SUNMATRIX module, we emphasize that the IDA user does not need to know detailed usage of linear solver functions by the IDA code modules in order to use IDA. The information is presented as an implementation detail for the interested reader.

The linear solver functions listed below are marked with ✓ to indicate that they are required, or with † to indicate that they are only called if they are non-NULL in the SUNLINSOL implementation that is being used. Note:

1. Although IDALS does not call SUNLinSolLastFlag directly, this routine is available for users to query linear solver issues directly.

2. Although IDALS does not call SUNLinSolFree directly, this routine should be available for users to call when cleaning up from a simulation.

Since there are a wide range of potential SUNLINSOL use cases, the following subsections describe some details of the IDALS interface, in the case that interested users wish to develop custom SUNLINSOL modules.

#### 10.4.1 Lagged matrix information

If the SUNLINSOL object self-identifies as having type SUNLINEARSOLVER\_DIRECT or
Table 10.3: List of linear solver function usage in the idals interface

<table>
<thead>
<tr>
<th>Function</th>
<th>DIRECT</th>
<th>ITERATIVE</th>
<th>MATRIX-ITERATIVE</th>
</tr>
</thead>
<tbody>
<tr>
<td>SUNLinSolGetType</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>SUNLinSolSetATimes</td>
<td>†</td>
<td>✓</td>
<td>†</td>
</tr>
<tr>
<td>SUNLinSolPreconditioner</td>
<td>†</td>
<td>†</td>
<td>†</td>
</tr>
<tr>
<td>SUNLinSolSetScalingVectors</td>
<td>†</td>
<td>†</td>
<td>†</td>
</tr>
<tr>
<td>SUNLinSolInitialize</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>SUNLinSolSetup</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>SUNLinSolSolve</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>SUNLinSolResid</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>'SUNLinSolLastFlag</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>'SUNLinSolFree</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SUNLinSolSpace</td>
<td>†</td>
<td>†</td>
<td>†</td>
</tr>
</tbody>
</table>

SUNLINEARSOLVER-MATRIX ITERATIVE, then the SUNLINSOL object solves a linear system defined by a SUNMATRIX object. IDALS will update the matrix information infrequently according to the strategies outlined in §2.1. To this end, we differentiate between the desired linear system $Mx = b$ and the actual linear system $\bar{M}\bar{x} = b$. Since IDALS updates the SUNMATRIX object infrequently, it is likely that $\alpha \neq \bar{\alpha}$, and in turn $M \neq \bar{M}$. Therefore, after calling the SUNLINSOL-provided SUNLinSolSolve routine, we test whether $\alpha/\bar{\alpha} \neq 1$, and if this is the case we scale the solution $\bar{x}$ to correct the linear system solution $x$ via

$$x = \frac{2}{1 + \alpha/\bar{\alpha}} \bar{x}. \quad (10.3)$$

The motivation for this selection of the scaling factor $c = 2/(1 + \alpha/\bar{\alpha})$ is discussed in detail in [6, 28]. In short, if we consider a stationary iteration for the linear system as consisting of a solve with $\bar{M}$ followed by scaling by $c$, then for a linear constant-coefficient problem, the error in the solution vector will be reduced at each iteration by the error matrix $E = I - c\bar{M}^{-1}M$, with a convergence rate given by the spectral radius of $E$. Assuming that stiff systems have a spectrum spread widely over the left half-plane, $c$ is chosen to minimize the magnitude of the eigenvalues of $E$.

### 10.4.2 Iterative linear solver tolerance

If the SUNLINSOL object self-identifies as having type SUNLINEARSOLVER_ITERATIVE or SUNLINEARSOLVER-MATRIX_ITERATIVE then IDALS will set the input tolerance delta as described in §2.1. However, if the iterative linear solver does not support scaling matrices (i.e., the SUNLinSolSetScalingVectors routine is NULL), then IDALS will attempt to adjust the linear solver tolerance to account for this lack of functionality. To this end, the following assumptions are made:

1. All solution components have similar magnitude; hence the error weight vector $W$ used in the WRMS norm (see §2.1) should satisfy the assumption

   $$W_i \approx W_{\text{mean}}, \quad \text{for} \quad i = 0, \ldots, n - 1.$$

2. The SUNLINSOL object uses a standard 2-norm to measure convergence.
Since IDA uses identical left and right scaling matrices, $S_1 = S_2 = S = \text{diag}(W)$, then the linear solver convergence requirement is converted as follows (using the notation from equations (10.1)-(10.2)):

$$
\| \tilde{b} - \tilde{Ax} \|_2 < \text{tol} \iff \| SP_1^{-1}b - SP_1^{-1}Ax \|_2 < \text{tol} \\
\iff \sum_{i=0}^{n-1} [W_i \left( P_1^{-1}(b - Ax) \right)_i]^2 < \text{tol}^2 \\
\iff W_{\text{mean}}^2 \sum_{i=0}^{n-1} [(P_1^{-1}(b - Ax))_i]^2 < \text{tol}^2 \\
\iff \sum_{i=0}^{n-1} [(P_1^{-1}(b - Ax))_i]^2 < \left( \frac{\text{tol}}{W_{\text{mean}}} \right)^2 \\
\iff \| P_1^{-1}(b - Ax) \|_2 < \frac{\text{tol}}{W_{\text{mean}}}
$$

Therefore the tolerance scaling factor

$$
W_{\text{mean}} = \| W \|_2 / \sqrt{n}
$$

is computed and the scaled tolerance $\text{delta} = \text{tol}/W_{\text{mean}}$ is supplied to the SUNLINSOL object.

10.5 The SUNLinearSolver_Dense implementation

This section describes the SUNLINSOL implementation for solving dense linear systems. The SUNLINSOL_DENSE module is designed to be used with the corresponding SUNMATRIX_DENSE matrix type, and one of the serial or shared-memory NVVECTOR implementations (NVVECTOR_SERIAL, NVVECTOR_OPENMP, or NVVECTOR_PTHREADS).

To access the SUNLINSOL_DENSE module, include the header file `sunlinsol/sunlinsol_dense.h`. We note that the SUNLINSOL_DENSE module is accessible from SUNDIALS packages without separately linking to the `libsundials_sunlinsoldense` module library.

10.5.1 SUNLinearSolver_Dense description

This solver is constructed to perform the following operations:

- The “setup” call performs a $LU$ factorization with partial (row) pivoting ($O(N^3)$ cost), $PA = LU$, where $P$ is a permutation matrix, $L$ is a lower triangular matrix with 1’s on the diagonal, and $U$ is an upper triangular matrix. This factorization is stored in-place on the input SUNMATRIX_DENSE object $A$, with pivoting information encoding $P$ stored in the pivots array.
- The “solve” call performs pivoting and forward and backward substitution using the stored pivots array and the $LU$ factors held in the SUNMATRIX_DENSE object ($O(N^2)$ cost).

10.5.2 SUNLinearSolver_Dense functions

The SUNLINSOL_DENSE module provides the following user-callable constructor for creating a SUNLinearSolver object.

```c
SUNLinSol_Dense
Call LS = SUNLinSol_Dense(y, A);
```
Description

The function SUNLinSol_Dense creates and allocates memory for a dense SUNLinearSolver object.

Arguments

- y (N_Vector) a template for cloning vectors needed within the solver
- A (SUNMatrix) a SUNMATRIX_DENSE matrix template for cloning matrices needed within the solver

Return value

This returns a SUNLinearSolver object. If either A or y are incompatible then this routine will return NULL.

Notes

This routine will perform consistency checks to ensure that it is called with consistent NVECTOR and SUNMATRIX implementations. These are currently limited to the SUNMATRIX_DENSE matrix type and the NVECTOR_SERIAL, NVECTOR_OPENMP, and NVECTOR_PTHREADS vector types. As additional compatible matrix and vector implementations are added to SUNDIALS, these will be included within this compatibility check.

Deprecated Name

For backward compatibility, the wrapper function SUNDenseLinearSolver with identical input and output arguments is also provided.

F2003 Name

FSUNLinSol_Dense

The SUNLINSOL_DENSE module defines implementations of all “direct” linear solver operations listed in Sections 10.1.1 – 10.1.3:

- SUNLinSolGetType_Dense
- SUNLinSolInitialize_Dense – this does nothing, since all consistency checks are performed at solver creation.
- SUNLinSolSetup_Dense – this performs the LU factorization.
- SUNLinSolSolve_Dense – this uses the LU factors and pivots array to perform the solve.
- SUNLinSolLastFlag_Dense
- SUNLinSolSpace_Dense – this only returns information for the storage within the solver object, i.e. storage for N, last_flag, and pivots.
- SUNLinSolFree_Dense

All of the listed operations are callable via the FORTRAN 2003 interface module by prepending an ‘F’ to the function name.

10.5.3 SUNLinearSolver_Dense Fortran interfaces

The SUNLINSOL_DENSE module provides a FORTRAN 2003 module as well as FORTRAN 77 style interface functions for use from FORTRAN applications.

FORTRAN 2003 interface module

The fsunlinsol_dense_mod FORTRAN module defines interfaces to all SUNLINSOL_DENSE C functions using the intrinsic iso_c_binding module which provides a standardized mechanism for interoperating with C. As noted in the C function descriptions above, the interface functions are named after the corresponding C function, but with a leading ‘F’. For example, the function SUNLinSol_Dense is interfaced as FSUNLinSol_Dense.

The FORTRAN 2003 SUNLINSOL_DENSE interface module can be accessed with the use statement, i.e. use fsunlinsol_dense_mod, and linking to the library lib sundials_fsunlinsoldense_mod.lib in addition to the C library. For details on where the library and module file fsunlinsol_dense_mod.mod are installed see Appendix A. We note that the module is accessible from the FORTRAN 2003 SUNDIALS integrators without separately linking to the lib sundials_fsunlinsoldense_mod library.
10.6 The SUNLinearSolver_Band implementation

**FORTRAN 77 interface functions**

For solvers that include a FORTRAN 77 interface module, the SUNLINSOL_DENSE module also includes a Fortran-callable function for creating a SUNLinearSolver object.

```c
FSUNDENSELINSOLINIT
```

Call `FSUNDENSELINSOLINIT(code, ier)`

Description The function `FSUNDENSELINSOLINIT` can be called for Fortran programs to create a dense SUNLinearSolver object.

Arguments `code` (int*) is an integer input specifying the solver id (1 for CVODE, 2 for IDA, 3 for KINSOL, and 4 for ARKODE).

Return value `ier` is a return completion flag equal to 0 for a success return and ~1 otherwise. See printed message for details in case of failure.

Notes This routine must be called after both the **nvector** and **sunmatrix** objects have been initialized.

Additionally, when using ARKODE with a non-identity mass matrix, the SUNLINSOL_DENSE module includes a Fortran-callable function for creating a SUNLinearSolver mass matrix solver object.

```c
FSUNMASSDENSELINSOLINIT
```

Call `FSUNMASSDENSELINSOLINIT(ier)`

Description The function `FSUNMASSDENSELINSOLINIT` can be called for Fortran programs to create a dense SUNLinearSolver object for mass matrix linear systems.

Arguments None

Return value `ier` is an int return completion flag equal to 0 for a success return and ~1 otherwise. See printed message for details in case of failure.

Notes This routine must be called after both the **nvector** and **sunmatrix** mass-matrix objects have been initialized.

**10.5.4 SUNLinearSolver_Dense content**

The SUNLINSOL_DENSE module defines the *content* field of a SUNLinearSolver as the following structure:

```c
struct _SUNLinearSolverContent_Dense {
    sunindextype N;
    sunindextype *pivots;
    sunindextype last_flag;
};
```

These entries of the *content* field contain the following information:

- `N` - size of the linear system,
- `pivots` - index array for partial pivoting in LU factorization,
- `last_flag` - last error return flag from internal function evaluations.

**10.6 The SUNLinearSolver_Band implementation**

This section describes the SUNLINSOL implementation for solving banded linear systems. The SUNLINSOL_BAND module is designed to be used with the corresponding SUNMATRIX_BAND matrix type, and one of the serial or shared-memory **nvector** implementations (**nvector_serial**, **nvector_openmp**, or **nvector_pthreads**).
To access the SUNLinearSolver module, include the header file `sunlinsol/sunlinsol_band.h`. We note that the SUNLinearSolver module is accessible from SUNDIALS packages without separately linking to the `libsundials_sunlinsolband` module library.

### 10.6.1 SUNLinearSolver_Band description

This solver is constructed to perform the following operations:

- The “setup” call performs a \( LU \) factorization with partial (row) pivoting, \( PA = LU \), where \( P \) is a permutation matrix, \( L \) is a lower triangular matrix with 1’s on the diagonal, and \( U \) is an upper triangular matrix. This factorization is stored in-place on the input SUNMATRIX_BAND object \( A \), with pivoting information encoding \( P \) stored in the `pivots` array.

- The “solve” call performs pivoting and forward and backward substitution using the stored `pivots` array and the \( LU \) factors held in the SUNMATRIX_BAND object.

- \( A \) must be allocated to accommodate the increase in upper bandwidth that occurs during factorization. More precisely, if \( A \) is a band matrix with upper bandwidth \( \mu \) and lower bandwidth \( \mu_l \), then the upper triangular factor \( U \) can have upper bandwidth as big as \( \text{smu} = \text{MIN}(N-1,\mu+\mu_l) \). The lower triangular factor \( L \) has lower bandwidth \( \mu_l \).

### 10.6.2 SUNLinearSolver_Band functions

The SUNLINSOL_BAND module provides the following user-callable constructor for creating a SUNLinearSolver object.

```c
SUNLinSol_Band LS = SUNLinSol_Band(y, A);
```

**Description**
The function `SUNLinSol_Band` creates and allocates memory for a band SUNLinearSolver object.

**Arguments**
- \( y \) ([N_Vector]) a template for cloning vectors needed within the solver
- \( A \) ([SUNMatrix]) a SUNMATRIX_BAND matrix template for cloning matrices needed within the solver

**Return value**
This returns a SUNLinearSolver object. If either \( A \) or \( y \) are incompatible then this routine will return NULL.

**Notes**
This routine will perform consistency checks to ensure that it is called with consistent NVECTOR and SUNMATRIX implementations. These are currently limited to the SUNMATRIX_BAND matrix type and the NVECTOR_SERIAL, NVECTOR_OPENMP, and NVECTOR_PTHREADS vector types. As additional compatible matrix and vector implementations are added to SUNDIALS, these will be included within this compatibility check.

Additionally, this routine will verify that the input matrix \( A \) is allocated with appropriate upper bandwidth storage for the \( LU \) factorization.

**Deprecated Name**
For backward compatibility, the wrapper function `SUNBandLinearSolver` with identical input and output arguments is also provided.

**F2003 Name**
`FSUNLinSol_Band`

The SUNLINSOL_BAND module defines band implementations of all “direct” linear solver operations listed in Sections 10.1.1 – 10.1.3:

- `SUNLinSolGetType_Band`
- `SUNLinSolInitialize_Band` – this does nothing, since all consistency checks are performed at solver creation.
10.6 The SUNLinearSolver_Band implementation

- **SUNLinSolSetup_Band** – this performs the \( LU \) factorization.
- **SUNLinSolSolve_Band** – this uses the \( LU \) factors and \texttt{pivots} array to perform the solve.
- **SUNLinSolLastFlag_Band**
- **SUNLinSolSpace_Band** – this only returns information for the storage \textit{within} the solver object, i.e. storage for \( N \), \texttt{last_flag}, and \texttt{pivots}.
- **SUNLinSolFree_Band**

All of the listed operations are callable via the FORTRAN 2003 interface module by prepending an ‘F’ to the function name.

10.6.3 SUNLinearSolver_Band Fortran interfaces

The \texttt{sunlinsol\_band} module provides a FORTRAN 2003 module as well as FORTRAN 77 style interface functions for use from FORTRAN applications.

**FORTRAN 2003 interface module**

The \texttt{fsunlinsol\_band\_mod} FORTRAN module defines interfaces to all \texttt{sunlinsol\_band} C functions using the intrinsic \texttt{iso\_c\_binding} module which provides a standardized mechanism for interoperating with C. As noted in the C function descriptions above, the interface functions are named after the corresponding C function, but with a leading ‘F’. For example, the function \texttt{SUNLinSol\_Band} is interfaced as \texttt{FSUNLinSol\_Band}.

The FORTRAN 2003 \texttt{sunlinsol\_band} interface module can be accessed with the \texttt{use} statement, i.e. \texttt{use fsunlinsol\_band\_mod}, and linking to the library \texttt{libsundials\_fsunlinsol\_band\_mod.lib} in addition to the C library. For details on where the library and module file \texttt{fsunlinsol\_band\_mod.mod} are installed see Appendix A. We note that the module is accessible from the FORTRAN 2003 SUNDIALS integrators \textit{without} separately linking to the \texttt{libsundials\_fsunlinsol\_band\_mod} library.

**FORTRAN 77 interface functions**

For solvers that include a FORTRAN 77 interface module, the \texttt{sunlinsol\_band} module also includes a Fortran-callable function for creating a \texttt{SUNLinearSolver} object.

```
FSUNBANDLINSOLINIT
Call FSUNBANDLINSOLINIT(code, ier)
Description The function FSUNBANDLINSOLINIT can be called for Fortran programs to create a band SUNLinearSolver object.
Arguments code (int*) is an integer input specifying the solver id (1 for CVODE, 2 for IDA, 3 for KINSOL, and 4 for ARKODE).
Return value ier is a return completion flag equal to 0 for a success return and \(-1\) otherwise. See printed message for details in case of failure.
Notes This routine must be called \textit{after} both the \texttt{nvector} and \texttt{sunmatrix} objects have been initialized.
```

Additionally, when using ARKODE with a non-identity mass matrix, the \texttt{sunlinsol\_band} module includes a Fortran-callable function for creating a \texttt{SUNLinearSolver} mass matrix solver object.
FSUNMASSBANDLINSOLINIT

Call FSUNMASSBANDLINSOLINIT(ier)

Description The function FSUNMASSBANDLINSOLINIT can be called for Fortran programs to create a band SUNLinearSolver object for mass matrix linear systems.

Arguments None

Return value ier is a int return completion flag equal to 0 for a success return and -1 otherwise. See printed message for details in case of failure.

Notes This routine must be called after both the nvector and sunmatrix mass-matrix objects have been initialized.

10.6.4 SUNLinearSolver_Band content

The SUNLINSOL_BAND module defines the content field of a SUNLinearSolver as the following structure:

```c
struct _SUNLinearSolverContent_Band {
    sunindextype N;
    sunindextype *pivots;
    sunindextype last_flag;
};
```

These entries of the content field contain the following information:

- **N** - size of the linear system,
- **pivots** - index array for partial pivoting in LU factorization,
- **last_flag** - last error return flag from internal function evaluations.

10.7 The SUNLinearSolver_LapackDense implementation

This section describes the SUNLINSOL implementation for solving dense linear systems with LAPACK. The SUNLINSOL_LAPACKDENSE module is designed to be used with the corresponding SUNMATRIX_DENSE matrix type, and one of the serial or shared-memory nvector implementations (nvector_serial, nvector_openmp, or nvector_pthreads).

To access the SUNLINSOL_LAPACKDENSE module, include the header file sunlinsol/sunlinsol_lapackdense.h. The installed module library to link to is lib sundials sunlinsollapackdense. lib where .lib is typically .so for shared libraries and .a for static libraries.

The SUNLINSOL_LAPACKDENSE module is a SUNLINSOL wrapper for the LAPACK dense matrix factorization and solve routines, *GETRF* and *GETRS*, where * is either D or S, depending on whether SUNDIALS was configured to have realtype set to double or single, respectively (see Section 4.2). In order to use the SUNLINSOL_LAPACKDENSE module it is assumed that LAPACK has been installed on the system prior to installation of SUNDIALS, and that SUNDIALS has been configured appropriately to link with LAPACK (see Appendix A for details). We note that since there do not exist 128-bit floating-point factorization and solve routines in LAPACK, this interface cannot be compiled when using extended precision for realtype. Similarly, since there do not exist 64-bit integer LAPACK routines, the SUNLINSOL_LAPACKDENSE module also cannot be compiled when using 64-bit integers for the sunindextype.

10.7.1 SUNLinearSolver_LapackDense description

This solver is constructed to perform the following operations:
10.7 The SunLinearSolver_LapackDense Implementation

- The “setup” call performs a LU factorization with partial (row) pivoting ($O(N^3)$ cost), $PA = LU$, where $P$ is a permutation matrix, $L$ is a lower triangular matrix with 1’s on the diagonal, and $U$ is an upper triangular matrix. This factorization is stored in-place on the input SUNMatrix_Dense object $A$, with pivoting information encoding $P$ stored in the pivots array.

- The “solve” call performs pivoting and forward and backward substitution using the stored pivots array and the $LU$ factors held in the SUNMatrix_Dense object ($O(N^2)$ cost).

10.7.2 SunLinearSolver_LapackDense Functions

The sunlinsol_lapackdense module provides the following user-callable constructor for creating a SunLinearSolver object.

```c
SUNLinSol_LapackDense Call LS = SUNLinSol_LapackDense(y, A);
```

Description The function SUNLinSol_LapackDense creates and allocates memory for a LAPACK-based, dense SunLinearSolver object.

Arguments
- $y$ ($N$_Vector) a template for cloning vectors needed within the solver
- $A$ (SUNMatrix) a SUNMatrix_Dense matrix template for cloning matrices needed within the solver

Return value This returns a SunLinearSolver object. If either $A$ or $y$ are incompatible then this routine will return NULL.

Notes This routine will perform consistency checks to ensure that it is called with consistent nvector and SunMatrix implementations. These are currently limited to the SunMatrix_Dense matrix type and the nvector_serial, nvector_openmp, and nvector_pthreads vector types. As additional compatible matrix and vector implementations are added to Sundials, these will be included within this compatibility check.

Deprecated Name For backward compatibility, the wrapper function SUNLapackDense with identical input and output arguments is also provided.

The sunlinsol_lapackdense module defines dense implementations of all “direct” linear solver operations listed in Sections 10.1.1 – 10.1.3:

- SUNLinSolGetType_LapackDense
- SUNLinSolInitialize_LapackDense – this does nothing, since all consistency checks are performed at solver creation.
- SUNLinSolSetup_LapackDense – this calls either DGETRF or SGETRF to perform the $LU$ factorization.
- SUNLinSolSolve_LapackDense – this calls either DGETRS or SGETRS to use the $LU$ factors and pivots array to perform the solve.
- SUNLinSolLastFlag_LapackDense
- SUNLinSolSpace_LapackDense – this only returns information for the storage within the solver object, i.e. storage for $N$, last_flag, and pivots.
- SUNLinSolFree_LapackDense

10.7.3 SunLinearSolver_LapackDense Fortran Interfaces

For solvers that include a Fortran 77 interface module, the sunlinsol_lapackdense module also includes a Fortran-callable function for creating a SunLinearSolver object.
**FSUNLAPACKDENSEINIT**

Call

FSUNLAPACKDENSEINIT(code, ier)

Description

The function FSUNLAPACKDENSEINIT can be called for Fortran programs to create a LAPACK-based dense SUNLinearSolver object.

Arguments

code (int*) is an integer input specifying the solver id (1 for cvode, 2 for IDA, 3 for KINSOL, and 4 for ARKODE).

Return value

ier is a return completion flag equal to 0 for a success return and -1 otherwise. See printed message for details in case of failure.

Notes

This routine must be called after both the nvector and sunmatrix objects have been initialized.

Additionally, when using ARKODE with a non-identity mass matrix, the SUNLINSOL_LAPACKDENSE module includes a Fortran-callable function for creating a SUNLinearSolver mass matrix solver object.

**FSUNMASSLAPACKDENSEINIT**

Call

FSUNMASSLAPACKDENSEINIT(ier)

Description

The function FSUNMASSLAPACKDENSEINIT can be called for Fortran programs to create a LAPACK-based, dense SUNLinearSolver object for mass matrix linear systems.

Arguments

None

Return value

ier is an int return completion flag equal to 0 for a success return and -1 otherwise. See printed message for details in case of failure.

Notes

This routine must be called after both the nvector and sunmatrix mass-matrix objects have been initialized.

### 10.7.4 SUNLinearSolver_LapackDense content

The SUNLINSOL_LAPACKDENSE module defines the content field of a SUNLinearSolver as the following structure:

```c
struct _SUNLinearSolverContent_Dense {
    sunindextype N;
    sunindextype *pivots;
    sunindextype last_flag;
};
```

These entries of the content field contain the following information:

- **N**: size of the linear system,
- **pivots**: index array for partial pivoting in LU factorization,
- **last_flag**: last error return flag from internal function evaluations.

### 10.8 The SUNLinearSolver_LapackBand implementation

This section describes the SUNLINSOL implementation for solving banded linear systems with LAPACK. The SUNLINSOL_LAPACKBAND module is designed to be used with the corresponding SUNMATRIX_BAND matrix type, and one of the serial or shared-memory NVECTOR implementations (NVECTOR_SERIAL, NVECTOR_OPENMP, or NVECTOR_PTHREADS).

To access the SUNLINSOL_LAPACKBAND module, include the header file sunlinsol/sunlinsol_lapackband.h. The installed module library to link to is libsundials_sunlinsollapackband.lib where .lib is typically .so for shared libraries and .a for static libraries.
10.8 The SUNLinearSolver LapackBand implementation

The SUNLINSOL_LAPACKBAND module is a SUNLINSOL wrapper for the LAPACK band matrix factorization and solve routines, *GBTRF and *GBTRS, where * is either D or S, depending on whether SUNDIALS was configured to have realtype set to double or single, respectively (see Section 4.2). In order to use the SUNLINSOL_LAPACKBAND module it is assumed that LAPACK has been installed on the system prior to installation of SUNDIALS, and that SUNDIALS has been configured appropriately to link with LAPACK (see Appendix A for details). We note that since there do not exist 128-bit floating-point factorization and solve routines in LAPACK, this interface cannot be compiled when using extended precision for realtype. Similarly, since there do not exist 64-bit integer LAPACK routines, the SUNLINSOL_LAPACKBAND module also cannot be compiled when using 64-bit integers for the sunindextype.

10.8.1 SUNLinearSolver_LapackBand description

This solver is constructed to perform the following operations:

- The “setup” call performs a LU factorization with partial (row) pivoting, PA = LU, where P is a permutation matrix, L is a lower triangular matrix with 1’s on the diagonal, and U is an upper triangular matrix. This factorization is stored in-place on the input SUNMATRIX_BAND object A, with pivoting information encoding P stored in the pivots array.

- The “solve” call performs pivoting and forward and backward substitution using the stored pivots array and the LU factors held in the SUNMATRIX_BAND object.

- A must be allocated to accommodate the increase in upper bandwidth that occurs during factorization. More precisely, if A is a band matrix with upper bandwidth mu and lower bandwidth ml, then the upper triangular factor U can have upper bandwidth as big as smu = MIN(N-1,mu+ml). The lower triangular factor L has lower bandwidth ml.

10.8.2 SUNLinearSolver_LapackBand functions

The SUNLINSOL_LAPACKBAND module provides the following user-callable constructor for creating a SUNLinearSolver object.

```c
SUNLinSolLapackBand
Call LS = SUNLinSolLapackBand(y, A);
Description The function SUNLinSolLapackBand creates and allocates memory for a LAPACK-based, band SUNLinearSolver object.
Arguments y (N_Vector) a template for cloning vectors needed within the solver
A (SUNMatrix) a SUNMATRIX_BAND matrix template for cloning matrices needed within the solver
Return value This returns a SUNLinearSolver object. If either A or y are incompatible then this routine will return NULL.
Notes This routine will perform consistency checks to ensure that it is called with consistent NVECTOR and SUNMATRIX implementations. These are currently limited to the SUNMATRIX_BAND matrix type and the NVECTOR_SERIAL, NVECTOR_OPENMP, and NVECTOR_PTHREADS vector types. As additional compatible matrix and vector implementations are added to SUNDIALS, these will be included within this compatibility check.
Additionally, this routine will verify that the input matrix A is allocated with appropriate upper bandwidth storage for the LU factorization.
Deprecated Name For backward compatibility, the wrapper function SUNLapackBand with identical input and output arguments is also provided.
```
The **SUNLINSOL\_LAPACKBAND** module defines band implementations of all “direct” linear solver operations listed in Sections 10.1.1 – 10.1.3:

- **SUNLinSolGetType\_LapackBand**
- **SUNLinSolInitialize\_LapackBand** – this does nothing, since all consistency checks are performed at solver creation.
- **SUNLinSolSetup\_LapackBand** – this calls either DGBTRF or SGBTRF to perform the LU factorization.
- **SUNLinSolSolve\_LapackBand** – this calls either DGBTRS or SGBTRS to use the LU factors and pivots array to perform the solve.
- **SUNLinSolLastFlag\_LapackBand**
- **SUNLinSolSpace\_LapackBand** – this only returns information for the storage within the solver object, i.e. storage for \( N \), last_flag, and pivots.
- **SUNLinSolFree\_LapackBand**

### 10.8.3 **SUNLinearSolver\_LapackBand** Fortran interfaces

For solvers that include a Fortran 77 interface module, the **SUNLINSOL\_LAPACKBAND** module also includes a Fortran-callable function for creating a SUNLinearSolver object.

**FSUNLAPACKDENSEINIT**

**Call**

\[
\text{FSUNLAPACKBANDINIT}(\text{code}, \text{ier})
\]

**Description**

The function **FSUNLAPACKBANDINIT** can be called for Fortran programs to create a LAPACK-based band SUNLinearSolver object.

**Arguments**

- **code** (int) is an integer input specifying the solver id (1 for CVODE, 2 for IDA, 3 for KINSOL, and 4 for ARKODE).

**Return value**

\( \text{ier} \) is a return completion flag equal to 0 for a success return and \(-1\) otherwise. See printed message for details in case of failure.

**Notes**

This routine must be called after both the NVECTOR and SUNMATRIX objects have been initialized. Additionally, when using ARKODE with a non-identity mass matrix, the **SUNLINSOL\_LAPACKBAND** module includes a Fortran-callable function for creating a SUNLinearSolver mass matrix solver object.

**FSUNMASSLAPACKBANDINIT**

**Call**

\[
\text{FSUNMASSLAPACKBANDINIT}(\text{ier})
\]

**Description**

The function **FSUNMASSLAPACKBANDINIT** can be called for Fortran programs to create a LAPACK-based, band SUNLinearSolver object for mass matrix linear systems.

**Arguments**

None

**Return value**

\( \text{ier} \) is a int return completion flag equal to 0 for a success return and \(-1\) otherwise. See printed message for details in case of failure.

**Notes**

This routine must be called after both the NVECTOR and SUNMATRIX mass-matrix objects have been initialized.
10.8.4 SUNLinearSolver_LapackBand content

The SUNLINSOL_LAPACKBAND module defines the content field of a SUNLinearSolver as the following structure:

```c
struct _SUNLinearSolverContent_Band {
    sunindextype N;
    sunindextype *pivots;
    sunindextype last_flag;
};
```

These entries of the content field contain the following information:
- `N` - size of the linear system,
- `pivots` - index array for partial pivoting in LU factorization,
- `last_flag` - last error return flag from internal function evaluations.

10.9 The SUNLinearSolver_KLU implementation

This section describes the SUNLINSOL implementation for solving sparse linear systems with KLU. The SUNLINSOL_KLU module is designed to be used with the corresponding SUNMATRIX_SPARSE matrix type, and one of the serial or shared-memory NVECTOR implementations (NVECTOR_SERIAL, NVECTOR_OPENMP, or NVECTOR_PTHREADS).

The header file to include when using this module is `sunlinsol/sunlinsol_klu.h`. The installed module library to link to is `libsundials_sunlinsolklu`.lib where .lib is typically .so for shared libraries and .a for static libraries.

The SUNLINSOL_KLU module is a SUNLINSOL wrapper for the KLU sparse matrix factorization and solver library written by Tim Davis [1, 19]. In order to use the SUNLINSOL_KLU interface to KLU, it is assumed that KLU has been installed on the system prior to installation of SUNDIALS, and that SUNDIALS has been configured appropriately to link with KLU (see Appendix A for details). Additionally, this wrapper only supports double-precision calculations, and therefore cannot be compiled if SUNDIALS is configured to have realtype set to either extended or single (see Section 4.2). Since the KLU library supports both 32-bit and 64-bit integers, this interface will be compiled for either of the available sunindextype options.

10.9.1 SUNLinearSolver_KLU description

The KLU library has a symbolic factorization routine that computes the permutation of the linear system matrix to block triangular form and the permutations that will pre-order the diagonal blocks (the only ones that need to be factored) to reduce fill-in (using AMD, COLAMD, CHOLAMD, natural, or an ordering given by the user). Of these ordering choices, the default value in the SUNLINSOL_KLU module is the COLAMD ordering.

KLU breaks the factorization into two separate parts. The first is a symbolic factorization and the second is a numeric factorization that returns the factored matrix along with final pivot information. KLU also has a refactor routine that can be called instead of the numeric factorization. This routine will reuse the pivot information. This routine also returns diagnostic information that a user can examine to determine if numerical stability is being lost and a full numerical factorization should be done instead of the refactor.

Since the linear systems that arise within the context of SUNDIALS calculations will typically have identical sparsity patterns, the SUNLINSOL_KLU module is constructed to perform the following operations:

- The first time that the “setup” routine is called, it performs the symbolic factorization, followed by an initial numerical factorization.
• On subsequent calls to the “setup” routine, it calls the appropriate KLU “refactor” routine, followed by estimates of the numerical conditioning using the relevant “rcond”, and if necessary “condest”, routine(s). If these estimates of the condition number are larger than $\varepsilon^{-2/3}$ (where $\varepsilon$ is the double-precision unit roundoff), then a new factorization is performed.

• The module includes the routine SUNKLUReInit, that can be called by the user to force a full or partial refactorization at the next “setup” call.

• The “solve” call performs pivoting and forward and backward substitution using the stored KLU data structures. We note that in this solve KLU operates on the native data arrays for the right-hand side and solution vectors, without requiring costly data copies.

10.9.2 SUNLinearSolver_KLU functions

The SUNLINSOL_KLU module provides the following user-callable constructor for creating a SUNLinearSolver object.

```
SUNLinSol_KLU
Call LS = SUNLinSol_KLU(y, A);
Description The function SUNLinSol_KLU creates and allocates memory for a KLU-based SUNLinearSolver object.
Arguments y (N_Vector) a template for cloning vectors needed within the solver
A (SUNMatrix) a SUNMATRIX_SPARSE matrix template for cloning matrices needed within the solver
Return value This returns a SUNLinearSolver object. If either A or y are incompatible then this routine will return NULL.
Notes This routine will perform consistency checks to ensure that it is called with consistent NVVECTOR and SUNMATRIX implementations. These are currently limited to the SUNMATRIX_SPARSE matrix type (using either CSR or CSC storage formats) and the NVVECTOR_SERIAL, NVVECTOR_OPENMP, and NVVECTOR_PTHREADS vector types. As additional compatible matrix and vector implementations are added to SUNDIALS, these will be included within this compatibility check.
Deprecated Name For backward compatibility, the wrapper function SUNKLU with identical input and output arguments is also provided.
F2003 Name FSUNLinSol_KLU
```

The SUNLINSOL_KLU module defines implementations of all “direct” linear solver operations listed in Sections 10.1.1 – 10.1.3:

• SUNLinSolGetType_KLU

• SUNLinSolInitialize_KLU – this sets the first_factorize flag to 1, forcing both symbolic and numerical factorizations on the subsequent “setup” call.

• SUNLinSolSetup_KLU – this performs either a $LU$ factorization or refactorization of the input matrix.

• SUNLinSolSolve_KLU – this calls the appropriate KLU solve routine to utilize the $LU$ factors to solve the linear system.

• SUNLinSolLastFlag_KLU

• SUNLinSolSpace_KLU – this only returns information for the storage within the solver interface, i.e. storage for the integers last_flag and first_factorize. For additional space requirements, see the KLU documentation.
10.9 The SUNLinearSolver_KLU implementation

- SUNLinSolFree_KLU

All of the listed operations are callable via the FORTRAN 2003 interface module by prepending an ‘F’ to the function name.

The SUNLINSOL_KLU module also defines the following additional user-callable functions.

SUNLinSol_KLUReInit

Call

```
retval = SUNLinSol_KLUReInit(LS, A, nnz, reinit_type);
```

Description

The function SUNLinSol_KLUReInit reinitializes memory and flags for a new factorization (symbolic and numeric) to be conducted at the next solver setup call. This routine is useful in the cases where the number of nonzeroes has changed or if the structure of the linear system has changed which would require a new symbolic (and numeric factorization).

Arguments

- LS (SUNLinearSolver) a template for cloning vectors needed within the solver
- A (SUNMatrix) a SUNMATRIX_SPARSE matrix template for cloning matrices needed within the solver
- nnz (sunindextype) the new number of nonzeros in the matrix
- reinit_type (int) flag governing the level of reinitialization. The allowed values are:
  - SUNKLU_REINIT_FULL – The Jacobian matrix will be destroyed and a new one will be allocated based on the nnz value passed to this call. New symbolic and numeric factorizations will be completed at the next solver setup.
  - SUNKLU_REINIT_PARTIAL – Only symbolic and numeric factorizations will be completed. It is assumed that the Jacobian size has not exceeded the size of nnz given in the sparse matrix provided to the original constructor routine (or the previous SUNLinSol_KLUReInit call).

Return value

The return values from this function are SUNLS_MEM_NULL (either S or A are NULL), SUNLS_ILL_INPUT (A does not have type SUNMATRIX_SPARSE or reinit_type is invalid), SUNLS_MEM_FAIL (reallocation of the sparse matrix failed) or SUNLS_SUCCESS.

Notes

This routine will perform consistency checks to ensure that it is called with consistent nvector and SUNMATRIX implementations. These are currently limited to the SUNMATRIX_SPARSE matrix type (using either CSR or CSC storage formats) and the NV_VECTOR_SERIAL, NV_VECTOR_OPENMP, and NV_VECTOR_PTHREADS vector types. As additional compatible matrix and vector implementations are added to SUNDIALS, these will be included within this compatibility check.

This routine assumes no other changes to solver use are necessary.

Deprecated Name

For backward compatibility, the wrapper function SUNKLUReInit with identical input and output arguments is also provided.

F2003 Name

FSUNLinSol_KLUReInit

SUNLinSol_KLUSetOrdering

Call

```
retval = SUNLinSol_KLUSetOrdering(LS, ordering);
```

Description

This function sets the ordering used by KLU for reducing fill in the linear solve.

Arguments

- LS (SUNLinearSolver) the SUNLINSOL_KLU object
- ordering (int) flag indicating the reordering algorithm to use, the options are:
  - 0 AMD,
Description of the SUNLinearSolver module

1 COLAMD, and
2 the natural ordering.

The default is 1 for COLAMD.

Return value The return values from this function are SUNLS_MEM_NULL (S is NULL), SUNLS_ILL_INPUT (invalid ordering choice), or SUNLS_SUCCESS.

Deprecated Name For backward compatibility, the wrapper function SUNKLUSetOrdering with identical input and output arguments is also provided.

F2003 Name FSUNLinSol_KLUSetOrdering

**SUNLinSol_KLUGetSymbolic**

Call `symbolic = SUNLinSol_KLUGetSymbolic(LS);`

Description This function returns a pointer to the KLU symbolic factorization stored in the SUNLINSOL_KLU content structure.

Arguments LS (SUNLinearSolver) the SUNLINSOL_KLU object

Return value The return type from this function is sun_klu_symbolic.

Notes When SUNDIALS is compiled with 32-bit indices (SUNDIALS_INDEX_SIZE=32), sun_klu_symbolic is mapped to the KLU type klu_symbolic; when SUNDIALS is compiled with 64-bit indices (SUNDIALS_INDEX_SIZE=64) this is mapped to the KLU type klu_l_symbolic.

**SUNLinSol_KLUGetNumeric**

Call `numeric = SUNLinSol_KLUGetNumeric(LS);`

Description This function returns a pointer to the KLU numeric factorization stored in the SUNLINSOL_KLU content structure.

Arguments LS (SUNLinearSolver) the SUNLINSOL_KLU object

Return value The return type from this function is sun_klu_numeric.

Notes When SUNDIALS is compiled with 32-bit indices (SUNDIALS_INDEX_SIZE=32), sun_klu_numeric is mapped to the KLU type klu_numeric; when SUNDIALS is compiled with 64-bit indices (SUNDIALS_INDEX_SIZE=64) this is mapped to the KLU type klu_l_numeric.

**SUNLinSol_KLUGetCommon**

Call `common = SUNLinSol_KLUGetCommon(LS);`

Description This function returns a pointer to the KLU common structure stored within in the SUNLINSOL_KLU content structure.

Arguments LS (SUNLinearSolver) the SUNLINSOL_KLU object

Return value The return type from this function is sun_klu_common.

Notes When SUNDIALS is compiled with 32-bit indices (SUNDIALS_INDEX_SIZE=32), sun_klu_common is mapped to the KLU type klu_common; when SUNDIALS is compiled with 64-bit indices (SUNDIALS_INDEX_SIZE=64) this is mapped to the KLU type klu_l_common.

10.9.3 SUNLinearSolver_KLU Fortran interfaces

The sunlinsol_klu module provides a FORTRAN 2003 module as well as FORTRAN 77 style interface functions for use from FORTRAN applications.
10.9 The SUNLinearSolver_KLU implementation

FORTRAN 2003 interface module

The fsunlinsol_klu_mod FORTRAN module defines interfaces to all SUNLINSOL_KLU C functions using the intrinsic iso_c_binding module which provides a standardized mechanism for interoperating with C. As noted in the C function descriptions above, the interface functions are named after the corresponding C function, but with a leading ‘F’. For example, the function SUNLinSol_klu is interfaced as FSUNLinSol_klu.

The FORTRAN 2003 SUNLINSOL_KLU interface module can be accessed with the use statement, i.e. use fsunlinsol_klu_mod, and linking to the library lib sundials_fsunlinsolklu_mod.lib in addition to the C library. For details on where the library and module file fsunlinsol_klu_mod.mod are installed see Appendix A.

FORTRAN 77 interface functions

For solvers that include a FORTRAN 77 interface module, the SUNLINSOL_KLU module also includes a Fortran-callable function for creating a SUNLinearSolver object.

**FSUNKLUINIT**

Call        FSUNKLUINIT(code, ier)

Description The function FSUNKLUINIT can be called for Fortran programs to create a sunlinsol_klu object.

Arguments 

- **code** (int*) is an integer input specifying the solver id (1 for CVODE, 2 for IDA, 3 for KINSOL, and 4 for ARKODE).

Return value ier is a return completion flag equal to 0 for a success return and -1 otherwise. See printed message for details in case of failure.

Notes  This routine must be called after both the NVECTOR and SUNMATRIX objects have been initialized.

Additionally, when using ARKODE with a non-identity mass matrix, the SUNLINSOL_KLU module includes a Fortran-callable function for creating a SUNLinearSolver mass matrix solver object.

**FSUNMASSKLUINIT**

Call        FSUNMASSKLUINIT(ier)

Description The function FSUNMASSKLUINIT can be called for Fortran programs to create a KLU-based SUNLinearSolver object for mass matrix linear systems.

Arguments None

Return value ier is a int return completion flag equal to 0 for a success return and -1 otherwise. See printed message for details in case of failure.

Notes  This routine must be called after both the NVECTOR and SUNMATRIX mass-matrix objects have been initialized.

The SUNLinSol_KLUReInit and SUNLinSol_KLUSetOrdering routines also support FORTRAN interfaces for the system and mass matrix solvers:

**FSUNKLUREINIT**

Call        FSUNKLUREINIT(code, nnz, reinit_type, ier)

Description The function FSUNKLUREINIT can be called for Fortran programs to re-initialize a SUNLINSOL_KLU object.

Arguments 

- **code** (int*) is an integer input specifying the solver id (1 for CVODE, 2 for IDA, 3 for KINSOL, and 4 for ARKODE).
- **nnz** (sunindextype*) the new number of nonzeros in the matrix
reinit_type (int*) flag governing the level of reinitialization. The allowed values are:
1 – The Jacobian matrix will be destroyed and a new one will be allocated based on the nnz value passed to this call. New symbolic and numeric factorizations will be completed at the next solver setup.
2 – Only symbolic and numeric factorizations will be completed. It is assumed that the Jacobian size has not exceeded the size of nnz given in the sparse matrix provided to the original constructor routine (or the previous SUNLinSol_KLUReInit call).

Return value ier is a int return completion flag equal to 0 for a success return and -1 otherwise. See printed message for details in case of failure.

Notes See SUNLinSol_KLUReInit for complete further documentation of this routine.

FSUNMASSKLUREINIT
Call FSUNMASSKLUREINIT(nnz, reinit_type, ier)
Description The function FSUNMASSKLUREINIT can be called for Fortran programs to re-initialize a SUNLINSOL_KLU object for mass matrix linear systems.
Arguments The arguments are identical to FSUNKLUREINIT above, except that code is not needed since mass matrix linear systems only arise in ARKODE.
Return value ier is a int return completion flag equal to 0 for a success return and -1 otherwise. See printed message for details in case of failure.
Notes See SUNLinSol_KLUReInit for complete further documentation of this routine.

FSUNKLUSETORDERING
Call FSUNKLUSETORDERING(code, ordering, ier)
Description The function FSUNKLUSETORDERING can be called for Fortran programs to change the reordering algorithm used by KLU.
Arguments code (int*) is an integer input specifying the solver id (1 for CVODE, 2 for IDA, 3 for KINSOL, and 4 for ARKODE).
ordering (int*) flag indication the reordering algorithm to use. Options include:
   0 AMD,
   1 COLAMD, and
   2 the natural ordering.
   The default is 1 for COLAMD.
Return value ier is a int return completion flag equal to 0 for a success return and -1 otherwise. See printed message for details in case of failure.
Notes See SUNLinSol_KLUSetOrdering for complete further documentation of this routine.

FSUNMASSKLUSETORDERING
Call FSUNMASSKLUSETORDERING(ier)
Description The function FSUNMASSKLUSETORDERING can be called for Fortran programs to change the reordering algorithm used by KLU for mass matrix linear systems.
Arguments The arguments are identical to FSUNKLUSETORDERING above, except that code is not needed since mass matrix linear systems only arise in ARKODE.
Return value ier is a int return completion flag equal to 0 for a success return and -1 otherwise. See printed message for details in case of failure.
Notes See SUNLinSol_KLUSetOrdering for complete further documentation of this routine.
10.9.4 SUNLinearSolver_KLU content

The SUNLINSOL_KLU module defines the content field of a SUNLinearSolver as the following structure:

```c
struct _SUNLinearSolverContent_KLU {
    int last_flag;
    int first_factorize;
    sun_klu_symbolic *symbolic;
    sun_klu_numeric *numeric;
    sun_klu_common common;
    sunindextype (*klu_solver)(sun_klu_symbolic*, sun_klu_numeric*,
                              sunindextype, sunindextype,
                              double*, sun_klu_common*);
};
```

These entries of the content field contain the following information:
- last_flag - last error return flag from internal function evaluations,
- first_factorize - flag indicating whether the factorization has ever been performed,
- symbolic - KLU storage structure for symbolic factorization components, with underlying type klu_symbolic or klu_l_symbolic, depending on whether SUNDIALS was installed with 32-bit versus 64-bit indices, respectively,
- numeric - KLU storage structure for numeric factorization components, with underlying type klu_numeric or klu_l_numeric, depending on whether SUNDIALS was installed with 32-bit versus 64-bit indices, respectively,
- common - storage structure for common KLU solver components, with underlying type klu_common or klu_l_common, depending on whether SUNDIALS was installed with 32-bit versus 64-bit indices, respectively,
- klu_solver - pointer to the appropriate KLU solver function (depending on whether it is using a CSR or CSC sparse matrix, and on whether SUNDIALS was installed with 32-bit or 64-bit indices).

10.10 The SUNLinearSolver_SuperLUDIST implementation

The SuperLU_DIST implementation of the SUNLINSOL module provided with SUNDIALS, SUNLINSOL_SUPERLUDIST, is designed to be used with the corresponding SUNMATRIX_SLUNRLOC matrix type, and one of the serial, threaded or parallel NVVECTOR implementations (NVVECTOR_SERIAL, NVVECTOR_OPENMP, NVVECTOR_PTHREADS, NVVECTOR_PARALLEL, or NVVECTOR_PARHYPER). The header file to include when using this module is sunlinsol/sunlinsol_superludist.h. The installed module library to link to is libsundials_sunlinsolsuperludist.lib where .lib is typically .so for shared libraries and .a for static libraries.

10.10.1 SUNLinearSolver_SuperLUDIST description

The SUNLINSOL_SUPERLUDIST module is a SUNLINSOL adapter for the SuperLU_DIST sparse matrix factorization and solver library written by X. Sherry Li [3, 25, 40, 41]. The package uses a SPMD parallel programming model and multithreading to enhance efficiency in distributed-memory parallel environments with multicore nodes and possibly GPU accelerators. It uses MPI for communication, OpenMP for threading, and CUDA for GPU support. In order to use the SUNLINSOL_SUPERLUDIST interface to SuperLU_DIST, it is assumed that SuperLU_DIST has been installed on the system prior to installation of SUNDIALS, and that SUNDIALS has been configured appropriately to link with SuperLU_DIST (see Appendix A for details). Additionally, the adapter only supports double-precision calculations, and therefore cannot be compiled if SUNDIALS is configured to use single or extended precision. Moreover, since the SuperLU_DIST library may be installed to support either 32-bit or...
64-bit integers, it is assumed that the SuperLU_DIST library is installed using the same integer size as SUNDIALS.

The SuperLU_DIST library provides many options to control how a linear system will be solved. These options may be set by a user on an instance of the superlu_dist_options_t struct, and then it may be provided as an argument to the SUNDIALS_SUNLINSOL_SUPERLUDIST constructor. The SUNDIALS_SUNLINSOL_SUPERLUDIST module will respect all options set except for Fact – this option is necessarily modified by the SUNDIALS_SUNLINSOL_SUPERLUDIST module in the setup and solve routines.

Since the linear systems that arise within the context of SUNDIALS calculations will typically have identical sparsity patterns, the SUNDIALS_SUNLINSOL_SUPERLUDIST module is constructed to perform the following operations:

- The first time that the “setup” routine is called, it sets the SuperLU_DIST option Fact to DOFACT so that a subsequent call to the “solve” routine will perform a symbolic factorization, followed by an initial numerical factorization before continuing to solve the system.

- On subsequent calls to the “setup” routine, it sets the SuperLU_DIST option Fact to SamePattern so that a subsequent call to “solve” will perform factorization assuming the same sparsity pattern as prior, i.e. it will reuse the column permutation vector.

- If “setup” is called prior to the “solve” routine, then the “solve” routine will perform a symbolic factorization, followed by an initial numerical factorization before continuing to the sparse triangular solves, and, potentially, iterative refinement. If “setup” is not called prior, “solve” will skip to the triangular solve step. We note that in this solve SuperLU_DIST operates on the native data arrays for the right-hand side and solution vectors, without requiring costly data copies.

10.10.2 SUNLinearSolver_SuperLUDIST functions

The SUNDIALS_SUNLINSOL_SUPERLUDIST module defines implementations of all “direct” linear solver operations listed in Sections 10.1.1-10.1.3:

- SUNLinSolGetType_SuperLUDIST

- SUNLinSolInitialize_SuperLUDIST – this sets the first_factorize flag to 1 and resets the internal SuperLU_DIST statistics variables.

- SUNLinSolSetup_SuperLUDIST – this sets the appropriate SuperLU_DIST options so that a subsequent solve will perform a symbolic and numerical factorization before proceeding with the triangular solves

- SUNLinSolSolve_SuperLUDIST – this calls the SuperLU_DIST solve routine to perform factorization (if the setup routine was called prior) and then use the LU factors to solve the linear system.

- SUNLinSolLastFlag_SuperLUDIST

- SUNLinSolSpace_SuperLUDIST – this only returns information for the storage within the solver interface, i.e. storage for the integers last_flag and first_factorize. For additional space requirements, see the SuperLU_DIST documentation.

- SUNLinSolFree_SuperLUDIST

In addition, the module SUNDIALS_SUNLINSOL_SUPERLUDIST provides the following user-callable routines:
SUNLinSol_SuperLUDIST

Call

\[
\text{LS} = \text{SUNLinSol\_SuperLUDIST}(y, A, \text{grid}, \text{lu}, \text{scaleperm}, \text{solve}, \text{stat}, \text{options});
\]

Description

The function SUNLinSol\_SuperLUDIST creates and allocates memory for a SUNLIN- SOL\_SUPERLUDIST object.

Arguments

- \(y\) (N\_Vector) a template for cloning vectors needed within the solver
- \(A\) (SUNMatrix) a SUNMATRIX\_SLUNRLOC matrix template for cloning matrices needed within the solver
- \(\text{grid}\) (gridinfo_t*)
- \(\text{lu}\) (LUstruct_t*)
- \(\text{scaleperm}\) (ScalePermstruct_t*)
- \(\text{solve}\) (SOLVEstruct_t*)
- \(\text{stat}\) (SuperLUStat_t*)
- \(\text{options}\) (superlu\_dist\_options_t*)

Return value

This returns a SUNLinearSolver object. If either \(A\) or \(y\) are incompatible then this routine will return NULL.

Notes

This routine analyzes the input matrix and vector to determine the linear system size and to assess compatibility with the SuperLU\_DIST library.

This routine will perform consistency checks to ensure that it is called with consistent NVECTOR and SUNMATRIX implementations. These are currently limited to the SUNMATRIX\_SLUNRLOC matrix type and the NVECTOR\_SERIAL, NVECTOR\_PARALLEL, NVECTOR\_PARHYPER, NVECTOR\_OPENMP, and NVECTOR\_PTHREADS vector types. As additional compatible matrix and vector implementations are added to SUNDIALS, these will be included within this compatibility check.

The \(\text{grid}\), \(\text{lu}\), \(\text{scaleperm}\), \(\text{solve}\), and \(\text{options}\) arguments are not checked and are passed directly to SuperLU\_DIST routines.

Some struct members of the \(\text{options}\) argument are modified internally by the SUNLIN- SOL\_SUPERLUDIST solver. Specifically the member \text{Fact}, is modified in the setup and solve routines.

SUNLinSol\_SuperLUDIST\_GetBerr

Call

\[
\text{realtype berr} = \text{SUNLinSol\_SuperLUDIST\_GetBerr}(\text{LS});
\]

Description

The function SUNLinSol\_SuperLUDIST\_GetBerr returns the componentwise relative backward error of the computed solution.

Arguments

- \(\text{LS}\) (SUNLinearSolver) the SUNLIN\_SOL\_SUPERLUDIST object

Return value

realtype

Notes

SUNLinSol\_SuperLUDIST\_GetGridinfo

Call

\[
\text{gridinfo}_t* \text{grid} = \text{SUNLinSol\_SuperLUDIST\_GetGridinfo}(\text{LS});
\]

Description

The function SUNLinSol\_SuperLUDIST\_GetGridinfo returns the SuperLU\_DIST structure that contains the 2D process grid.

Arguments

- \(\text{LS}\) (SUNLinearSolver) the SUNLIN\_SOL\_SUPERLUDIST object

Return value

gridinfo_t*

Notes
**SUNLinSol_SuperLUDIST_GetLUstruct**

Call: `LUstruct_t *lu = SUNLinSol_SuperLUDIST_GetLUstruct(LS);`

Description: The function `SUNLinSol_SuperLUDIST_GetLUstruct` returns the SuperLU_DIST structure that contains the distributed $L$ and $U$ factors.

Arguments: `LS` (SUNLinearSolver) the SUNLINSOL_SUPERLUDIST object

Return value: `LUstruct_t*`

Notes

**SUNLinSol_SuperLUDIST_GetSuperLUOptions**

Call: `superlu_dist_options_t *opts = SUNLinSol_SuperLUDIST_GetSuperLUOptions(LS);`

Description: The function `SUNLinSol_SuperLUDIST_GetSuperLUOptions` returns the SuperLU_DIST structure that contains the options which control how the linear system is factorized and solved.

Arguments: `LS` (SUNLinearSolver) the SUNLINSOL_SUPERLUDIST object

Return value: `superlu_dist_options_t*`

Notes

**SUNLinSol_SuperLUDIST_GetScalePermstruct**

Call: `ScalePermstruct_t *sp = SUNLinSol_SuperLUDIST_GetScalePermstruct(LS);`

Description: The function `SUNLinSol_SuperLUDIST_GetScalePermstruct` returns the SuperLU_DIST structure that contains the vectors that describe the transformations done to the matrix, $A$.

Arguments: `LS` (SUNLinearSolver) the SUNLINSOL_SUPERLUDIST object

Return value: `ScalePermstruct_t*`

Notes

**SUNLinSol_SuperLUDIST_GetSOLVEstruct**

Call: `SOLVEstruct_t *solve = SUNLinSol_SuperLUDIST_GetSOLVEstruct(LS);`

Description: The function `SUNLinSol_SuperLUDIST_GetSOLVEstruct` returns the SuperLU_DIST structure that contains information for communication during the solution phase.

Arguments: `LS` (SUNLinearSolver) the SUNLINSOL_SUPERLUDIST object

Return value: `SOLVEstruct_t*`

Notes

**SUNLinSol_SuperLUDIST_GetSuperLUStat**

Call: `SuperLUStat_t *stat = SUNLinSol_SuperLUDIST_GetSuperLUStat(LS);`

Description: The function `SUNLinSol_SuperLUDIST_GetSuperLUStat` returns the SuperLU_DIST structure that stores information about runtime and flop count.

Arguments: `LS` (SUNLinearSolver) the SUNLINSOL_SUPERLUDIST object

Return value: `SuperLUStat_t*`

Notes
10.10.3 SUNLinearSolver_SuperLUDIST content

The SUNLINSOL_SUPERLUDIST module defines the content field of a SUNLinearSolver to be the following structure:

```c
struct _SUNLinearSolverContent_SuperLUDIST {
    boolean type first_factorize;
    int last_flag;
    realtype berr;
    gridinfo_t *grid;
    LUstruct_t *lu;
    superlu_dist_options_t *options;
    ScalePermstruct_t *scaleperm;
    SOLVEstruct_t *solve;
    SuperLUStat_t *stat;
    sunindextype N;
};
```

These entries of the content field contain the following information:

- **first_factorize** - flag indicating whether the factorization has ever been performed,
- **last_flag** - last error return flag from calls to internal routines,
- **berr** - the componentwise relative backward error of the computed solution,
- **grid** - pointer to the SuperLU_DIST structure that stores the 2D process grid,
- **lu** - pointer to the SuperLU_DIST structure that stores the distributed \( L \) and \( U \) factors,
- **options** - pointer to SuperLU_DIST options structure,
- **scaleperm** - pointer to the SuperLU_DIST structure that stores vectors describing the transformations done to the matrix, \( A \),
- **solve** - pointer to the SuperLU_DIST solve structure,
- **stat** - pointer to the SuperLU_DIST structure that stores information about runtime and flop count,
- **N** - the number of equations in the system

10.11 The SUNLinearSolver_SuperLUMT implementation

This section describes the SUNLINSOL implementation for solving sparse linear systems with SuperLU_MT. The SUPERLUMT module is designed to be used with the corresponding SUNMATRIX_SPARSE matrix type, and one of the serial or shared-memory NVECTOR implementations (NVECTOR_SERIAL, NVECTOR_OPENMP, or NVECTOR_PTHREADS). While these are compatible, it is not recommended to use a threaded vector module with SUNLINSOL_SUPERLUMT unless it is the NVECTOR_OPENMP module and the SUPERLUMT library has also been compiled with OpenMP.

The header file to include when using this module is `sunlinsol/sunlinsol_superlumt.h`. The installed module library to link to is `libsundials_sunlinsolsuperlumt.lib` where `.lib` is typically `.so` for shared libraries and `.a` for static libraries.

The SUNLINSOL_SUPERLUMT module is a SUNLINSOL wrapper for the SUPERLUMT sparse matrix factorization and solver library written by X. Sherry Li [4, 39, 21]. The package performs matrix factorization using threads to enhance efficiency in shared memory parallel environments. It should be noted that threads are only used in the factorization step. In order to use the SUNLINSOL_SUPERLUMT interface to SUPERLUMT, it is assumed that SUPERLUMT has been installed on the system prior to installation of Sundials, and that Sundials has been configured appropriately to link with SUPERLUMT.
(see Appendix A for details). Additionally, this wrapper only supports single- and double-precision calculations, and therefore cannot be compiled if SUNDIALS is configured to have realtype set to extended (see Section 4.2). Moreover, since the SUPERLUMT library may be installed to support either 32-bit or 64-bit integers, it is assumed that the SUPERLUMT library is installed using the same integer precision as the SUNDIALS sunindextype option.

10.11.1 SUNLinearSolver_SuperLUMT description

The SUPERLUMT library has a symbolic factorization routine that computes the permutation of the linear system matrix to reduce fill-in on subsequent LU factorizations (using COLAMD, minimal degree ordering on $A^T \cdot A$, minimal degree ordering on $A^T + A$, or natural ordering). Of these ordering choices, the default value in the SUNLINSOL_SUPERLUMT module is the COLAMD ordering.

Since the linear systems that arise within the context of SUNDIALS calculations will typically have identical sparsity patterns, the SUNLINSOL_SUPERLUMT module is constructed to perform the following operations:

- The first time that the “setup” routine is called, it performs the symbolic factorization, followed by an initial numerical factorization.
- On subsequent calls to the “setup” routine, it skips the symbolic factorization, and only refactors the input matrix.
- The “solve” call performs pivoting and forward and backward substitution using the stored SUPERLUMT data structures. We note that in this solve SUPERLUMT operates on the native data arrays for the right-hand side and solution vectors, without requiring costly data copies.

10.11.2 SUNLinearSolver_SuperLUMT functions

The module SUNLINSOL_SUPERLUMT provides the following user-callable constructor for creating a SUNLinearSolver object.

```
SUNLinSol_SuperLUMT
```

Call

```
LS = SUNLinSol_SuperLUMT(y, A, num_threads);
```

Description

The function SUNLinSol_SuperLUMT creates and allocates memory for a SuperLU_MT-based SUNLinearSolver object.

Arguments

- $y$ (N_Vector) a template for cloning vectors needed within the solver
- $A$ (SUNMatrix) a SUNMATRIX_SPARSE matrix template for cloning matrices needed within the solver
- num_threads (int) desired number of threads (OpenMP or Pthreads, depending on how SUPERLUMT was installed) to use during the factorization steps

Return value

This returns a SUNLinearSolver object. If either $A$ or $y$ are incompatible then this routine will return NULL.

Notes

This routine analyzes the input matrix and vector to determine the linear system size and to assess compatibility with the SUPERLUMT library. This routine will perform consistency checks to ensure that it is called with consistent NVECTOR and SUNMATRIX implementations. These are currently limited to the SUNMATRIX_SPARSE matrix type (using either CSR or CSC storage formats) and the NVECTOR_SERIAL, NVECTOR_OPENMP, and NVECTOR_PTHREADS vector types. As additional compatible matrix and vector implementations are added to SUNDIALS, these will be included within this compatibility check.

The num_threads argument is not checked and is passed directly to SUPERLUMT routines.
10.11 The SUNLinearSolver_SuperLUMT implementation

Deprecated Name For backward compatibility, the wrapper function SUNSuperLUMT with identical input and output arguments is also provided.

The sunlinsol_superlumt module defines implementations of all “direct” linear solver operations listed in Sections 10.1.1 – 10.1.3:

- **SUNLinSolGetType_SuperLUMT**
- **SUNLinSolInitialize_SuperLUMT** – this sets the first_factorize flag to 1 and resets the internal SUPERLUMT statistics variables.
- **SUNLinSolSetup_SuperLUMT** – this performs either a LU factorization or refactorization of the input matrix.
- **SUNLinSolSolve_SuperLUMT** – this calls the appropriate SUPERLUMT solve routine to utilize the LU factors to solve the linear system.
- **SUNLinSolLastFlag_SuperLUMT**
- **SUNLinSolSpace_SuperLUMT** – this only returns information for the storage within the solver interface, i.e. storage for the integers last_flag and first_factorize. For additional space requirements, see the SUPERLUMT documentation.
- **SUNLinSolFree_SuperLUMT**

The sunlinsol_superlumt module also defines the following additional user-callable function.

```
SUNLinSol_SuperLUMTSetOrdering
```

Call `retval = SUNLinSol_SuperLUMTSetOrdering(LS, ordering);`

Description This function sets the ordering used by SUPERLUMT for reducing fill in the linear solve.

Arguments `LS` (SUNLinearSolver) the SUNLINSOL_SUPERLUMT object
`ordering` (int) a flag indicating the ordering algorithm to use, the options are:

- 0 natural ordering
- 1 minimal degree ordering on $A^T A$
- 2 minimal degree ordering on $A^T + A$
- 3 COLAMD ordering for unsymmetric matrices

The default is 3 for COLAMD.

Return value The return values from this function are SUNLS_MEM_NULL (S is NULL), SUNLS_ILL_INPUT (invalid ordering choice), or SUNLS_SUCCESS.

Deprecated Name For backward compatibility, the wrapper function SUNSuperLUMTSetOrdering with identical input and output arguments is also provided.

10.11.3 SUNLinearSolver_SuperLUMT Fortran interfaces

For solvers that include a Fortran interface module, the SUNLINSOL_SUPERLUMT module also includes a Fortran-callable function for creating a SUNLinearSolver object.

```
FSUNSUPERLUMTINIT
```

Call `FSUNSUPERLUMTINIT(code, num_threads, ier)`

Description The function FSUNSUPERLUMTINIT can be called for Fortran programs to create a SUNLINSOL_KLU object.

Arguments `code` (int*) is an integer input specifying the solver id (1 for CVODE, 2 for IDA, 3 for KINSOL, and 4 for ARKODE).
num_threads (int*) desired number of threads (OpenMP or Pthreads, depending on how SUPERLUMT was installed) to use during the factorization steps.

Return value ier is a return completion flag equal to 0 for a success return and -1 otherwise. See printed message for details in case of failure.

Notes
This routine must be called after both the NVVECTOR and SUNMATRIX objects have been initialized.

Additionally, when using ARKODE with a non-identity mass matrix, the SUNLINSOL_SUPERLUMT module includes a Fortran-callable function for creating a SUNLinearSolver mass matrix solver object.

FSUNMASSSUPERLUMTINIT
Call
FSUNMASSSUPERLUMTINIT(num_threads, ier)

Description
The function FSUNMASSSUPERLUMTINIT can be called for Fortran programs to create a SuperLU_MT-based SUNLinearSolver object for mass matrix linear systems.

Arguments
num_threads (int*) desired number of threads (OpenMP or Pthreads, depending on how SUPERLUMT was installed) to use during the factorization steps.

Return value ier is an int return completion flag equal to 0 for a success return and -1 otherwise. See printed message for details in case of failure.

Notes
This routine must be called after both the NVVECTOR and SUNMATRIX mass-matrix objects have been initialized.

The SUNLinSol_SuperLUMTSetOrdering routine also supports Fortran interfaces for the system and mass matrix solvers:

FSUNSUPERLUMTSETORDERING
Call
FSUNSUPERLUMTSETORDERING(code, ordering, ier)

Description
The function FSUNSUPERLUMTSETORDERING can be called for Fortran programs to update the ordering algorithm in a SUNLINSOL_SUPERLUMT object.

Arguments
code (int*) is an integer input specifying the solver id (1 for CVODE, 2 for IDA, 3 for KINSOL, and 4 for ARKODE).
ordering (int*) a flag indicating the ordering algorithm, options are:
   0 natural ordering
   1 minimal degree ordering on $A^T A$
   2 minimal degree ordering on $A^T + A$
   3 COLAMD ordering for unsymmetric matrices
   The default is 3 for COLAMD.

Return value ier is an int return completion flag equal to 0 for a success return and -1 otherwise. See printed message for details in case of failure.

Notes
See SUNLinSol_SuperLUMTSetOrdering for complete further documentation of this routine.

FSUNMASSSUPERLUMTSETORDERING
Call
FSUNMASSSUPERLUMTSETORDERING(ordering, ier)

Description
The function FSUNMASSSUPERLUMTSETORDERING can be called for Fortran programs to update the ordering algorithm in a SUNLINSOL_SUPERLUMT object for mass matrix linear systems.

Arguments
ordering (int*) a flag indicating the ordering algorithm, options are:
   0 natural ordering
10.12 The SUNLinearSolver_cuSolverSp_batchQR implementation

1 minimal degree ordering on $A^T A$
2 minimal degree ordering on $A^T + A$
3 COLAMD ordering for unsymmetric matrices

The default is 3 for COLAMD.

Return value ier is a int return completion flag equal to 0 for a success return and -1 otherwise. See printed message for details in case of failure.

Notes See SUNLinSol_SuperLUMTSetOrdering for complete further documentation of this routine.

10.11.4 SUNLinearSolver_SuperLUMT content

The SUNLINSOL_SUPERLUMT module defines the content field of a SUNLinearSolver as the following structure:

```c
struct _SUNLinearSolverContent_SuperLUMT {
    int last_flag;
    int first_factorize;
    Gstat_t *Gstat;
    sunindextype *perm_r, *perm_c;
    sunindextype N;
    int num_threads;
    realtype diag_pivot_thresh;
    int ordering;
    superlumt_options_t *options;
};
```

These entries of the content field contain the following information:

- last_flag - last error return flag from internal function evaluations,
- first_factorize - flag indicating whether the factorization has ever been performed,
- A, AC, L, U, B - SuperMatrix pointers used in solve,
- Gstat - GStat_t object used in solve,
- perm_r, perm_c - permutation arrays used in solve,
- N - size of the linear system,
- num_threads - number of OpenMP/Pthreads threads to use,
- diag_pivot_thresh - threshold on diagonal pivoting,
- ordering - flag for which reordering algorithm to use,
- options - pointer to SUPERLUMT options structure.

10.12 The SUNLinearSolver_cuSolverSp_batchQR implementation

The SUNLinearSolver_cuSolverSp_batchQR implementation of the SUNLINSOL API is designed to be used with the SUNMATRIX SPARSE matrix type, and the NVVECTOR CUDA vector type with managed memory. The header file to include when using this module is sunlinsol/sunlinsol_cusolversp_batchqr.h. The installed library to link to is libsundials_sunlinsolcusolversp_batchqr.lib where .lib is typically .so for shared libraries and .a for static libraries.

The SUNLinearSolver_cuSolverSp_batchQR module is experimental and subject to change.
10.12.1 SUNLinearSolver_cuSolverSp\_batchQR description

The SUNLinearSolver_cuSolverSp\_batchQR implementation provides an interface to the batched sparse QR factorization method provided by the NVIDIA cuSOLVER library [2]. The module is designed for solving block diagonal linear systems of the form

\[
A = \begin{bmatrix}
A_1 & 0 & \cdots & 0 \\
0 & A_2 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & A_n
\end{bmatrix}
\]

where all block matrices \(A_j\) share the same sparsity pattern. The matrix must be in the CSR storage format. For further details about the method itself, review the NVIDIA documentation.

10.12.2 SUNLinearSolver_cuSolverSp\_batchQR functions

The SUNLinearSolver_cuSolverSp\_batchQR module defines implementations of all “direct” linear solver operations listed in Sections 10.1.1-10.1.3:

- SUNLinSolGetType_cuSolverSp\_batchQR
- SUNLinSolInitialize_cuSolverSp\_batchQR – this sets the first\_factorize flag to 1
- SUNLinSolSetup_cuSolverSp\_batchQR – this always copies the relevant SUNMATRIX\_SPARSE data to the GPU; if this is the first setup it will perform symbolic analysis on the system
- SUNLinSolSolve_cuSolverSp\_batchQR – this calls the cusolverSpXcsrqrsvBatched routine to perform factorization
- SUNLinSolLastFlag_cuSolverSp\_batchQR
- SUNLinSolFree_cuSolverSp\_batchQR

In addition, the module provides the following user-callable routines:

```c
SUNLinSol_cuSolverSp\_batchQR
```

**Call**

\[ LS = \text{SUNLinSol\_cuSolverSp\_batchQR}(y, A, nsubsys, subsys\_size, subsys\_nnz); \]

**Description**

The function SUNLinSol\_cuSolverSp\_batchQR creates and allocates memory for a SUNLINSOL object.

**Arguments**

- \(y\) (N\_Vector) a NV\_VECTOR\_CUDA vector for checking compatibility with the solver
- \(A\) (SUN\_Matrix) a SUN\_MATRIX\_SPARSE matrix for checking compatibility with the solver
- \(nsubsys\) (int) the number of subsystems, i.e., the number of blocks in the matrix
- \(subsys\_size\) (int) the number of rows/columns in a block
- \(subsys\_nnz\) (int) the number of nonzeros in a block

**Return value**

This returns a SUNLinearSolver object. If either \(A\) or \(y\) are incompatible then this routine will return NULL.

**Notes**

This routine analyzes the input matrix and vector to determine the linear system size and to assess compatibility with the solver.

This routine will perform consistency checks to ensure that it is called with consistent NV\_VECTOR and SUN\_MATRIX implementations. These are currently limited to the SUN\_MATRIX\_SPARSE matrix type and the NV\_VECTOR\_CUDA vector type. Since the SUN\_MATRIX\_SPARSE matrix type is only compatible with the NV\_VECTOR\_CUDA when using...
managed memory, the restriction is also in place for the linear solver. As additional compatible matrix and vector implementations are added to SUNDIALS, these will be included within this compatibility check.

```c
SUNLinSol_cuSolverSp_batchQR_GetDescription
Call SUNLinSol_cuSolverSp_batchQR_GetDescription(LS, &desc);
Description The function SUNLinSol_cuSolverSp_batchQR_GetDescription accesses the string description of the object (empty by default).
Arguments LS (SUNLinearSolver) a SUNLinSol_cuSolverSp_batchQR object
desc (char **) the string description of the linear solver
Return value None
```

```c
SUNLinSol_cuSolverSp_batchQR_SetDescription
Call SUNLinSol_cuSolverSp_batchQR_SetDescription(LS, desc);
Description The function SUNLinSol_cuSolverSp_batchQR_SetDescription sets the string description of the object (empty by default).
Arguments LS (SUNLinearSolver) a SUNLinSol_cuSolverSp_batchQR object
desc (const char *) the string description of the linear solver
Return value None
```

### 10.12.3 SUNLinearSolver_cuSolverSp_batchQR content

The SUNLinearSolver_cuSolverSp_batchQR module defines the `content` field of a SUNLinearSolver to be the following structure:

```c
struct _SUNLinearSolverContent_cuSolverSp_batchQR {
  int nsubsys; /* number of subsystems */
  int subsys_size; /* size of each subsystem */
  int subsys_nnz; /* number of nonzeros per subsystem */
  int last_flag; /* last return flag */
  boolean first_factorize; /* is this the first factorization? */
  size_t internal_size; /* size of cusolver internal buffer for Q and R */
  size_t workspace_size; /* size of cusolver memory block for num. factorization */
  cusolverSpHandle_t cusolver_handle; /* cuSolverSp context */
  csrqrInfo_t info; /* opaque cusolver data structure */
  size_t d_values; /* device array of matrix A values */
  size_t d_rowptr; /* device array of rowptrs for a subsystem */
  size_t d_colind; /* device array of column indices for a subsystem */
  void* workspace; /* memory block used by cusolver */
  const char* desc; /* description of this linear solver */
};
```

### 10.13 The SUNLinearSolver_SPGMR implementation

This section describes the SUNLINSOL implementation of the SPGMR (Scaled, Preconditioned, Generalized Minimum Residual [46]) iterative linear solver. The SUNLINSOL_SPGMR module is designed to be compatible with any NVector implementation that supports a minimal subset of operations (`NVClone`, `NVDotProd`, `NVScale`, `NLinearSum`, `NProd`, `NConst`, `NDiv`, and `NDestroy`).
When using Classical Gram-Schmidt, the optional function `N_VDotProdMulti` may be supplied for increased efficiency.

To access the `SUNLINSOL_SPGMR` module, include the header file `sunlinsol/sunlinsol_spgmr.h`. We note that the `SUNLINSOL_SPGMR` module is accessible from SUNDIALS packages without separately linking to the `lib sundials_sunlinsolspgmr` module library.

### 10.13.1 SUNLinearSolver_SPGMR description

This solver is constructed to perform the following operations:

- During construction, the `xcor` and `vtemp` arrays are cloned from a template `NVECTOR` that is input, and default solver parameters are set.
- User-facing “set” routines may be called to modify default solver parameters.
- Additional “set” routines are called by the SUNDIALS solver that interfaces with `SUNLINSOL_SPGMR` to supply the `ATimes`, `PSetup`, and `Psolve` function pointers and `s1` and `s2` scaling vectors.
- In the “initialize” call, the remaining solver data is allocated (`V`, `Hes`, `givens`, and `yg`)
- In the “setup” call, any non-`NULL` `PSetup` function is called. Typically, this is provided by the SUNDIALS solver itself, that translates between the generic `PSetup` function and the solver-specific routine (solver-supplied or user-supplied).
- In the “solve” call, the GMRES iteration is performed. This will include scaling, preconditioning, and restarts if those options have been supplied.

### 10.13.2 SUNLinearSolver_SPGMR functions

The `SUNLINSOL_SPGMR` module provides the following user-callable constructor for creating a `SUNLinearSolver` object.

```c
SUNLinSol_SPGMR
Call               LS = SUNLinSol_SPGMR(y, pretype, maxl);
Description        The function `SUNLinSol_SPGMR` creates and allocates memory for a `SPGMR` `SUNLinearSolver` object.
Arguments          y    (N_Vector) a template for cloning vectors needed within the solver
                   pretype (int) flag indicating the desired type of preconditioning, allowed values are:
                   • PREC_NONE (0)
                   • PREC_LEFT (1)
                   • PREC_RIGHT (2)
                   • PREC_BOTH (3)
                   Any other integer input will result in the default (no preconditioning).
                   maxl    (int) the number of Krylov basis vectors to use. Values ≤ 0 will result in the default value (5).
Return value       This returns a `SUNLinearSolver` object. If either `y` is incompatible then this routine will return NULL.
Notes              This routine will perform consistency checks to ensure that it is called with a consistent `NVECTOR` implementation (i.e. that it supplies the requisite vector operations). If `y` is incompatible, then this routine will return NULL.
We note that some SUNDIALS solvers are designed to only work with left preconditioning (IDA and IDAS) and others with only right preconditioning (KINSOL). While
it is possible to configure a SUNLINSOL_SPGMR object to use any of the preconditioning options with these solvers, this use mode is not supported and may result in inferior performance.

Deprecated Name For backward compatibility, the wrapper function SUNSPGMR with identical input and output arguments is also provided.

F2003 Name FSUNLinSol_SPGMR

The SUNLINSOL_SPGMR module defines implementations of all “iterative” linear solver operations listed in Sections 10.1.1 – 10.1.3:

- SUNLinSolGetType_SPGMR
- SUNLinSolInitialize_SPGMR
- SUNLinSolSetATimes_SPGMR
- SUNLinSolSetPreconditioner_SPGMR
- SUNLinSolSetScalingVectors_SPGMR
- SUNLinSolSetup_SPGMR
- SUNLinSolSolve_SPGMR
- SUNLinSolNumIters_SPGMR
- SUNLinSolResNorm_SPGMR
- SUNLinSolResid_SPGMR
- SUNLinSolLastFlag_SPGMR
- SUNLinSolSpace_SPGMR
- SUNLinSolFree_SPGMR

All of the listed operations are callable via the FORTRAN 2003 interface module by prepending an ‘F’ to the function name.

The SUNLINSOL_SPGMR module also defines the following additional user-callable functions.

SUNLinSol_SPGMRSetPrecType

Call retval = SUNLinSol_SPGMRSetPrecType(LS, pretype);

Description The function SUNLinSol_SPGMRSetPrecType updates the type of preconditioning to use in the SUNLINSOL_SPGMR object.

Arguments LS (SUNLinearSolver) the SUNLINSOL_SPGMR object to update
pretype (int) flag indicating the desired type of preconditioning, allowed values match those discussed in SUNLinSol_SPGMR.

Return value This routine will return with one of the error codes SUNLS_Ill_INPUT (illegal pretype), SUNLS_MEM_NULL (S is NULL) or SUNLS_SUCCESS.

Deprecated Name For backward compatibility, the wrapper function SUNSPGMRSetPrecType with identical input and output arguments is also provided.

F2003 Name FSUNLinSol_SPGMRSetPrecType
### SUNLinSol_SPGMRSetGSType

**Call**
```
retval = SUNLinSol_SPGMRSetGSType(LS, gstype);
```

**Description**
The function `SUNLinSol_SPGMRSetGSType` sets the type of Gram-Schmidt orthogonalization to use in the `SUNLINSOL_SPGMR` object.

**Arguments**
- `LS` *(SUNLinearSolver)*: the `SUNLINSOL_SPGMR` object to update.
- `gstype` *(int)*: flag indicating the desired orthogonalization algorithm; allowed values are:
  - `MODIFIED_GS` (1)
  - `CLASSICAL_GS` (2)

Any other integer input will result in a failure, returning error code `SUNLS_Ill_INPUT`.

**Return value**
This routine will return with one of the error codes `SUNLS_Ill_INPUT` (illegal pretype), `SUNLS_MEM_NULL` (S is NULL) or `SUNLS_SUCCESS`.

**Deprecated Name**
For backward compatibility, the wrapper function `SUNSPGMRSetGSType` with identical input and output arguments is also provided.

**F2003 Name**
`FSUNLinSol_SPGMRSetGSType`

### SUNLinSol_SPGMRSetMaxRestarts

**Call**
```
retval = SUNLinSol_SPGMRSetMaxRestarts(LS, maxrs);
```

**Description**
The function `SUNLinSol_SPGMRSetMaxRestarts` sets the number of GMRES restarts to allow in the `SUNLINSOL_SPGMR` object.

**Arguments**
- `LS` *(SUNLinearSolver)*: the `SUNLINSOL_SPGMR` object to update.
- `maxrs` *(int)*: integer indicating number of restarts to allow. A negative input will result in the default of 0.

**Return value**
This routine will return with one of the error codes `SUNLS_MEM_NULL` (S is NULL) or `SUNLS_SUCCESS`.

**Deprecated Name**
For backward compatibility, the wrapper function `SUNSPGMRSetMaxRestarts` with identical input and output arguments is also provided.

**F2003 Name**
`FSUNLinSol_SPGMRSetMaxRestarts`

### 10.13.3 SUNLinearSolver_SPGMR Fortran interfaces

The `SUNLINSOL_SPGMR` module provides a FORTRAN 2003 module as well as FORTRAN 77 style interface functions for use from FORTRAN applications.

**FORTRAN 2003 interface module**

The `fsunlinsol_spgrm.mod` FORTRAN module defines interfaces to all `SUNLINSOL_SPGMR` C functions using the intrinsic `iso_c_binding` module which provides a standardized mechanism for interoperating with C. As noted in the C function descriptions above, the interface functions are named after the corresponding C function, but with a leading ‘F’. For example, the function `SUNLinSol_SPGMR` is interfaced as `FSUNLinSol_SPGMR`.

The FORTRAN 2003 `SUNLINSOL_SPGMR` interface module can be accessed with the `use` statement, i.e. `use fsunlinsol_spgrm.mod`, and linking to the library `libsundials_fsunlinsolspgmr_mod.lib` in addition to the C library. For details on where the library and module file `fsunlinsol_spgrm.mod.mod` are installed see Appendix A. We note that the module is accessible from the FORTRAN 2003 SUNDIALS integrators without separately linking to the `libsundials_fsunlinsolspgmr_mod` library.
FORTRAN 77 interface functions

For solvers that include a FORTRAN 77 interface module, the SUNLINSOL_SPGMR module also includes a Fortran-callable function for creating a SUNLinearSolver object.

FSUNSPGMRINIT

Call FSUNSPGMRINIT(code, pretype, maxl, ier)

Description The function FSUNSPGMRINIT can be called for Fortran programs to create a SUNLINSOL_SPGMR object.

Arguments code (int*) is an integer input specifying the solver id (1 for CVODE, 2 for IDA, 3 for KINSOL, and 4 for ARKODE).

pretype (int*) flag indicating desired preconditioning type

maxl (int*) flag indicating Krylov subspace size

Return value ier is a return completion flag equal to 0 for a success return and -1 otherwise. See printed message for details in case of failure.

Notes This routine must be called after the NVECTOR object has been initialized.

Allowable values for pretype and maxl are the same as for the C function SUNLinSol_SPGMR.

Additionally, when using ARKODE with a non-identity mass matrix, the SUNLINSOL_SPGMR module includes a Fortran-callable function for creating a SUNLinearSolver mass matrix solver object.

FSUNMASSSPGMRINIT

Call FSUNMASSSPGMRINIT(pretype, maxl, ier)

Description The function FSUNMASSSPGMRINIT can be called for Fortran programs to create a SUNLINSOL_SPGMR object for mass matrix linear systems.

Arguments pretype (int*) flag indicating desired preconditioning type

maxl (int*) flag indicating Krylov subspace size

Return value ier is an int return completion flag equal to 0 for a success return and -1 otherwise. See printed message for details in case of failure.

Notes This routine must be called after the NVECTOR object has been initialized.

Allowable values for pretype and maxl are the same as for the C function SUNLinSol_SPGMR.

The SUNLinSol_SPGMRSetPrecType, SUNLinSol_SPGMRSetGSType and SUNLinSol_SPGMRSetMaxRestarts routines also support Fortran interfaces for the system and mass matrix solvers.

FSUNSPGMRSETGSTYPE

Call FSUNSPGMRSETGSTYPE(code, gstype, ier)

Description The function FSUNSPGMRSETGSTYPE can be called for Fortran programs to change the Gram-Schmidt orthogonalization algorithm.

Arguments code (int*) is an integer input specifying the solver id (1 for CVODE, 2 for IDA, 3 for KINSOL, and 4 for ARKODE).

gstype (int*) flag indicating the desired orthogonalization algorithm.

Return value ier is an int return completion flag equal to 0 for a success return and -1 otherwise. See printed message for details in case of failure.

Notes See SUNLinSol_SPGMRSetGSType for complete further documentation of this routine.
**FSUNMASSSPGMRSETGSTYPE**

Call: `FSUNMASSSPGMRSETGSTYPE(gstype, ier)`

Description: The function `FSUNMASSSPGMRSETGSTYPE` can be called for Fortran programs to change the Gram-Schmidt orthogonalization algorithm for mass matrix linear systems.

Arguments: The arguments are identical to `FSUNSPGMRSETGSTYPE` above, except that `code` is not needed since mass matrix linear systems only arise in ARKODE.

Return value: `ier` is an int return completion flag equal to 0 for a success return and -1 otherwise. See printed message for details in case of failure.

Notes: See `SUNLinSol_SPGMRSetGSType` for complete further documentation of this routine.

**FSUNSPGMRSETPRECTYPE**

Call: `FSUNSPGMRSETPRECTYPE(code, pretype, ier)`

Description: The function `FSUNSPGMRSETPRECTYPE` can be called for Fortran programs to change the type of preconditioning to use.

Arguments: `code` (int*) is an integer input specifying the solver id (1 for CVODE, 2 for IDA, 3 for KINSOL, and 4 for ARKODE).

`pretype` (int*) flag indicating the type of preconditioning to use.

Return value: `ier` is an int return completion flag equal to 0 for a success return and -1 otherwise. See printed message for details in case of failure.

Notes: See `SUNLinSol_SPGMRSetPrecType` for complete further documentation of this routine.

**FSUNMASSSPGMRSETPRECTYPE**

Call: `FSUNMASSSPGMRSETPRECTYPE(pretype, ier)`

Description: The function `FSUNMASSSPGMRSETPRECTYPE` can be called for Fortran programs to change the type of preconditioning for mass matrix linear systems.

Arguments: The arguments are identical to `FSUNSPGMRSETPRECTYPE` above, except that `code` is not needed since mass matrix linear systems only arise in ARKODE.

Return value: `ier` is an int return completion flag equal to 0 for a success return and -1 otherwise. See printed message for details in case of failure.

Notes: See `SUNLinSol_SPGMRSetPrecType` for complete further documentation of this routine.

**FSUNSPGMRSETMAXRS**

Call: `FSUNSPGMRSETMAXRS(code, maxrs, ier)`

Description: The function `FSUNSPGMRSETMAXRS` can be called for Fortran programs to change the maximum number of restarts allowed for SPGMR.

Arguments: `code` (int*) is an integer input specifying the solver id (1 for CVODE, 2 for IDA, 3 for KINSOL, and 4 for ARKODE).

`maxrs` (int*) maximum allowed number of restarts.

Return value: `ier` is an int return completion flag equal to 0 for a success return and -1 otherwise. See printed message for details in case of failure.

Notes: See `SUNLinSol_SPGMRSetMaxRestarts` for complete further documentation of this routine.
10.13 The SUNLinearSolver_SPGMR implementation

FSUNMASSSPGMRSETMAXRS

Call
FSUNMASSSPGMRSETMAXRS(maxrs, ier)

Description
The function FSUNMASSSPGMRSETMAXRS can be called for Fortran programs to change
the maximum number of restarts allowed for SPGMR for mass matrix linear systems.

Arguments
The arguments are identical to FSUNSPGMRSETMAXRS above, except that code is not
needed since mass matrix linear systems only arise in ARKODE.

Return value
ier is a int return completion flag equal to 0 for a success return and -1 otherwise.
See printed message for details in case of failure.

Notes
See SUNLinSol_SPGMRSetMaxRestarts for complete further documentation of this rou-
tine.

10.13.4 SUNLinearSolver_SPGMR content

The SUNLINSOL_SPGMR module defines the content field of a SUNLinearSolver as the following struc-
ture:

struct _SUNLinearSolverContent_SPGMR {
    int maxl;
    int pretype;
    int gstype;
    int max_restarts;
    int numiters;
    realtype resnorm;
    int last_flag;
    ATimesFn ATimes;
    void* ATData;
    PSetupFn Psetup;
    PSolveFn Psolve;
    void* PData;
    N_Vector s1;
    N_Vector s2;
    N_Vector *V;
    realtype **Hes;
    realtype *givens;
    N_Vector xcor;
    realtype *yg;
    N_Vector vtemp;
};

These entries of the content field contain the following information:
maxl       - number of GMRES basis vectors to use (default is 5),
pretype    - flag for type of preconditioning to employ (default is none),
gstype     - flag for type of Gram-Schmidt orthogonalization (default is modified Gram-Schmidt),
max_restarts - number of GMRES restarts to allow (default is 0),
umiters    - number of iterations from the most-recent solve,
resnorm    - final linear residual norm from the most-recent solve,
last_flag  - last error return flag from an internal function,
ATimes     - function pointer to perform $Av$ product,
ATData     - pointer to structure for ATimes,
Psetup     - function pointer to preconditioner setup routine,
Description of the SUNLinearSolver module

Psolve - function pointer to preconditioner solve routine,
PData  - pointer to structure for Psetup and Psolve,
s1, s2 - vector pointers for supplied scaling matrices (default is NULL),
V     - the array of Krylov basis vectors $v_1, \ldots, v_{maxl+1}$, stored in $V[0], \ldots, V[maxl]$. Each $v_i$ is a vector of type NVeCTOR.,
Hes   - the $(maxl + 1) \times maxl$ Hessenberg matrix. It is stored row-wise so that the $(i,j)$th element is given by $Hes[i][j],$
givens - a length $2*maxl$ array which represents the Givens rotation matrices that arise in the GMRES algorithm. These matrices are $F_0, F_1, \ldots, F_j$, where

$$F_i = \begin{bmatrix}
1 & & & & \\
& \ddots & & & \\
& & 1 & -s_i & \\
& & s_i & c_i & \\
& & & \ddots & \\
& & & & 1
\end{bmatrix},$$

are represented in the givens vector as $givens[0] = c_0, givens[1] = s_0, givens[2] = c_1, givens[3] = s_1, \ldots, givens[2j] = c_j, givens[2j+1] = s_j$,
xcor  - a vector which holds the scaled, preconditioned correction to the initial guess,
yg    - a length $(maxl+1)$ array of realtype values used to hold “short” vectors (e.g. $y$ and $g$),
vtemp - temporary vector storage.

10.14 The SUNLinearSolver_SPFGMR implementation

This section describes the SUNLINSOL implementation of the SPFGMR (Scaled, Preconditioned, Flexible, Generalized Minimum Residual [45]) iterative linear solver. The SUNLINSOL_SPFGMR module is designed to be compatible with any NVeCTOR implementation that supports a minimal subset of operations (NVClone, NVDotProd, NVScale, NVLinearSum, NVProd, NVConst, NVDiv, and NVDestroy). When using Classical Gram-Schmidt, the optional function NVDotProdMulti may be supplied for increased efficiency. Unlike the other Krylov iterative linear solvers supplied with SUNDIALS, SPFGMR is specifically designed to work with a changing preconditioner (e.g. from an iterative method).

To access the SUNLINSOL_SPFGMR module, include the header file sunlinsol/sunlinsol_spfgmr.h. We note that the SUNLINSOL_SPFGMR module is accessible from SUNDIALS packages without separately linking to the libsundials_sunlinsolspfgmr module library.

10.14.1 SUNLinearSolver_SPFGMR description

This solver is constructed to perform the following operations:

- During construction, the xcor and vtemp arrays are cloned from a template NVeCTOR that is input, and default solver parameters are set.
- User-facing “set” routines may be called to modify default solver parameters.
- Additional “set” routines are called by the SUNDIALS solver that interfaces with SUNLINSOL_SPFGMR to supply the ATimes, PSetup, and Psolve function pointers and s1 and s2 scaling vectors.
• In the “initialize” call, the remaining solver data is allocated \((V, Hes, givens, \text{and } yg)\)

• In the “setup” call, any non-NULL PSetup function is called. Typically, this is provided by the SUNDIALS solver itself, that translates between the generic PSetup function and the solver-specific routine (solver-supplied or user-supplied).

• In the “solve” call, the FGMRES iteration is performed. This will include scaling, preconditioning, and restarts if those options have been supplied.

### 10.14.2 SUNLinearSolver_SPFGMR functions

The sunlinsol_spfgmr module provides the following user-callable constructor for creating a SUNLinearSolver object.

```c
SUNLinSol SPFGMR
```

**Call**

\[ LS = \text{SUNLinSol SPFGMR}(y, \text{pretype}, \text{maxl}); \]

**Description**

The function SUNLinSol SPFGMR creates and allocates memory for a SPFGMR SUNLinearSolver object.

**Arguments**

- \(y\) (N_Vector) a template for cloning vectors needed within the solver
- \(\text{pretype}\) (int) flag indicating the desired type of preconditioning, allowed values are:
  - PREC_NONE (0)
  - PREC_LEFT (1)
  - PREC_RIGHT (2)
  - PREC_BOTH (3)
  
Any other integer input will result in the default (no preconditioning).

- \(\text{maxl}\) (int) the number of Krylov basis vectors to use. Values \(\leq 0\) will result in the default value (5).

**Return value**

This returns a SUNLinearSolver object. If either \(y\) is incompatible then this routine will return NULL.

**Notes**

This routine will perform consistency checks to ensure that it is called with a consistent NVECTOR implementation (i.e. that it supplies the requisite vector operations). If \(y\) is incompatible, then this routine will return NULL.

We note that some SUNDIALS solvers are designed to only work with left preconditioning (IDA and IDAS) and others with only right preconditioning (KINSOL). While it is possible to configure a SUNLINSOL SPFGMR object to use any of the preconditioning options with these solvers, this use mode is not supported and may result in inferior performance.

**F2003 Name**

FSUNLinSol_SPFGMR

SUNSFGMR The sunlinsol_spfgmr module defines implementations of all “iterative” linear solver operations listed in Sections 10.1.1 – 10.1.3:
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- SUNLinSolNumIters_SPFGMRR
- SUNLinSolResNorm_SPFGMRR
- SUNLinSolResid_SPFGMRR
- SUNLinSolLastFlag_SPFGMRR
- SUNLinSolSpace_SPFGMRR
- SUNLinSolFree_SPFGMRR

All of the listed operations are callable via the FORTRAN 2003 interface module by prepending an ‘F’ to the function name.

The SUNLINSOL_SPFGMRR module also defines the following additional user-callable functions.

### SUNLinSol_SPFGMRRSetPrecType

**Call**

```fortran
retval = SUNLinSol_SPFGMRRSetPrecType(LS, pretype);
```

**Description**
The function SUNLinSol_SPFGMRRSetPrecType updates the type of preconditioning to use in the SUNLINSOL_SPFGMRR object.

**Arguments**
- LS (SUNLinearSolver) the SUNLINSOL_SPFGMRR object to update
- pretype (int) flag indicating the desired type of preconditioning, allowed values match those discussed in SUNLinSol_SPFGMRR.

**Return value**
This routine will return with one of the error codes SUNLS_ILL_INPUT (illegal pretype), SUNLS_MEM_NULL (S is NULL) or SUNLS_SUCCESS.

**Deprecated Name**
For backward compatibility, the wrapper function SUNSPFGMRRSetPrecType with identical input and output arguments is also provided.

**F2003 Name**
FSUNLinSol_SPFGMRRSetPrecType

### SUNLinSol_SPFGMRRSetGSType

**Call**

```fortran
retval = SUNLinSol_SPFGMRRSetGSType(LS, gstype);
```

**Description**
The function SUNLinSol_SPFGMRRSetGSType sets the type of Gram-Schmidt orthogonalization to use in the SUNLINSOL_SPFGMRR object.

**Arguments**
- LS (SUNLinearSolver) the SUNLINSOL_SPFGMRR object to update
- gstype (int) flag indicating the desired orthogonalization algorithm; allowed values are:
  - MODIFIED_GS (1)
  - CLASSICAL_GS (2)

Any other integer input will result in a failure, returning error code SUNLS_ILL_INPUT.

**Return value**
This routine will return with one of the error codes SUNLS_ILL_INPUT (illegal pretype), SUNLS_MEM_NULL (S is NULL) or SUNLS_SUCCESS.

**Deprecated Name**
For backward compatibility, the wrapper function SUNSPFGMRRSetGSType with identical input and output arguments is also provided.

**F2003 Name**
FSUNLinSol_SPFGMRRSetGSType
10.14 The SUNLinearSolver_SPFGMR implementation

**SUNLinSol_SPFGMRSetMaxRestarts**

**Call**

```
retval = SUNLinSol_SPFGMRSetMaxRestarts(LS, maxrs);
```

**Description**
The function `SUNLinSol_SPFGMRSetMaxRestarts` sets the number of GMRES restarts to allow in the SUNLinearSolver object.

**Arguments**
- `LS` (SUNLinearSolver) the SUNLinearSolver object to update
- `maxrs` (int) integer indicating number of restarts to allow. A negative input will result in the default of 0.

**Return value**
This routine will return with one of the error codes `SUNLS_MEM_NULL` (S is NULL) or `SUNLS_SUCCESS`.

**Deprecated Name**
For backward compatibility, the wrapper function `SUNSPFGMRSetMaxRestarts` with identical input and output arguments is also provided.

**F2003 Name**
`FSUNLinSol_SPFGMRSetMaxRestarts`

10.14.3 SUNLinearSolver_SPFGMR Fortran interfaces

The SUNLinearSolver_SPFGMR module provides a Fortran 2003 module as well as Fortran 77 style interface functions for use from Fortran applications.

**FORTRAN 2003 interface module**

The `fsunlinsol_spfgmr_mod` Fortran module defines interfaces to all SUNLinearSolver_SPFGMR C functions using the intrinsic `iso_c_binding` module which provides a standardized mechanism for interoperating with C. As noted in the C function descriptions above, the interface functions are named after the corresponding C function, but with a leading 'F'. For example, the function `SUNLinSol_SPFGMR` is interfaced as `FSUNLinSol_SPFGMR`.

The Fortran 2003 SUNLinearSolver_SPFGMR interface module can be accessed with the `use` statement, i.e. `use fsunlinsol_spfgmr_mod`, and linking to the library `libsundials_fsunlinsolspfgmr_mod.lib` in addition to the C library. For details on where the library and module file `fsunlinsol_spfgmr_mod.mod` are installed see Appendix A. We note that the module is accessible from the FORTRAN 2003 SUNDIALS integrators without separately linking to the `libsundials_fsunlinsolspfgmr_mod` library.

**FORTRAN 77 interface functions**

For solvers that include a Fortran 77 interface module, the SUNLinearSolver_SPFGMR module also includes a Fortran-callable function for creating a SUNLinearSolver object.

**FSUNSPFGMRINIT**

**Call**

```
FSUNSPFGMRINIT(code, pretype, maxl, ier)
```

**Description**
The function `FSUNSPFGMRINIT` can be called for Fortran programs to create a SUNLinearSolver object.

**Arguments**
- `code` (int*) is an integer input specifying the solver id (1 for CVODE, 2 for IDA, 3 for KINSOL, and 4 for ARKODE).
- `pretype` (int*) flag indicating desired preconditioning type
- `maxl` (int*) flag indicating Krylov subspace size

**Return value**
`ier` is a return completion flag equal to 0 for a success return and -1 otherwise. See printed message for details in case of failure.

**Notes**
This routine must be called after the NVECTOR object has been initialized.

Allowable values for `pretype` and `maxl` are the same as for the C function `SUNLinSol_SPFGMR`.
Additionally, when using ARKODE with a non-identity mass matrix, the SUNLINSOL_SPFGRM module includes a Fortran-callable function for creating a SUNLinearSolver mass matrix solver object.

\textbf{FSUNMASSSPFGRMInit}

\texttt{Call \textbf{FSUNMASSSPFGRMInit}(pretype, maxl, ier)}

\textbf{Description} The function \textbf{FSUNMASSSPFGRMInit} can be called for Fortran programs to create a SUNLINSOL_SPFGRM object for mass matrix linear systems.

\textbf{Arguments} \texttt{pretype (int*)} flag indicating desired preconditioning type \texttt{maxl (int*)} flag indicating Krylov subspace size

\textbf{Return value} \texttt{ier} is a \texttt{int} return completion flag equal to 0 for a success return and -1 otherwise. See printed message for details in case of failure.

\textbf{Notes} This routine must be called \textit{after} the NVector object has been initialized.

Allowable values for \texttt{pretype} and \texttt{maxl} are the same as for the C function SUNLinSol_SPFGRM.

The SUNLinSol_SPFGRMSetPrecType, SUNLinSol_SPFGRMSetGSType and SUNLinSol_SPFGRMSetMaxRestarts routines also support Fortran interfaces for the system and mass matrix solvers.

\textbf{FSUNSPFGRMSetGSType}

\texttt{Call \textbf{FSUNSPFGRMSetGSType}(code, gstype, ier)}

\textbf{Description} The function \textbf{FSUNSPFGRMSetGSType} can be called for Fortran programs to change the Gram-Schmidt orthogonalization algorithm.

\textbf{Arguments} \texttt{code (int*)} is an integer input specifying the solver id (1 for \texttt{cvode}, 2 for \texttt{ida}, 3 for \texttt{kinsol}, and 4 for \texttt{ARKODE}). \texttt{gstype (int*)} flag indicating the desired orthogonalization algorithm.

\textbf{Return value} \texttt{ier} is a \texttt{int} return completion flag equal to 0 for a success return and -1 otherwise. See printed message for details in case of failure.

\textbf{Notes} See SUNLinSol_SPFGRMSetGSType for complete further documentation of this routine.

\textbf{FSUNMASSSPFGRMSetGSType}

\texttt{Call \textbf{FSUNMASSSPFGRMSetGSType}(gstype, ier)}

\textbf{Description} The function \textbf{FSUNMASSSPFGRMSetGSType} can be called for Fortran programs to change the Gram-Schmidt orthogonalization algorithm for mass matrix linear systems.

\textbf{Arguments} The arguments are identical to \textbf{FSUNSPFGRMSetGSType} above, except that \texttt{code} is not needed since mass matrix linear systems only arise in ARKODE.

\textbf{Return value} \texttt{ier} is a \texttt{int} return completion flag equal to 0 for a success return and -1 otherwise. See printed message for details in case of failure.

\textbf{Notes} See SUNLinSol_SPFGRMSetGSType for complete further documentation of this routine.

\textbf{FSUNSPFGRMSetPrecType}

\texttt{Call \textbf{FSUNSPFGRMSetPrecType}(code, pretype, ier)}

\textbf{Description} The function \textbf{FSUNSPFGRMSetPrecType} can be called for Fortran programs to change the type of preconditioning to use.

\textbf{Arguments} \texttt{code (int*)} is an integer input specifying the solver id (1 for \texttt{cvode}, 2 for \texttt{ida}, 3 for \texttt{kinsol}, and 4 for \texttt{ARKODE}). \texttt{pretype (int*)} flag indicating the type of preconditioning to use.
Return value \( ier \) is a int return completion flag equal to 0 for a success return and -1 otherwise. See printed message for details in case of failure.

Notes See SUNLinSol_SPFGMRSetPrecType for complete further documentation of this routine.

### FSUNMASSSPFGMRSETPRECTYPE

**Call**

FSUNMASSSPFGMRSETPRECTYPE(pretype, ier)

**Description**
The function FSUNMASSSPFGMRSETPRECTYPE can be called for Fortran programs to change the type of preconditioning for mass matrix linear systems.

**Arguments**
The arguments are identical to FSUNSPFGMRSETPRECTYPE above, except that code is not needed since mass matrix linear systems only arise in ARKODE.

**Return value**
\( ier \) is a int return completion flag equal to 0 for a success return and -1 otherwise. See printed message for details in case of failure.

**Notes**
See SUNLinSol_SPFGMRSetPrecType for complete further documentation of this routine.

### FSUNSPFGMRSETMAXRS

**Call**

FSUNSPFGMRSETMAXRS(code, maxrs, ier)

**Description**
The function FSUNSPFGMRSETMAXRS can be called for Fortran programs to change the maximum number of restarts allowed for SPFMR.

**Arguments**
- code (int*) is an integer input specifying the solver id (1 for CVODE, 2 for IDA, 3 for KINSOL, and 4 for ARKODE).
- maxrs (int*) maximum allowed number of restarts.

**Return value**
\( ier \) is a int return completion flag equal to 0 for a success return and -1 otherwise. See printed message for details in case of failure.

**Notes**
See SUNLinSol_SPFGMRSetMaxRestarts for complete further documentation of this routine.

### FSUNMASSSPFGMRSETMAXRS

**Call**

FSUNMASSSPFGMRSETMAXRS(maxrs, ier)

**Description**
The function FSUNMASSSPFGMRSETMAXRS can be called for Fortran programs to change the maximum number of restarts allowed for SPFMR for mass matrix linear systems.

**Arguments**
The arguments are identical to FSUNSPFGMRSETMAXRS above, except that code is not needed since mass matrix linear systems only arise in ARKODE.

**Return value**
\( ier \) is a int return completion flag equal to 0 for a success return and -1 otherwise. See printed message for details in case of failure.

**Notes**
See SUNLinSol_SPFGMRSetMaxRestarts for complete further documentation of this routine.

### 10.14.4 SUNLinearSolver_SPFGMR content

The SUNLINSOL_SPFGMR module defines the content field of a SUNLinearSolver as the following structure:

```c
struct _SUNLinearSolverContent_SPFGMR {
    int maxl;
    int pretype;
    int gstype;
    int max_restarts;
    int numiters;
};
```
Description of the SUNLinearSolver module

realtype resnorm;
int last_flag;
ATimesFn ATimes;
void* ATData;
PSsetupFn Psetup;
PSolveFn Psolve;
void* PData;
N_Vector s1;
N_Vector s2;
N_Vector *V;
N_Vector *Z;
realtype **Hes;
realtype *givens;
N_Vector xcor;
realtype *yg;
N_Vector vtemp;
};

These entries of the content field contain the following information:

maxl - number of FGMRES basis vectors to use (default is 5),
pretype - flag for type of preconditioning to employ (default is none),
gstype - flag for type of Gram-Schmidt orthogonalization (default is modified Gram-Schmidt),
max restarts - number of FGMRES restarts to allow (default is 0),
umiters - number of iterations from the most-recent solve,
resnorm - final linear residual norm from the most-recent solve,
last_flag - last error return flag from an internal function,
ATimes - function pointer to perform $A v$ product,
ATData - pointer to structure for ATimes,
Psetup - function pointer to preconditioner setup routine,
Psolve - function pointer to preconditioner solve routine,
PData - pointer to structure for Psetup and Psolve,
s1, s2 - vector pointers for supplied scaling matrices (default is NULL),
V - the array of Krylov basis vectors $v_1, \ldots, v_{\text{maxl}+1}$, stored in V[0], ..., V[\text{maxl}]. Each $v_i$ is a vector of type NVECTOR.,
Z - the array of preconditioned Krylov basis vectors $z_1, \ldots, z_{\text{maxl}+1}$, stored in Z[0], ..., Z[\text{maxl}]. Each $z_i$ is a vector of type NVECTOR.,
Hes - the $(\text{maxl} + 1) \times \text{maxl}$ Hessenberg matrix. It is stored row-wise so that the (i,j)th element is given by $\text{Hes}[i][j]$,,
givens - a length 2*\text{maxl} array which represents the Givens rotation matrices that arise in the FGMRES algorithm. These matrices are $F_0, F_1, \ldots, F_j$, where

$$F_i = \begin{bmatrix} 1 & & & & & \\ & \ddots & & & & \\ & & 1 & -s_i & c_i & \\ & & s_i & c_i & & \\ & & & \ddots & & \\ & & & & 1 & \end{bmatrix},$$

- $xcor$ - a vector which holds the scaled, preconditioned correction to the initial guess,
- $yg$ - a length $(\maxl+1)$ array of realtype values used to hold “short” vectors (e.g. $y$ and $g$),
- $vtemp$ - temporary vector storage.

### 10.15 The SUNLinearSolver.SPBCGS implementation

This section describes the SUNLINSOL implementation of the SPBCGS (Scaled, Preconditioned, Bi-Conjugate Gradient, Stabilized [49]) iterative linear solver. The SUNLINSOL.SPBCGS module is designed to be compatible with any NVECTOR implementation that supports a minimal subset of operations ($\text{N} \text{VClone, NVDotProd, NVScale, NVLinearSum, NVProd, NVDiv, and NVDestroy}$). Unlike the SPGMR and SPFGMR algorithms, SPBCGS requires a fixed amount of memory that does not increase with the number of allowed iterations.

To access the SUNLINSOL.SPBCGS module, include the header file `sunlinsol/sunlinsol_spbcgs.h`. We note that the SUNLINSOL.SPBCGS module is accessible from SUNDIALS packages without separately linking to the `libsundials` module library.

#### 10.15.1 SUNLinearSolver.SPBCGS description

This solver is constructed to perform the following operations:

- During construction all NVECTOR solver data is allocated, with vectors cloned from a template NVECTOR that is input, and default solver parameters are set.
- User-facing “set” routines may be called to modify default solver parameters.
- Additional “set” routines are called by the SUNDIALS solver that interfaces with SUNLINSOL.SPBCGS to supply the $\text{ATimes}$, $\text{PSetup}$, and $\text{Psolve}$ function pointers and $s1$ and $s2$ scaling vectors.
- In the “initialize” call, the solver parameters are checked for validity.
- In the “setup” call, any non-NULL $\text{PSetup}$ function is called. Typically, this is provided by the SUNDIALS solver itself, that translates between the generic $\text{PSetup}$ function and the solver-specific routine (solver-supplied or user-supplied).
- In the “solve” call the SPBCGS iteration is performed. This will include scaling and preconditioning if those options have been supplied.

#### 10.15.2 SUNLinearSolver.SPBCGS functions

The SUNLINSOL.SPBCGS module provides the following user-callable constructor for creating a SUNLinearSolver object.

```c
SUNLinSol_SPBCGS
Call LS = SUNLinSol_SPBCGS(y, pretype, maxl);
Description The function SUNLinSol_SPBCGS creates and allocates memory for a SPBCGS SUNLinearSolver object.
Arguments y (N_Vector) a template for cloning vectors needed within the solver
pretype (int) flag indicating the desired type of preconditioning, allowed values are:
  * PREC_NONE (0)
```
Description of the SUNLinearSolver module

- PREC_LEFT (1)
- PREC_RIGHT (2)
- PREC_BOTH (3)

Any other integer input will result in the default (no preconditioning).

```
maxl (int) the number of linear iterations to allow. Values ≤ 0 will result in the default value (5).
```

Return value

This returns a SUNLinearSolver object. If either y is incompatible then this routine will return NULL.

Notes

This routine will perform consistency checks to ensure that it is called with a consistent NVECTOR implementation (i.e. that it supplies the requisite vector operations). If y is incompatible, then this routine will return NULL.

We note that some SUNDIALS solvers are designed to only work with left preconditioning (IDA and IDAS) and others with only right preconditioning (KINSOL). While it is possible to configure a SUNLINSOL_SPBCGS object to use any of the preconditioning options with these solvers, this use mode is not supported and may result in inferior performance.

Deprecated Name

For backward compatibility, the wrapper function SUNSPBCGS with identical input and output arguments is also provided.

F2003 Name: FSUNLinSol_SPBCGS

The SUNLINSOL_SPBCGS module defines implementations of all “iterative” linear solver operations listed in Sections 10.1.1 – 10.1.3:

- SUNLinSolGetType_SPBCGS
- SUNLinSolInitialize_SPBCGS
- SUNLinSolSetATimes_SPBCGS
- SUNLinSolSetPreconditioner_SPBCGS
- SUNLinSolSetScalingVectors_SPBCGS
- SUNLinSolSetup_SPBCGS
- SUNLinSolSolve_SPBCGS
- SUNLinSolNumIters_SPBCGS
- SUNLinSolResNorm_SPBCGS
- SUNLinSolResid_SPBCGS
- SUNLinSolLastFlag_SPBCGS
- SUNLinSolSpace_SPBCGS
- SUNLinSolFree_SPBCGS

All of the listed operations are callable via the FORTRAN 2003 interface module by prepending an ‘F’ to the function name.

The SUNLINSOL_SPBCGS module also defines the following additional user-callable functions.
10.15 The SUNLinearSolver_SPBCGS implementation

**SUNLinSol_SPBCGSSetPrecType**

Call: 
```c
retval = SUNLinSol_SPBCGSSetPrecType(LS, pretype);
```

Description: The function `SUNLinSol_SPBCGSSetPrecType` updates the type of preconditioning to use in the `SUNLINSOL_SPBCGS` object.

Arguments:
- **LS**: `SUNLinearSolver` the `SUNLINSOL_SPBCGS` object to update
- **pretype**: `int` flag indicating the desired type of preconditioning, allowed values match those discussed in `SUNLinSol_SPBCGS`.

Return value: This routine will return with one of the error codes `SUNLS_ILL_INPUT` (illegal `pretype`), `SUNLS_MEM_NULL` (`S` is `NULL`) or `SUNLS_SUCCESS`.

Deprecated Name: For backward compatibility, the wrapper function `SUNSPBCGSSetPrecType` with identical input and output arguments is also provided.

F2003 Name: `FSUNLinSol_SPBCGSSetPrecType`

**SUNLinSol_SPBCGSSetMaxl**

Call: 
```c
retval = SUNLinSol_SPBCGSSetMaxl(LS, maxl);
```

Description: The function `SUNLinSol_SPBCGSSetMaxl` updates the number of linear solver iterations to allow.

Arguments:
- **LS**: `SUNLinearSolver` the `SUNLINSOL_SPBCGS` object to update
- **maxl**: `int` flag indicating the number of iterations to allow. Values ≤ 0 will result in the default value (5).

Return value: This routine will return with one of the error codes `SUNLS_MEM_NULL` (`S` is `NULL`) or `SUNLS_SUCCESS`.

Deprecated Name: For backward compatibility, the wrapper function `SUNSPBCGSSetMaxl` with identical input and output arguments is also provided.

F2003 Name: `FSUNLinSol_SPBCGSSetMaxl`

10.15.3 SUNLinearSolver_SPBCGS Fortran interfaces

The `SUNLINSOL_SPBCGS` module provides a FORTRAN 2003 module as well as FORTRAN 77 style interface functions for use from FORTRAN applications.

**FORTRAN 2003 interface module**

The `fsunlinsol_spbcgs_mod` FORTRAN module defines interfaces to all `SUNLINSOL_SPBCGS` C functions using the intrinsic `iso_c_binding` module which provides a standardized mechanism for interoperating with C. As noted in the C function descriptions above, the interface functions are named after the corresponding C function, but with a leading 'F'. For example, the function `SUNLinSol_SPBCGS` is interfaced as `FSUNLinSol_SPBCGS`.

The FORTRAN 2003 `SUNLINSOL_SPBCGS` interface module can be accessed with the `use` statement, i.e., `use fsunlinsol_spbcgs_mod`, and linking to the library `libsundials_fsunlinsolspbcgs_mod.lib` in addition to the C library. For details on where the library and module file `fsunlinsol_spbcgs_mod.mod` are installed see Appendix A. We note that the module is accessible from the FORTRAN 2003 SUNDIALS integrators without separately linking to the `libsundials_fsunlinsolspbcgs_mod` library.

**FORTRAN 77 interface functions**

For solvers that include a FORTRAN 77 interface module, the `SUNLINSOL_SPBCGS` module also includes a Fortran-callable function for creating a `SUNLinearSolver` object.
FSUNSPBCGSINIT
Call FSUNSPBCGSINIT(code, pretype, maxl, ier)
Description The function FSUNSPBCGSINIT can be called for Fortran programs to create a SUNLINSOL_SPBCGS object.
Arguments code (int*) is an integer input specifying the solver id (1 for CVODE, 2 for IDA, 3 for KINSOL, and 4 for ARKODE).
pretype (int*) flag indicating desired preconditioning type
maxl (int*) flag indicating number of iterations to allow
Return value ier is a return completion flag equal to 0 for a success return and -1 otherwise. See printed message for details in case of failure.
Notes This routine must be called after the NVECTOR object has been initialized. Allowable values for pretype and maxl are the same as for the C function SUNLinSol_SPBCGS.
Additionally, when using ARKODE with a non-identity mass matrix, the SUNLINSOL_SPBCGS module includes a Fortran-callable function for creating a SUNLinearSolver mass matrix solver object.

FSUNMASSSPBCGSINIT
Call FSUNMASSSPBCGSINIT(pretype, maxl, ier)
Description The function FSUNMASSSPBCGSINIT can be called for Fortran programs to create a SUNLINSOL_SPBCGS object for mass matrix linear systems.
Arguments pretype (int*) flag indicating desired preconditioning type
maxl (int*) flag indicating number of iterations to allow
Return value ier is a int return completion flag equal to 0 for a success return and -1 otherwise. See printed message for details in case of failure.
Notes This routine must be called after the NVECTOR object has been initialized. Allowable values for pretype and maxl are the same as for the C function SUNLinSol_SPBCGS.
The SUNLinSol_SPBCGSSetPrecType and SUNLinSol_SPBCGSSetMaxl routines also support Fortran interfaces for the system and mass matrix solvers.

FSUNSPBCGSSETPRECTYPE
Call FSUNSPBCGSSETPRECTYPE(code, pretype, ier)
Description The function FSUNSPBCGSSETPRECTYPE can be called for Fortran programs to change the type of preconditioning to use.
Arguments code (int*) is an integer input specifying the solver id (1 for CVODE, 2 for IDA, 3 for KINSOL, and 4 for ARKODE).
pretype (int*) flag indicating the type of preconditioning to use.
Return value ier is a int return completion flag equal to 0 for a success return and -1 otherwise. See printed message for details in case of failure.
Notes See SUNLinSol_SPBCGSSetPrecType for complete further documentation of this routine.

FSUNMASSSPBCGSSETPRECTYPE
Call FSUNMASSSPBCGSSETPRECTYPE(pretype, ier)
Description The function FSUNMASSSPBCGSSETPRECTYPE can be called for Fortran programs to change the type of preconditioning for mass matrix linear systems.
Arguments: The arguments are identical to \texttt{FSUNSPBCGSSETPRECTYPE} above, except that \texttt{code} is not needed since mass matrix linear systems only arise in ARKODE.

Return value: \texttt{ier} is an int return completion flag equal to 0 for a success return and -1 otherwise. See printed message for details in case of failure.

Notes: See \texttt{SUNLinSol\_SPBCGSSetPrecType} for complete further documentation of this routine.

\textbf{FSUNSPBCGSSETMAXL}

\textbf{Call} \quad \texttt{FSUNSPBCGSSETMAXL(code, maxl, ier)}

\textbf{Description} \quad The function \texttt{FSUNSPBCGSSETMAXL} can be called for Fortran programs to change the maximum number of iterations to allow.

\textbf{Arguments} \quad \texttt{code} (int*) is an integer input specifying the solver id (1 for CVODE, 2 for IDA, 3 for KINSOL, and 4 for ARKODE).

\texttt{maxl} (int*) the number of iterations to allow.

\textbf{Return value} \quad \texttt{ier} is an int return completion flag equal to 0 for a success return and -1 otherwise. See printed message for details in case of failure.

\textbf{Notes} \quad See \texttt{SUNLinSol\_SPBCGSSetMaxl} for complete further documentation of this routine.

\textbf{FSUNMASSSPBCGSSETMAXL}

\textbf{Call} \quad \texttt{FSUNMASSSPBCGSSETMAXL(maxl, ier)}

\textbf{Description} \quad The function \texttt{FSUNMASSSPBCGSSETMAXL} can be called for Fortran programs to change the type of preconditioning for mass matrix linear systems.

\textbf{Arguments} \quad The arguments are identical to \texttt{FSUNSPBCGSSETMAXL} above, except that \texttt{code} is not needed since mass matrix linear systems only arise in ARKODE.

\textbf{Return value} \quad \texttt{ier} is an int return completion flag equal to 0 for a success return and -1 otherwise. See printed message for details in case of failure.

\textbf{Notes} \quad See \texttt{SUNLinSol\_SPBCGSSetMaxl} for complete further documentation of this routine.

10.15.4 SUNLinearSolver\_SPBCGS content

The \texttt{sunlinsol\_spbcgs} module defines the \texttt{content} field of a \texttt{SUNLinearSolver} as the following structure:

\begin{verbatim}
struct _SUNLinearSolverContent_SPBCGS {
    int maxl;
    int pretype;
    int numiters;
    realtype resnorm;
    int last_flag;
    ATimesFn ATimes;
    void* ATData;
    PSetupFn Psetup;
    PSolveFn Psolve;
    void* PData;
    N_Vector s1;
    N_Vector s2;
    N_Vector r;
    N_Vector r_star;
    N_Vector p;
    N_Vector q;
    N_Vector u;
}
\end{verbatim}
N_Vector Ap;
N_Vector vtemp;
};

These entries of the content field contain the following information:
maxl - number of spbcgs iterations to allow (default is 5),
pretype - flag for type of preconditioning to employ (default is none),
umiters - number of iterations from the most-recent solve,
resnorm - final linear residual norm from the most-recent solve,
last_flag - last error return flag from an internal function,
ATimes - function pointer to perform \( Av \) product,
ATData - pointer to structure for ATimes,
Psetup - function pointer to preconditioner setup routine,
Psolve - function pointer to preconditioner solve routine,
PData - pointer to structure for Psetup and Psolve,
s1, s2 - vector pointers for supplied scaling matrices (default is NULL),
r - a NVVECTOR which holds the current scaled, preconditioned linear system residual,
r_star - a NVVECTOR which holds the initial scaled, preconditioned linear system residual,
p, q, u, Ap, vtemp - NVECTORS used for workspace by the SPBCGS algorithm.

10.16 The SUNLinearSolver_SPTFQMR implementation

This section describes the SUNLINSOL implementation of the SPTFQMR (Scaled, Preconditioned, Transpose-Free Quasi-Minimum Residual [24]) iterative linear solver. The SUNLINSOL_SPTFQMR module is designed to be compatible with any NVVECTOR implementation that supports a minimal subset of operations (N_VClone, N_VDotProd, N_VScale, N_VLinearSum, N_VProd, N_VConst, N_VDiv, and N_VDestroy). Unlike the SPGMR and SPFGMR algorithms, SPTFQMR requires a fixed amount of memory that does not increase with the number of allowed iterations.

To access the SUNLINSOL_SPTFQMR module, include the header file
sunlinsol/sunlinsol_sptfqmr.h. We note that the SUNLINSOL_SPTFQMR module is accessible from SUNDIALS packages without separately linking to the libsundials_sunlinsolsptfqmr module library.

10.16.1 SUNLinearSolver_SPTFQMR description

This solver is constructed to perform the following operations:

- During construction all NVVECTOR solver data is allocated, with vectors cloned from a template NVVECTOR that is input, and default solver parameters are set.
- User-facing “set” routines may be called to modify default solver parameters.
- Additional “set” routines are called by the SUNDIALS solver that interfaces with SUNLINSOL_SPTFQMR to supply the ATimes, PSetup, and Psolve function pointers and s1 and s2 scaling vectors.
- In the “initialize” call, the solver parameters are checked for validity.
- In the “setup” call, any non-NULL PSetup function is called. Typically, this is provided by the SUNDIALS solver itself, that translates between the generic PSetup function and the solver-specific routine (solver-supplied or user-supplied).
- In the “solve” call the TFQMR iteration is performed. This will include scaling and preconditioning if those options have been supplied.
10.16 The SUNLinearSolver_SPTFQMR implementation

10.16.2 SUNLinearSolver_SPTFQMR functions

The SUNLINSOL_SPTFQMR module provides the following user-callable constructor for creating a SUNLinearSolver object.

```
SUNLinSol_SPTFQMR
Call      LS = SUNLinSol_SPTFQMR(y, pretype, maxl);
Description The function SUNLinSol_SPTFQMR creates and allocates memory for a SPTFQMR SUNLinearSolver object.
Arguments  
y (N_Vector) a template for cloning vectors needed within the solver
pretype (int) flag indicating the desired type of preconditioning, allowed values are:
  • PREC_NONE (0)
  • PREC_LEFT (1)
  • PREC_RIGHT (2)
  • PREC_BOTH (3)
  Any other integer input will result in the default (no preconditioning).
maxl (int) the number of linear iterations to allow. Values \( \leq 0 \) will result in the default value (5).
Return value This returns a SUNLinearSolver object. If either y is incompatible then this routine will return NULL.
Notes      This routine will perform consistency checks to ensure that it is called with a consistent NVECTOR implementation (i.e. that it supplies the requisite vector operations). If y is incompatible, then this routine will return NULL.
We note that some SUNDIALS solvers are designed to only work with left preconditioning (IDA and IDAS) and others with only right preconditioning (KINSOL). While it is possible to configure a SUNLINSOL_SPTFQMR object to use any of the preconditioning options with these solvers, this use mode is not supported and may result in inferior performance.
Deprecated Name For backward compatibility, the wrapper function SUNSPTFQMR with identical input and output arguments is also provided.
F2003 Name  FSUNLinSol_SPTFQMR
```

The SUNLINSOL_SPTFQMR module defines implementations of all “iterative” linear solver operations listed in Sections 10.1.1 – 10.1.3:

- SUNLinSolGetType_SPTFQMR
- SUNLinSolInitialize_SPTFQMR
- SUNLinSolSetATimes_SPTFQMR
- SUNLinSolSetPreconditioner_SPTFQMR
- SUNLinSolSetScalingVectors_SPTFQMR
- SUNLinSolSetup_SPTFQMR
- SUNLinSolSolve_SPTFQMR
- SUNLinSolNumIters_SPTFQMR
- SUNLinSolResNorm_SPTFQMR
- SUNLinSolResid_SPTFQMR
Description of the SUNLinearSolver module

- SUNLinSolLastFlag_SPTFQMR
- SUNLinSolSpace_SPTFQMR
- SUNLinSolFree_SPTFQMR

All of the listed operations are callable via the FORTRAN 2003 interface module by prepending an ‘F’ to the function name.

The SUNLINSOL_SPTFQMR module also defines the following additional user-callable functions.

```fortran
SUNLinSol_SPTFQMRSetPrecType
Call retval = SUNLinSol_SPTFQMRSetPrecType(LS, pretype);
Description The function SUNLinSol_SPTFQMRSetPrecType updates the type of preconditioning to use in the SUNLINSOL_SPTFQMR object.
Arguments LS (SUNLinearSolver) the SUNLINSOL_SPTFQMR object to update
pretype (int) flag indicating the desired type of preconditioning, allowed values match those discussed in SUNLinSol_SPTFQMR.
Return value This routine will return with one of the error codes SUNLS_ILL_INPUT (illegal pretype), SUNLS_MEM_NULL (S is NULL) or SUNLS_SUCCESS.
Deprecated Name For backward compatibility, the wrapper function SUNSPTFQMRSetPrecType with identical input and output arguments is also provided.
F2003 Name FSUNLinSol_SPTFQMRSetPrecType
```

```fortran
SUNLinSol_SPTFQMRSetMaxl
Call retval = SUNLinSol_SPTFQMRSetMaxl(LS, maxl);
Description The function SUNLinSol_SPTFQMRSetMaxl updates the number of linear solver iterations to allow.
Arguments LS (SUNLinearSolver) the SUNLINSOL_SPTFQMR object to update
maxl (int) flag indicating the number of iterations to allow; values ≤ 0 will result in the default value (5)
Return value This routine will return with one of the error codes SUNLS_MEM_NULL (S is NULL) or SUNLS_SUCCESS.
F2003 Name FSUNLinSol_SPTFQMRSetMaxl
```

10.16.3 SUNLinearSolver_SPTFQMR Fortran interfaces

The sunlinsol_sptfqmr module provides a FORTRAN 2003 module as well as FORTRAN 77 style interface functions for use from FORTRAN applications.

FORTRAN 2003 interface module

The fsunlinsol_sptfqmr_mod FORTRAN module defines interfaces to all SUNLINSOL_SPTFQMR C functions using the intrinsic iso_c_binding module which provides a standardized mechanism for inter-operating with C. As noted in the C function descriptions above, the interface functions are named after the corresponding C function, but with a leading ‘F’. For example, the function SUNLinSol_SPTFQMR is interfaced as FSUNLinSol_SPTFQMR.

The FORTRAN 2003 SUNLINSOL_SPTFQMR interface module can be accessed with the use statement, i.e. use fsunlinsol_sptfqmr_mod, and linking to the library libsundials_fsunlinsolsptfqmr_mod.lib in addition to the C library. For details on where the library and module file fsunlinsol_sptfqmr_mod.mod are installed see Appendix A. We note that the module is accessible
from the Fortran 2003 Sundials integrators without separately linking to the 
libsundials_fsunlinolsptfqmr_mod library.

FORTRAN 77 interface functions

For solvers that include a FORTRAN 77 interface module, the SUNLINSOL_SPTFQMR module also includes a Fortran-callable function for creating a SUNLinearSolver object.

**FSUNSPTFQMRINIT**

**Call**

FSUNSPTFQMRINIT(code, pretype, maxl, ier)

**Description**
The function FSUNSPTFQMRINIT can be called for Fortran programs to create a SUNLINSOL_SPTFQMR object.

**Arguments**
- `code` (int*) is an integer input specifying the solver id (1 for CVODE, 2 for IDA, 3 for KINSOL, and 4 for ARKODE).
- `pretype` (int*) flag indicating desired preconditioning type
- `maxl` (int*) flag indicating number of iterations to allow

**Return value**
- `ier` is a return completion flag equal to 0 for a success return and -1 otherwise. See printed message for details in case of failure.

**Notes**
- This routine must be called after the NVECTOR object has been initialized.
- Allowable values for `pretype` and `maxl` are the same as for the C function SUNLinSol_SPTFQMR.

Additionally, when using ARKODE with a non-identity mass matrix, the SUNLINSOL_SPTFQMR module includes a Fortran-callable function for creating a SUNLinearSolver mass matrix solver object.

**FSUNMASSSPTFQMRINIT**

**Call**

FSUNMASSSPTFQMRINIT(pretype, maxl, ier)

**Description**
The function FSUNMASSSPTFQMRINIT can be called for Fortran programs to create a SUNLINSOL_SPTFQMR object for mass matrix linear systems.

**Arguments**
- `pretype` (int*) flag indicating desired preconditioning type
- `maxl` (int*) flag indicating number of iterations to allow

**Return value**
- `ier` is an int return completion flag equal to 0 for a success return and -1 otherwise. See printed message for details in case of failure.

**Notes**
- This routine must be called after the NVECTOR object has been initialized.
- Allowable values for `pretype` and `maxl` are the same as for the C function SUNLinSol_SPTFQMR.

The SUNLinSol_SPTFQMRSetPrecType and SUNLinSol_SPTFQMRSetMaxl routines also support Fortran interfaces for the system and mass matrix solvers.

**FSUNSPTFQMRSETPRECTYPE**

**Call**

FSUNSPTFQMRSETPRECTYPE(code, pretype, ier)

**Description**
The function FSUNSPTFQMRSETPRECTYPE can be called for Fortran programs to change the type of preconditioning to use.

**Arguments**
- `code` (int*) is an integer input specifying the solver id (1 for CVODE, 2 for IDA, 3 for KINSOL, and 4 for ARKODE).
- `pretype` (int*) flag indicating the type of preconditioning to use.

**Return value**
- `ier` is an int return completion flag equal to 0 for a success return and -1 otherwise. See printed message for details in case of failure.
340 Description of the SUNLinearSolver module

Notes See SUNLinSol_SPTFQMRSetPrecType for complete further documentation of this routine.

**FSUNMASSSPTFQMRSETPRECTYPE**

Call FSUNMASSSPTFQMRSETPRECTYPE(pretype, ier)

Description The function FSUNMASSSPTFQMRSETPRECTYPE can be called for Fortran programs to change the type of preconditioning for mass matrix linear systems.

Arguments The arguments are identical to FSUNSPTFQMRSETPRECTYPE above, except that code is not needed since mass matrix linear systems only arise in ARKODE.

Return value ier is a int return completion flag equal to 0 for a success return and -1 otherwise. See printed message for details in case of failure.

Notes See SUNLinSol_SPTFQMRSetPrecType for complete further documentation of this routine.

**FSUNSPTFQMRSETMAXL**

Call FSUNSPTFQMRSETMAXL(code, maxl, ier)

Description The function FSUNSPTFQMRSETMAXL can be called for Fortran programs to change the maximum number of iterations to allow.

Arguments code (int*) is an integer input specifying the solver id (1 for CVODE, 2 for IDA, 3 for KINSOL, and 4 for ARKODE).

maxl (int*) the number of iterations to allow.

Return value ier is a int return completion flag equal to 0 for a success return and -1 otherwise. See printed message for details in case of failure.

Notes See SUNLinSol_SPTFQMRSetMaxl for complete further documentation of this routine.

**FSUNMASSSPTFQMRSETMAXL**

Call FSUNMASSSPTFQMRSETMAXL(maxl, ier)

Description The function FSUNMASSSPTFQMRSETMAXL can be called for Fortran programs to change the type of preconditioning for mass matrix linear systems.

Arguments The arguments are identical to FSUNSPTFQMRSETMAXL above, except that code is not needed since mass matrix linear systems only arise in ARKODE.

Return value ier is a int return completion flag equal to 0 for a success return and -1 otherwise. See printed message for details in case of failure.

Notes See SUNLinSol_SPTFQMRSetMaxl for complete further documentation of this routine.

10.16.4 SUNLinearSolver_SPTFQMR content

The SUNLINSOL_SPTFQMR module defines the content field of a SUNLinearSolver as the following structure:

```c
struct _SUNLinearSolverContent_SPTFQMR {
    int maxl;
    int pretype;
    int numiters;
    realtype resnorm;
    int last_flag;
    ATimesFn ATimes;
    void* ATData;
};
```
10.17 The SUNLinearSolver_PCG implementation

This section describes the SUNLinearSolver_PCG implementation of the PCG (Preconditioned Conjugate Gradient [26]) iterative linear solver. The SUNLinearSolver_PCG module is designed to be compatible with any NVECTOR implementation that supports a minimal subset of operations (N_VClone, N_VDotProd, N_VScale, N_VLinearSum, N_VProd, and N_VDestroy). Unlike the SPGMR and SPFGMR algorithms, PCG requires a fixed amount of memory that does not increase with the number of allowed iterations.

To access the SUNLinearSolver_PCG module, include the header file sunlinsol/sunlinsol_pcg.h. We note that the SUNLinearSolver_PCG module is accessible from SUNDIALS packages without separately linking to the libsundials_sunlinsolpcg module library.

10.17.1 SUNLinearSolver_PCG description

Unlike all of the other iterative linear solvers supplied with SUNDIALS, PCG should only be used on symmetric linear systems (e.g. mass matrix linear systems encountered in ARKODE). As a result, the
description of the role of scaling and preconditioning matrices given in general must be modified in this scenario. The PCG algorithm solves a linear system $Ax = b$ where $A$ is a symmetric ($A^T = A$), real-valued matrix. Preconditioning is allowed, and is applied in a symmetric fashion on both the right and left. Scaling is also allowed and is applied symmetrically. We denote the preconditioner and scaling matrices as follows:

- $P$ is the preconditioner (assumed symmetric),
- $S$ is a diagonal matrix of scale factors.

The matrices $A$ and $P$ are not required explicitly; only routines that provide $A$ and $P^{-1}$ as operators are required. The diagonal of the matrix $S$ is held in a single nvector, supplied by the user.

In this notation, PCG applies the underlying CG algorithm to the equivalent transformed system

$$\tilde{A}\tilde{x} = \tilde{b}$$

where

$$\tilde{A} = SP^{-1}AP^{-1}S,$$
$$\tilde{b} = SP^{-1}b,$$
$$\tilde{x} = S^{-1}Px.$$  \hspace{1cm} (10.5)

The scaling matrix must be chosen so that the vectors $SP^{-1}b$ and $S^{-1}Px$ have dimensionless components.

The stopping test for the PCG iterations is on the L2 norm of the scaled preconditioned residual:

$$\|\tilde{b} - \tilde{A}\tilde{x}\|_2 < \delta$$
$$\Leftrightarrow$$
$$\|SP^{-1}b - SP^{-1}Ax\|_2 < \delta$$
$$\Leftrightarrow$$
$$\|P^{-1}b - P^{-1}Ax\|_S < \delta$$

where $\|v\|_S = \sqrt{v^T S^T S v}$, with an input tolerance $\delta$. 

This solver is constructed to perform the following operations:

- During construction all nvector solver data is allocated, with vectors cloned from a template nvector that is input, and default solver parameters are set.
- User-facing “set” routines may be called to modify default solver parameters.
- Additional “set” routines are called by the SUNDIALS solver that interfaces with SUNLINSOL_PCG to supply the ATimes, PSetup, and Psolve function pointers and $S$ scaling vector.
- In the “initialize” call, the solver parameters are checked for validity.
- In the “setup” call, any non-NULL PSetup function is called. Typically, this is provided by the SUNDIALS solver itself, that translates between the generic PSetup function and the solver-specific routine (solver-supplied or user-supplied).
- In the “solve” call the PCG iteration is performed. This will include scaling and preconditioning if those options have been supplied.

### 10.17.2 SUNLinearSolver_PCG functions

The SUNLINSOL_PCG module provides the following user-callable constructor for creating a SUNLinearSolver object.
The SUNLinearSolver PCG implementation

**Call**

```c
LS = SUNLinSol_PC(y, pretype, maxl);
```

**Description**

The function `SUNLinSol_PC` creates and allocates memory for a PCG SUNLinearSolver object.

**Arguments**

- `y` ([N_Vector]) a template for cloning vectors needed within the solver
- `pretype` ([int]) flag indicating whether to use preconditioning. Since the PCG algorithm is designed to only support symmetric preconditioning, then any of the `pretype` inputs `PREC_LEFT` (1), `PREC_RIGHT` (2), or `PREC_BOTH` (3) will result in use of the symmetric preconditioner; any other integer input will result in the default (no preconditioning).
- `maxl` ([int]) the number of linear iterations to allow; values \( \leq 0 \) will result in the default value (5).

**Return value**

This returns a `SUNLinearSolver` object. If either `y` is incompatible then this routine will return `NULL`.

**Notes**

This routine will perform consistency checks to ensure that it is called with a consistent `NVECTOR` implementation (i.e. that it supplies the requisite vector operations). If `y` is incompatible, then this routine will return `NULL`.

Although some `SUNDIALS` solvers are designed to only work with left preconditioning (IDA and IDAS) and others with only right preconditioning (KINSOL), PCG should only be used with these packages when the linear systems are known to be symmetric. Since the scaling of matrix rows and columns must be identical in a symmetric matrix, symmetric preconditioning should work appropriately even for packages designed with one-sided preconditioning in mind.

**Deprecated Name**

For backward compatibility, the wrapper function `SUNPCG` with identical input and output arguments is also provided.

**F2003 Name**

`FSUNLinSol_PC`

The `sulinsol_pc` module defines implementations of all “iterative” linear solver operations listed in Sections 10.1.1 – 10.1.3:

- `SUNLinSolGetType_PC`  
- `SUNLinSolInitialize_PC`  
- `SUNLinSolSetATimes_PC`  
- `SUNLinSolSetPreconditioner_PC`  
- `SUNLinSolSetScalingVectors_PC` – since PCG only supports symmetric scaling, the second `NVECTOR` argument to this function is ignored  
- `SUNLinSolSetup_PC`  
- `SUNLinSolSolve_PC`  
- `SUNLinSolNumIters_PC`  
- `SUNLinSolResNorm_PC`  
- `SUNLinSolResid_PC`  
- `SUNLinSolLastFlag_PC`  
- `SUNLinSolSpace_PC`  
- `SUNLinSolFree_PC`  

All of the listed operations are callable via the FORTRAN 2003 interface module by prepending an ‘F’ to the function name.

The `sulinsol_pc` module also defines the following additional user-callable functions.
### SUNLinSol_PCGSetPrecType

**Call**

```c
retval = SUNLinSol_PCGSetPrecType(LS, pretype);
```

**Description**
The function `SUNLinSol_PCGSetPrecType` updates the flag indicating use of preconditioning in the SUNLINSOL_PCG object.

**Arguments**
- `LS` (SUNLinearSolver) the SUNLINSOL_PCG object to update
- `pretype` (int) flag indicating use of preconditioning, allowed values match those discussed in SUNLinSol_PCG.

**Return value**
This routine will return with one of the error codes `SUNLS_Ill_INPUT` (illegal `pretype`), `SUNLS_MEM_NULL` (S is NULL) or `SUNLS_SUCCESS`.

**Deprecated Name**
For backward compatibility, the wrapper function `SUNPCGSetPrecType` with identical input and output arguments is also provided.

**F2003 Name**
`FSUNLinSol_PCGSetPrecType`

### SUNLinSol_PCGSetMaxl

**Call**

```c
retval = SUNLinSol_PCGSetMaxl(LS, maxl);
```

**Description**
The function `SUNLinSol_PCGSetMaxl` updates the number of linear solver iterations to allow.

**Arguments**
- `LS` (SUNLinearSolver) the SUNLINSOL_PCG object to update
- `maxl` (int) flag indicating the number of iterations to allow; values \( \leq 0 \) will result in the default value (5)

**Return value**
This routine will return with one of the error codes `SUNLS_MEM_NULL` (S is NULL) or `SUNLS_SUCCESS`.

**Deprecated Name**
For backward compatibility, the wrapper function `SUNPCGSetMaxl` with identical input and output arguments is also provided.

**F2003 Name**
`FSUNLinSol_PCGSetMaxl`

## 10.17.3 SUNLinearSolver_PCG Fortran interfaces

The SUNLINSOL_PCG module provides a FORTRAN 2003 module as well as FORTRAN 77 style interface functions for use from FORTRAN applications.

### FORTRAN 2003 interface module

The `fsunlinsol_pcg_mod` FORTRAN module defines interfaces to all SUNLINSOL_PCG C functions using the intrinsic `iso_c_binding` module which provides a standardized mechanism for interoperating with C. As noted in the C function descriptions above, the interface functions are named after the corresponding C function, but with a leading ‘F’. For example, the function `SUNLinSol_PCG` is interfaced as `FSUNLinSol_PCG`.

The FORTRAN 2003 SUNLINSOL_PCG interface module can be accessed with the `use` statement, i.e. `use fsunlinsol_pcg_mod`, and linking to the library `libsundials_fsunlinsolpcg_mod.lib` in addition to the C library. For details on where the library and module file `fsunlinsol_pcg_mod.mod` are installed see Appendix A. We note that the module is accessible from the FORTRAN 2003 SUNDIALS integrators without separately linking to the `libsundials_fsunlinsolpcg_mod` library.

### FORTRAN 77 interface functions

For solvers that include a FORTRAN 77 interface module, the SUNLINSOL_PCG module also includes a Fortran-callable function for creating a SUNLinearSolver object.
10.17 The SUNLinearSolver_PCG implementation

**FSUNPCGINIT**

Call: `FSUNPCGINIT(code, pretype, maxl, ier)`

Description: The function `FSUNPCGINIT` can be called for Fortran programs to create a SUNLinearSolver_PCG object.

Arguments:
- `code` (int*) is an integer input specifying the solver id (1 for CVODE, 2 for IDA, 3 for KINSOL, and 4 for ARKODE).
- `pretype` (int*) flag indicating desired preconditioning type
- `maxl` (int*) flag indicating number of iterations to allow

Return value: `ier` is a return completion flag equal to 0 for a success return and -1 otherwise. See printed message for details in case of failure.

Notes:
- This routine must be called after the NVector object has been initialized.
- Allowable values for `pretype` and `maxl` are the same as for the C function SUNLinSol_PCG.

Additionally, when using ARKODE with a non-identity mass matrix, the SUNLINSOL_PCG module includes a Fortran-callable function for creating a SUNLinearSolver mass matrix solver object.

**FSUNMASSPCGINIT**

Call: `FSUNMASSPCGINIT(pretype, maxl, ier)`

Description: The function `FSUNMASSPCGINIT` can be called for Fortran programs to create a SUNLinearSolver_PCG object for mass matrix linear systems.

Arguments:
- `pretype` (int*) flag indicating desired preconditioning type
- `maxl` (int*) flag indicating number of iterations to allow

Return value: `ier` is an int return completion flag equal to 0 for a success return and -1 otherwise. See printed message for details in case of failure.

Notes:
- This routine must be called after the NVector object has been initialized.
- Allowable values for `pretype` and `maxl` are the same as for the C function SUNLinSol_PCG.

The SUNLinSol_PCGSetPrecType and SUNLinSol_PCGSetMax1 routines also support Fortran interfaces for the system and mass matrix solvers.

**FSUNPCGSETPRECTYPE**

Call: `FSUNPCGSETPRECTYPE(code, pretype, ier)`

Description: The function `FSUNPCGSETPRECTYPE` can be called for Fortran programs to change the type of preconditioning to use.

Arguments:
- `code` (int*) is an integer input specifying the solver id (1 for CVODE, 2 for IDA, 3 for KINSOL, and 4 for ARKODE).
- `pretype` (int*) flag indicating the type of preconditioning to use.

Return value: `ier` is an int return completion flag equal to 0 for a success return and -1 otherwise. See printed message for details in case of failure.

Notes:
- See SUNLinSol_PCGSetPrecType for complete further documentation of this routine.

**FSUNMASSPCGSETPRECTYPE**

Call: `FSUNMASSPCGSETPRECTYPE(pretype, ier)`

Description: The function `FSUNMASSPCGSETPRECTYPE` can be called for Fortran programs to change the type of preconditioning for mass matrix linear systems.

Arguments: The arguments are identical to `FSUNPCGSETPRECTYPE` above, except that `code` is not needed since mass matrix linear systems only arise in ARKODE.
Return value $\text{ier}$ is an int return completion flag equal to 0 for a success return and -1 otherwise. See printed message for details in case of failure.

Notes See SUNLinSol_PCGSetPrecType for complete further documentation of this routine.

\textbf{FSUNPCGSETMAXL}

\textbf{Call} \quad \text{FSUNPCGSETMAXL}(\text{code}, \text{maxl}, \text{ier})

\textbf{Description} \quad The function FSUNPCGSETMAXL can be called for Fortran programs to change the maximum number of iterations to allow.

\textbf{Arguments} \quad \text{code} (int*) is an integer input specifying the solver id (1 for \text{cvode}, 2 for \text{ida}, 3 for \text{kinsol}, and 4 for \text{arkode}).

\text{maxl} (int*) the number of iterations to allow.

Return value $\text{ier}$ is an int return completion flag equal to 0 for a success return and -1 otherwise. See printed message for details in case of failure.

Notes See SUNLinSol_PCGSetMaxl for complete further documentation of this routine.

\textbf{FSUNMASSPCGSETMAXL}

\textbf{Call} \quad \text{FSUNMASSPCGSETMAXL}(\text{maxl}, \text{ier})

\textbf{Description} \quad The function FSUNMASSPCGSETMAXL can be called for Fortran programs to change the type of preconditioning for mass matrix linear systems.

\textbf{Arguments} \quad The arguments are identical to FSUNPCGSETMAXL above, except that \text{code} is not needed since mass matrix linear systems only arise in \text{arkode}.

Return value $\text{ier}$ is an int return completion flag equal to 0 for a success return and -1 otherwise. See printed message for details in case of failure.

Notes See SUNLinSol_PCGSetMaxl for complete further documentation of this routine.

10.17.4 SUNLinearSolver_PCG content

The SUNLINSOL_PCG module defines the \textit{content} field of a SUNLinearSolver as the following structure:

```c
cstruct _SUNLinearSolverContent_PCG {
    int maxl;
    int pretype;
    int numiters;
    realtype resnorm;
    int last_flag;
    ATimesFn ATimes;
    void* ATData;
    PSetupFn Psetup;
    PSolveFn Psolve;
    void* PData;
    N_Vector s;
    N_Vector r;
    N_Vector p;
    N_Vector z;
    N_Vector Ap;
};
```

These entries of the \textit{content} field contain the following information:

maxl \quad - number of PCG iterations to allow (default is 5),

pretype \quad - flag for use of preconditioning (default is none),
numiters - number of iterations from the most-recent solve,
resnorm - final linear residual norm from the most-recent solve,
last_flag - last error return flag from an internal function,
ATimes - function pointer to perform $Av$ product,
ATData - pointer to structure for ATimes,
Psetup - function pointer to preconditioner setup routine,
Psolve - function pointer to preconditioner solve routine,
PData - pointer to structure for Psetup and Psolve,
s - vector pointer for supplied scaling matrix (default is NULL),
r - a NVECTOR which holds the preconditioned linear system residual,
p, z, Ap - NVECTORS used for workspace by the PCG algorithm.

10.18 SUNLinearSolver Examples

There are SUNLinearSolver examples that may be installed for each implementation; these make use of the functions in test_sunlinsol.c. These example functions show simple usage of the SUNLinearSolver family of functions. The inputs to the examples depend on the linear solver type, and are output to stdout if the example is run without the appropriate number of command-line arguments.

The following is a list of the example functions in test_sunlinsol.c:

- **Test_SUNLinSolGetType**: Verifies the returned solver type against the value that should be returned.
- **Test_SUNLinSolInitialize**: Verifies that SUNLinSolInitialize can be called and returns successfully.
- **Test_SUNLinSolSetup**: Verifies that SUNLinSolSetup can be called and returns successfully.
- **Test_SUNLinSolSolve**: Given a SUNMATRIX object $A$, NVECTOR objects $x$ and $b$ (where $Ax = b$) and a desired solution tolerance $tol$, this routine clones $x$ into a new vector $y$, calls SUNLinSolSolve to fill $y$ as the solution to $Ay = b$ (to the input tolerance), verifies that each entry in $x$ and $y$ match to within $10*tol$, and overwrites $x$ with $y$ prior to returning (in case the calling routine would like to investigate further).
- **Test_SUNLinSolSetATimes** (iterative solvers only): Verifies that SUNLinSolSetATimes can be called and returns successfully.
- **Test_SUNLinSolSetPreconditioner** (iterative solvers only): Verifies that SUNLinSolSetPreconditioner can be called and returns successfully.
- **Test_SUNLinSolSetScalingVectors** (iterative solvers only): Verifies that SUNLinSolSetScalingVectors can be called and returns successfully.
- **Test_SUNLinSolLastFlag**: Verifies that SUNLinSolLastFlag can be called, and outputs the result to stdout.
- **Test_SUNLinSolNumIters** (iterative solvers only): Verifies that SUNLinSolNumIters can be called, and outputs the result to stdout.
- **Test_SUNLinSolResNorm** (iterative solvers only): Verifies that SUNLinSolResNorm can be called, and that the result is non-negative.
- **Test_SUNLinSolResid** (iterative solvers only): Verifies that SUNLinSolResid can be called.
- **Test\_SUNLinSolSpace** verifies that **SUNLinSolSpace** can be called, and outputs the results to stdout.

We’ll note that these tests should be performed in a particular order. For either direct or iterative linear solvers, **Test\_SUNLinSolInitialize** must be called before **Test\_SUNLinSolSetup**, which must be called before **Test\_SUNLinSolSolve**. Additionally, for iterative linear solvers **Test\_SUNLinSolSetATimes**, **Test\_SUNLinSolSetPreconditioner** and **Test\_SUNLinSolSetScalingVectors** should be called before **Test\_SUNLinSolInitialize**; similarly **Test\_SUNLinSolNumIters**, **Test\_SUNLinSolResNorm** and **Test\_SUNLinSolResid** should be called after **Test\_SUNLinSolSolve**. These are called in the appropriate order in all of the example problems.
Chapter 11

Description of the SUNNonlinearSolver module

SUNDIALS time integration packages are written in terms of generic nonlinear solver operations defined by the SUNNONLINSOL API and implemented by a particular SUNNONLINSOL module of type SUNNonlinearSolver. Users can supply their own SUNNONLINSOL module, or use one of the modules provided with SUNDIALS. Depending on the package, nonlinear solver modules can either target system presented in a rootfinding \( F(y) = 0 \) or fixed-point \( G(y) = y \) formulation. For more information on the formulation of the nonlinear system(s) see section 11.2.

The time integrators in SUNDIALS specify a default nonlinear solver module and as such this chapter is intended for users that wish to use a non-default nonlinear solver module or would like to provide their own nonlinear solver implementation. Users interested in using a non-default solver module may skip the description of the SUNNONLINSOL API in section 11.1 and proceeded to the subsequent sections in this chapter that describe the SUNNONLINSOL modules provided with SUNDIALS.

For users interested in providing their own SUNNONLINSOL module, the following section presents the SUNNONLINSOL API and its implementation beginning with the definition of SUNNONLINSOL functions in sections 11.1.1 – 11.1.3. This is followed by the definition of functions supplied to a nonlinear solver implementation in section 11.1.4. A table of nonlinear solver return codes is given in section 11.1.5. The SUNNonlinearSolver type and the generic SUNNONLINSOL module are defined in section 11.1.6. Section 11.1.7 describes how SUNNONLINSOL models interface with SUNDIALS integrators providing sensitivity analysis capabilities (CVODES and IDAS). Finally, section 11.1.8 lists the requirements for supplying a custom SUNNONLINSOL module. Users wishing to supply their own SUNNONLINSOL module are encouraged to use the SUNNONLINSOL implementations provided with SUNDIALS as a template for supplying custom nonlinear solver modules.

11.1 The SUNNonlinearSolver API

The SUNNONLINSOL API defines several nonlinear solver operations that enable SUNDIALS integrators to utilize any SUNNONLINSOL implementation that provides the required functions. These functions can be divided into three categories. The first are the core nonlinear solver functions. The second group of functions consists of set routines to supply the nonlinear solver with functions provided by the SUNDIALS time integrators and to modify solver parameters. The final group consists of get routines for retrieving nonlinear solver statistics. All of these functions are defined in the header file sundials/sundials_nonlinearsolver.h.

11.1.1 SUNNonlinearSolver core functions

The core nonlinear solver functions consist of two required functions to get the nonlinear solver type (SUNNonlinSolGetType) and solve the nonlinear system (SUNNonlinSolSolve). The remaining three
functions for nonlinear solver initialization (SUNNonlinSolInitialization), setup (SUNNonlinSolSetup), and destruction (SUNNonlinSolFree) are optional.

```
SUNNonlinSolGetType
Call type = SUNNonlinSolGetType(NLS);
Description The required function SUNNonlinSolGetType returns nonlinear solver type.
Arguments NLS (SUNNonlinearSolver) a SUNNONLINSOL object.
Return value The return value type (of type int) will be one of the following:
SUNNONLINEARSOLVER_ROOTFIND 0, the SUNNONLINSOL module solves \( F(y) = 0 \).
SUNNONLINEARSOLVER_FIXEDPOINT 1, the SUNNONLINSOL module solves \( G(y) = y \).
F2003 Name FSUNNonlinSolGetType
```

```
SUNNonlinSolInitialize
Call retval = SUNNonlinSolInitialize(NLS);
Description The optional function SUNNonlinSolInitialize performs nonlinear solver initialization and may perform any necessary memory allocations.
Arguments NLS (SUNNonlinearSolver) a SUNNONLINSOL object.
Return value The return value retval (of type int) is zero for a successful call and a negative value for a failure.
Notes It is assumed all solver-specific options have been set prior to calling SUNNonlinSolInitialize. SUNNONLINSOL implementations that do not require initialization may set this operation to NULL.
F2003 Name FSUNNonlinSolInitialize
```

```
SUNNonlinSolSetup
Call retval = SUNNonlinSolSetup(NLS, y, mem);
Description The optional function SUNNonlinSolSetup performs any solver setup needed for a nonlinear solve.
Arguments NLS (SUNNonlinearSolver) a SUNNONLINSOL object.
y (N_Vector) the initial iteration passed to the nonlinear solver.
mem (void *) the SUNDIALS integrator memory structure.
Return value The return value retval (of type int) is zero for a successful call and a negative value for a failure.
Notes SUNDIALS integrators call SUNNonlinSolSetup before each step attempt. SUNNONLINSOL implementations that do not require setup may set this operation to NULL.
F2003 Name FSUNNonlinSolSetup
```

```
SUNNonlinSolSolve
Call retval = SUNNonlinSolSolve(NLS, y0, ycor, w, tol, callLSetup, mem);
Description The required function SUNNonlinSolSolve solves the nonlinear system \( F(y) = 0 \) or \( G(y) = y \).
Arguments NLS (SUNNonlinearSolver) a SUNNONLINSOL object.
y0 (N_Vector) the predicted value for the new solution state. This must remain unchanged throughout the solution process. See section 11.2 for more detail on the nonlinear system formulation.
```
11.1 The SUNNonlinearSolver API

ycor (N_Vector) on input the initial guess for the correction to the predicted state (zero) and on output the final correction to the predicted state. See section 11.2 for more detail on the nonlinear system formulation.

w (N_Vector) the solution error weight vector used for computing weighted error norms.

tol (realtype) the requested solution tolerance in the weighted root-mean-squared norm.

callLSetup (booleantype) a flag indicating that the integrator recommends for the linear solver setup function to be called.

mem (void *) the SUNDIALS integrator memory structure.

Return value The return value retval (of type int) is zero for a successful solve, a positive value for a recoverable error (i.e., the solve failed and the integrator should reduce the step size and reattempt the step), and a negative value for an unrecoverable error (i.e., the solve failed and the integrator should halt and return an error to the user).

F2003 Name FSUNNonlinSolSolve

SUNNonlinSolFree

Call retval = SUNNonlinSolFree(NLS);

Description The optional function SUNNonlinSolFree frees any memory allocated by the nonlinear solver.

Arguments NLS (SUNNonlinearSolver) a SUNNONLINSOL object.

Return value The return value retval (of type int) should be zero for a successful call, and a negative value for a failure. SUNNONLINSOL implementations that do not allocate data may set this operation to NULL.

F2003 Name FSUNNonlinSolFree

11.1.2 SUNNonlinearSolver set functions

The following set functions are used to supply nonlinear solver modules with functions defined by the SUNDIALS integrators and to modify solver parameters. Only the routine for setting the nonlinear system defining function (SUNNonlinSolSetSysFn is required. All other set functions are optional.

SUNNonlinSolSetSysFn

Call retval = SUNNonlinSolSetSysFn(NLS, SysFn);

Description The required function SUNNonlinSolSetSysFn is used to provide the nonlinear solver with the function defining the nonlinear system. This is the function \( F(y) \) in \( F(y) = 0 \) for SUNNONLINEARSOLVER_ROOTFIND modules or \( G(y) \) in \( G(y) = y \) for SUNNONLINEARSOLVER_FIXEDPOINT modules.

Arguments NLS (SUNNonlinearSolver) a SUNNONLINSOL object.

SysFn (SUNNonlinSolSysFn) the function defining the nonlinear system. See section 11.1.4 for the definition of SUNNonlinSolSysFn.

Return value The return value retval (of type int) should be zero for a successful call, and a negative value for a failure.

F2003 Name FSUNNonlinSolSetSysFn
352 Description of the SUNNonlinearSolver module

**SUNNonlinSolSetLSetupFn**

**Call**

```c
retval = SUNNonlinSolSetLSetupFn(NLS, LSetupFn);
```

**Description**
The optional function SUNNonlinSolSetLSetupFn is called by SUNDIALS integrators to provide the nonlinear solver with access to its linear solver setup function.

**Arguments**
- `NLS` (SUNNonlinearSolver) a SUNNONSOL object.
- `LSetupFn` (SUNNonlinSolLSetupFn) a wrapper function to the SUNDIALS integrator’s linear solver setup function. See section 11.1.4 for the definition of SUNNonlinSetupFn.

**Return value**
The return value `retval` (of type `int`) should be zero for a successful call, and a negative value for a failure.

**Notes**
The SUNNonlinLSetupFn function sets up the linear system $Ax = b$ where $A = \frac{\partial F}{\partial y}$ is the linearization of the nonlinear residual function $F(y) = 0$ (when using SUNNONSOL direct linear solvers) or calls the user-defined preconditioner setup function (when using SUNNONSOL iterative linear solvers). SUNNONSOL implementations that do not require solving this system, do not utilize SUNNONSOL linear solvers, or use SUNNONSOL linear solvers that do not require setup may set this operation to `NULL`.

F2003 Name FSUNNonlinSolSetLSetupFn

**SUNNonlinSolSetLSolveFn**

**Call**

```c
retval = SUNNonlinSolSetLSolveFn(NLS, LSolveFn);
```

**Description**
The optional function SUNNonlinSolSetLSolveFn is called by SUNDIALS integrators to provide the nonlinear solver with access to its linear solver solve function.

**Arguments**
- `NLS` (SUNNonlinearSolver) a SUNNONSOL object
- `L SolveFn` (SUNNonlinSolLSolveFn) a wrapper function to the SUNDIALS integrator’s linear solver solve function. See section 11.1.4 for the definition of SUNNonlinSolLSolveFn.

**Return value**
The return value `retval` (of type `int`) should be zero for a successful call, and a negative value for a failure.

**Notes**
The SUNNonlinLSolveFn function solves the linear system $Ax = b$ where $A = \frac{\partial F}{\partial y}$ is the linearization of the nonlinear residual function $F(y) = 0$. SUNNONSOL implementations that do not require solving this system or do not use SUNNONSOL linear solvers may set this operation to `NULL`.

F2003 Name FSUNNonlinSolSetLSolveFn

**SUNNonlinSolSetConvTestFn**

**Call**

```c
retval = SUNNonlinSolSetConvTestFn(NLS, CTestFn, ctest_data);
```

**Description**
The optional function SUNNonlinSolSetConvTestFn is used to provide the nonlinear solver with a function for determining if the nonlinear solver iteration has converged. This is typically called by SUNDIALS integrators to define their nonlinear convergence criteria, but may be replaced by the user.

**Arguments**
- `NLS` (SUNNonlinearSolver) a SUNNONSOL object
- `CTestFn` (SUNNonlineSolConvTestFn) a SUNDIALS integrator’s nonlinear solver convergence test function. See section 11.1.4 for the definition of SUNNonlinSolConvTestFn.
- `ctest_data` (void*) is a data pointer passed to `CTestFn` every time it is called.

**Return value**
The return value `retval` (of type `int`) should be zero for a successful call, and a negative value for a failure.
Notes SUNNONLINSOL implementations utilizing their own convergence test criteria may set this function to NULL.

F2003 Name FSUNNonlinSolSetConvTestFn

**SUNNonlinSolSetMaxIters**

**Call**

```c
retval = SUNNonlinSolSetMaxIters(NLS, maxiters);
```

**Description**
The optional function SUNNonlinSolSetMaxIters sets the maximum number of nonlinear solver iterations. This is typically called by SUNDIALS integrators to define their default iteration limit, but may be adjusted by the user.

**Arguments**
- `NLS` (SUNNonlinearSolver) a SUNNONLINSOL object.
- `maxiters` (int) the maximum number of nonlinear iterations.

**Return value**
The return value `retval` (of type `int`) should be zero for a successful call, and a negative value for a failure (e.g., `maxiters < 1`).

F2003 Name FSUNNonlinSolSetMaxIters

### 11.1.3 SUNNonlinearSolver get functions

The following get functions allow SUNDIALS integrators to retrieve nonlinear solver statistics. The routines to get the current total number of iterations (SUNNonlinSolGetNumIters) and number of convergence failures (SUNNonlinSolGetNumConvFails) are optional. The routine to get the current nonlinear solver iteration (SUNNonlinSolGetCurIter) is required when using the convergence test provided by the SUNDIALS integrator or by the ARKODE and CVODE linear solver interfaces. Otherwise, SUNNonlinSolGetCurIter is optional.

**SUNNonlinSolGetNumIters**

**Call**

```c
retval = SUNNonlinSolGetNumIters(NLS, numiters);
```

**Description**
The optional function SUNNonlinSolGetNumIters returns the total number of nonlinear solver iterations. This is typically called by the SUNDIALS integrator to store the nonlinear solver statistics, but may also be called by the user.

**Arguments**
- `NLS` (SUNNonlinearSolver) a SUNNONLINSOL object
- `numiters` (long int*) the total number of nonlinear solver iterations.

**Return value**
The return value `retval` (of type `int`) should be zero for a successful call, and a negative value for a failure.

F2003 Name FSUNNonlinSolGetNumIters

**SUNNonlinSolGetCurIter**

**Call**

```c
retval = SUNNonlinSolGetCurIter(NLS, iter);
```

**Description**
The function SUNNonlinSolGetCurIter returns the iteration index of the current nonlinear solve. This function is **required** when using SUNDIALS integrator-provided convergence tests or when using a SUNLINSOL spils linear solver; otherwise it is optional.

**Arguments**
- `NLS` (SUNNonlinearSolver) a SUNNONLINSOL object
- `iter` (int*) the nonlinear solver iteration in the current solve starting from zero.

**Return value**
The return value `retval` (of type `int`) should be zero for a successful call, and a negative value for a failure.

F2003 Name FSUNNonlinSolGetCurIter
Description of the SUNNonlinearSolver module

**SUNNonlinSolGetNumConvFails**

Call: \( \text{retval} = \text{SUNNonlinSolGetNumConvFails}(\text{NLS}, \text{nconvfails}); \)

Description: The optional function `SUNNonlinSolGetNumConvFails` returns the total number of nonlinear solver convergence failures. This may be called by the SUNDIALS integrator to store the nonlinear solver statistics, but may also be called by the user.

Arguments:
- `NLS` (SUNNonlinearSolver) a SUNNONLINSL object
- `nconvfails` (long int*) the total number of nonlinear solver convergence failures.

Return value: The return value `retval` (of type int) should be zero for a successful call, and a negative value for a failure.

F2003 Name: FSUNNonlinSolGetNumConvFails

### 11.1.4 Functions provided by SUNDIALS integrators

To interface with SUNNONLINSL modules, the SUNDIALS integrators supply a variety of routines for evaluating the nonlinear system, calling the SUNLINSOL setup and solve functions, and testing the nonlinear iteration for convergence. These integrator-provided routines translate between the user-supplied ODE or DAE systems and the generic interfaces to the nonlinear or linear systems of equations that result in their solution. The types for functions provided to a SUNNONLINSL module are defined in the header file `sundials/sundials_nonlinearsolver.h`, and are described below.

**SUNNonlinSolSysFn**

Definition: `typedef int (*SUNNonlinSolSysFn)(N_Vector ycor, N_Vector F, void* mem);`

Purpose: These functions evaluate the nonlinear system \( F(y) \) for SUNNONLINEARSOLVER_ROOTFIND type modules or \( G(y) \) for SUNNONLINEARSOLVER_FIXEDPOINT type modules. Memory for \( F \) must be allocated prior to calling this function. The vector \( ycor \) will be left unchanged.

Arguments:
- \( ycor \) is the current correction to the predicted state at which the nonlinear system should be evaluated. See section 11.2 for more detail on the nonlinear system formulation.
- \( F \) is the output vector containing \( F(y) \) or \( G(y) \), depending on the solver type.
- \( mem \) is the SUNDIALS integrator memory structure.

Return value: The return value `retval` (of type int) is zero for a successful solve, a positive value for a recoverable error, and a negative value for an unrecoverable error.

Notes: As discussed in section 11.2, SUNDIALS integrators formulate nonlinear systems as a function of the correction to the predicted solution. On each call to the nonlinear system function the integrator will compute and store the current solution based on the input correction. Additionally, the residual will store the value of the ODE right-hand side function or DAE residual used in computing the nonlinear system residual. These stored values are then directly used in the integrator-supplied linear solver setup and solve functions as applicable.

**SUNNonlinSolLSetupFn**

Definition: `typedef int (*SUNNonlinSolLSetupFn)(booleantype jbad, booleantype* jcur, void* mem);`

Purpose: These functions are wrappers to the SUNDIALS integrator’s function for setting up linear solves with SUNLINSOL modules.

Arguments:
- `jbad` is an input indicating whether the nonlinear solver believes that \( A \) has gone stale (SUNTRUE) or not (SUNFALSE).
11.1 The SUNNonlinearSolver API

jcur is an output indicating whether the routine has updated the Jacobian $A$ (SUNTRUE) or not (SUNFALSE).

mem is the SUNDIALS integrator memory structure.

Return value The return value retval (of type int) is zero for a successful solve, a positive value for a recoverable error, and a negative value for an unrecoverable error.

Notes The SUNNonlinSetupFn function sets up the linear system $Ax = b$ where $A = \frac{\partial F}{\partial y}$ is the linearization of the nonlinear residual function $F(y) = 0$ (when using SUNLINSOL direct linear solvers) or calls the user-defined preconditioner setup function (when using SUNLINSOL iterative linear solvers). SUNNONLINSOL implementations that do not require solving this system, do not utilize SUNLINSOL linear solvers, or use SUNLINSOL linear solvers that do not require setup may ignore these functions.

As discussed in the description of SUNNonlinSolSysFn, the linear solver setup function assumes that the nonlinear system function has been called prior to the linear solver setup function as the setup will utilize saved values from the nonlinear system evaluation (e.g., the updated solution).

**SUNNonlinSolLSolveFn**

Definition typedef int (*SUNNonlinSolLSolveFn)(N_Vector b, void* mem);

Purpose These functions are wrappers to the SUNDIALS integrator’s function for solving linear systems with SUNLINSOL modules.

Arguments b contains the right-hand side vector for the linear solve on input and the solution to the linear system on output.

mem is the SUNDIALS integrator memory structure.

Return value The return value retval (of type int) is zero for a successful solve, a positive value for a recoverable error, and a negative value for an unrecoverable error.

Notes The SUNNonlinLSolveFn function solves the linear system $Ax = b$ where $A = \frac{\partial F}{\partial y}$ is the linearization of the nonlinear residual function $F(y) = 0$. SUNNONLINSOL implementations that do not require solving this system or do not use SUNLINSOL linear solvers may ignore these functions.

As discussed in the description of SUNNonlinSolSysFn, the linear solver solve function assumes that the nonlinear system function has been called prior to the linear solver solve function as the solve may utilize saved values from the nonlinear system evaluation (e.g., the updated solution).

**SUNNonlinSolConvTestFn**

Definition typedef int (*SUNNonlinSolConvTestFn)(SUNNonlinearSolver NLS, N_Vector ycor, N_Vector del, realtype tol, N_Vector ewt, void* ctest_data);

Purpose These functions are SUNDIALS integrator-specific convergence tests for nonlinear solvers and are typically supplied by each SUNDIALS integrator, but users may supply custom problem-specific versions as desired.

Arguments NLS is the SUNNONLINSOL object.

ycor is the current correction (nonlinear iterate).

del is the difference between the current and prior nonlinear iterates.

tol is the nonlinear solver tolerance.

ewt is the weight vector used in computing weighted norms.

ctest_data is the data pointer provided to SUNNonlinSolSetConvTestFn.
Return value  The return value of this routine will be a negative value if an unrecoverable error occurred or one of the following:

- **SUN-NLS_SUCCESS**: the iteration is converged.
- **SUN-NLS_CONTINUE**: the iteration has not converged, keep iterating.
- **SUN-NLS_CONV_RECVR**: the iteration appears to be diverging, try to recover.

Notes  The tolerance passed to this routine by SUNDIALS integrators is the tolerance in a weighted root-mean-squared norm with error weight vector `ewt`. SUNNONLINSOL modules utilizing their own convergence criteria may ignore these functions.

### 11.1.5 SUNNonlinearSolver return codes

The functions provided to SUNNONLINSOL modules by each SUNDIALS integrator, and functions within the SUNDIALS-provided SUNNONLINSOL implementations utilize a common set of return codes, shown below in Table 11.1. Here, negative values correspond to non-recoverable failures, positive values to recoverable failures, and zero to a successful call.

<table>
<thead>
<tr>
<th>Name</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>SUN-NLS_SUCCESS</td>
<td>0</td>
<td>successful call or converged solve</td>
</tr>
<tr>
<td>SUN-NLS_CONTINUE</td>
<td>901</td>
<td>the nonlinear solver is not converged, keep iterating</td>
</tr>
<tr>
<td>SUN-NLS_CONV_RECVR</td>
<td>902</td>
<td>the nonlinear solver appears to be diverging, try to recover</td>
</tr>
<tr>
<td>SUN-NLS_MEM_NULL</td>
<td>-901</td>
<td>a memory argument is NULL</td>
</tr>
<tr>
<td>SUN-NLS_MEM_FAIL</td>
<td>-902</td>
<td>a memory access or allocation failed</td>
</tr>
<tr>
<td>SUN-NLS_ILL_INPUT</td>
<td>-903</td>
<td>an illegal input option was provided</td>
</tr>
<tr>
<td>SUN-NLSVECTOROP_ERR</td>
<td>-904</td>
<td>a NVECTOR operation failed</td>
</tr>
<tr>
<td>SUN-NLS_EXT_FAIL</td>
<td>-905</td>
<td>an external library call returned an error</td>
</tr>
</tbody>
</table>

### 11.1.6 The generic SUNNonlinearSolver module

SUNDIALS integrators interact with specific SUNNONLINSOL implementations through the generic SUNNONLINSOL module on which all other SUNNONLINSOL implementations are built. The SUNNonlinearSolver type is a pointer to a structure containing an implementation-dependent `content` field and an `ops` field. The type SUNNonlinearSolver is defined as follows:

```c
typedef struct _generic_SUNNonlinearSolver *SUNNonlinearSolver;

struct _generic_SUNNonlinearSolver {
    void *content;
    struct _generic_SUNNonlinearSolver_Ops *ops;
};
```

where the `_generic_SUNNonlinearSolver_Ops` structure is a list of pointers to the various actual nonlinear solver operations provided by a specific implementation. The `_generic_SUNNonlinearSolver_Ops` structure is defined as

```c
struct _generic_SUNNonlinearSolver_Ops {
    SUNNonlinearSolver_Type (*gettype)(SUNNonlinearSolver);
    int (*initialize)(SUNNonlinearSolver);
    int (*setup)(SUNNonlinearSolver, N_Vector, void*);
    int (*solve)(SUNNonlinearSolver, N_Vector, N_Vector, realtype, booleantype, void*);
};
```
The SUNNonlinearSolver API

```c
int (*free)(SUNNonlinearSolver);
int (*setsysfn)(SUNNonlinearSolver, SUNNonlinSolSysFn);
int (*setlsetupfn)(SUNNonlinearSolver, SUNNonlinSolLSetupFn);
int (*setlsolvefn)(SUNNonlinearSolver, SUNNonlinSolLSolveFn);
int (*setctestfn)(SUNNonlinearSolver, SUNNonlinSolConvTestFn,
                 void*);
int (*setmaxiters)(SUNNonlinearSolver, int);
int (*getnumiters)(SUNNonlinearSolver, long int*);
int (*getcuriter)(SUNNonlinearSolver, int*);
int (*getnumconvfails)(SUNNonlinearSolver, long int*);
};
```

The generic SUNNONLINSOL module defines and implements the nonlinear solver operations defined in Sections 11.1.1 – 11.1.3. These routines are in fact only wrappers to the nonlinear solver operations provided by a particular SUNNONLINSOL implementation, which are accessed through the `ops` field of the `SUNNonlinearSolver` structure. To illustrate this point we show below the implementation of a typical nonlinear solver operation from the generic SUNNONLINSOL module, namely `SUNNonlinSolSolve`, which solves the nonlinear system and returns a flag denoting a successful or failed solve:

```c
int SUNNonlinSolSolve(SUNNonlinearSolver NLS,
                       N_Vector y0, N_Vector y,
                       N_Vector w, realtype tol,
                       booleantype callLSetup, void* mem)
{
    return((int) NLS->ops->solve(NLS, y0, y, w, tol, callLSetup, mem));
}
```

The Fortran 2003 interface provides a `bind(C)` derived-type for the `generic_SUNNonlinearSolver` and the `generic_SUNNonlinearSolver_Ops` structures. Their definition is given below.

```fortran
type, bind(C), public :: SUNNonlinearSolver
    type(C_PTR), public :: content
    type(C_PTR), public :: ops
end type SUNNonlinearSolver

type, bind(C), public :: SUNNonlinearSolver_Ops
    type(C_FUNPTR), public :: gettype
    type(C_FUNPTR), public :: initialize
    type(C_FUNPTR), public :: setup
    type(C_FUNPTR), public :: solve
    type(C_FUNPTR), public :: free
    type(C_FUNPTR), public :: setsysfn
    type(C_FUNPTR), public :: setlsetupfn
    type(C_FUNPTR), public :: setlsolvefn
    type(C_FUNPTR), public :: setctestfn
    type(C_FUNPTR), public :: setmaxiters
    type(C_FUNPTR), public :: getnumiters
    type(C_FUNPTR), public :: getcuriter
    type(C_FUNPTR), public :: getnumconvfails
end type SUNNonlinearSolver_Ops
```

11.1.7 Usage with sensitivity enabled integrators

When used with SUNDIALS packages that support sensitivity analysis capabilities (e.g., CVODES and IDAS) a special NVECTOR module is used to interface with SUNNONLINSOL modules for solves involving...
sensitivity vectors stored in an NVECTOR array. As described below, the NVECTOR_SENSWRAPPER module is an NVECTOR implementation where the vector content is an NVECTOR array. This wrapper vector allows SUNNONLINSOL modules to operate on data stored as a collection of vectors.

For all SUNDIALS-provided SUNNONLINSOL modules a special constructor wrapper is provided so users do not need to interact directly with the NVECTOR_SENSWRAPPER module. These constructors follow the naming convention SUNNonlinSol_***Sens(count,...) where *** is the name of the SUNNONLINSOL module, count is the size of the vector wrapper, and ... are the module-specific constructor arguments.

The NVECTOR_SENSWRAPPER module

This section describes the NVECTOR_SENSWRAPPER implementation of an NVECTOR. To access the NVECTOR_SENSWRAPPER module, include the header file sundials/sundials_nvector_senswrapper.h.

The NVECTOR_SENSWRAPPER module defines an N_Vector implementing all of the standard vectors operations defined in Table 8.1.1 but with some changes to how operations are computed in order to accommodate operating on a collection of vectors.

1. Element-wise vector operations are computed on a vector-by-vector basis. For example, the linear sum of two wrappers containing \( n_v \) vectors of length \( n \), \( N_VLinearSum(a,x,b,y,z) \), is computed as
   \[
   z_{j,i} = ax_{j,i} + by_{j,i}, \quad i = 0, \ldots, n - 1, \quad j = 0, \ldots, n_v - 1.
   \]

2. The dot product of two wrappers containing \( n_v \) vectors of length \( n \) is computed as if it were the dot product of two vectors of length \( nn_v \). Thus \( d = N_VDotProd(x,y) \) is
   \[
   d = \sum_{j=0}^{n_v-1} \sum_{i=0}^{n-1} x_{j,i} y_{j,i}.
   \]

3. All norms are computed as the maximum of the individual norms of the \( n_v \) vectors in the wrapper. For example, the weighted root mean square norm \( m = N_VWrmsNorm(x, w) \) is
   \[
   m = \max_j \left( \frac{1}{n} \sum_{i=0}^{n-1} (x_{j,i} w_{j,i})^2 \right)^{1/2}.
   \]

To enable usage alongside other NVECTOR modules the NVECTOR_SENSWRAPPER functions implementing vector operations have _SensWrapper appended to the generic vector operation name.

The NVECTOR_SENSWRAPPER module provides the following constructors for creating an NVECTOR_SENSWRAPPER:

```
N_VNewEmpty_SensWrapper
Call w = N_VNewEmpty_SensWrapper(count);
Description The function N_VNewEmpty_SensWrapper creates an empty NVECTOR_SENSWRAPPER wrapper with space for count vectors.
Arguments count (int) the number of vectors the wrapper will contain.
Return value The return value w (of type N_Vector) will be a NVECTOR object if the constructor exits successfully, otherwise w will be NULL.
```

F2003 Name FN_VNewEmpty_SensWrapper
Call $w = \text{N\_VNew\_SensWrapper}(\text{count}, y);$.

Description The function \text{N\_VNew\_SensWrapper} creates an \text{NVector\_SENSWRAPPER} wrapper containing \text{count} vectors cloned from \text{y}.

Arguments \text{count} (\text{int}) the number of vectors the wrapper will contain.

\text{y} (\text{N\_Vector}) the template vectors to use in creating the vector wrapper.

Return value The return value \text{w} (of type \text{N\_Vector}) will be a \text{NVector} object if the constructor exits successfully, otherwise \text{w} will be \text{NULL}.

\text{F2003 Name FN\_VNew\_SensWrapper}

The \text{NVector\_SENSWRAPPER} implementation of the \text{NVector} module defines the \text{content} field of the \text{N\_Vector} to be a structure containing an \text{N\_Vector} array, the number of vectors in the vector array, and a boolean flag indicating ownership of the vectors in the vector array.

\text{struct _N\_VectorContent\_SensWrapper} {
    \text{N\_Vector* vecs;}
    \text{int nvecs;}
    \text{bool type own_vecs;}
};

The following macros are provided to access the content of an \text{NVector\_SENSWRAPPER} vector.

- \text{NV\_CONTENT\_SW(v)} - provides access to the content structure
- \text{NV\_VECS\_SW(v)} - provides access to the vector array
- \text{NV\_NVECS\_SW(v)} - provides access to the number of vectors
- \text{NV\_OWN\_VECS\_SW(v)} - provides access to the ownership flag
- \text{NV\_VEC\_SW(v,i)} - provides access to the \text{i}-th vector in the vector array

\section*{11.1.8 Implementing a Custom SUNNonlinearSolver Module}

A \text{SUNNONLINSOL} implementation \text{must} do the following:

1. Specify the content of the \text{SUNNONLINSOL} module.

2. Define and implement the required nonlinear solver operations defined in Sections 11.1.1 – 11.1.3.
   Note that the names of the module routines should be unique to that implementation in order to permit using more than one \text{SUNNONLINSOL} module (each with different \text{SUNNonlinearSolver} internal data representations) in the same code.

3. Define and implement a user-callable constructor to create a \text{SUNNonlinearSolver} object.

Additionally, a \text{SUNNonlinearSolver} implementation \text{may} do the following:

1. Define and implement additional user-callable “set” routines acting on the \text{SUNNonlinearSolver} object, e.g., for setting various configuration options to tune the performance of the nonlinear solve algorithm.

2. Provide additional user-callable “get” routines acting on the \text{SUNNonlinearSolver} object, e.g., for returning various solve statistics.

To aid in the creation of custom \text{SUNNONLINSOL} modules the generic \text{SUNNONLINSOL} module provides the utility functions \text{SUNNonlinSolNewEmpty} and \text{SUNNonlinSolFreeEmpty}. When used in custom \text{SUNNONLINSOL} constructors, the function \text{SUNNonlinSolNewEmpty} will ease the introduction of any new optional nonlinear solver operations to the \text{SUNNONLINSOL} API by ensuring only required operations need to be set.
Description of the SUNNonlinearSolver module

**SUNNonlinSolNewEmpty**

Call

```c
NLS = SUNNonlinSolNewEmpty();
```

Description

The function SUNNonlinSolNewEmpty allocates a new generic SUNNONLINSOL object and initializes its content pointer and the function pointers in the operations structure to NULL.

Arguments

None

Return value

This function returns a SUNNonlinearSolver object. If an error occurs when allocating the object, then this routine will return NULL.

F2003 Name

FSUNNonlinSolNewEmpty

**SUNNonlinSolFreeEmpty**

Call

```c
SUNNonlinSolFreeEmpty(NLS);
```

Description

This routine frees the generic SUNNonlinearSolver object, under the assumption that any implementation-specific data that was allocated within the underlying content structure has already been freed. It will additionally test whether the ops pointer is NULL, and, if it is not, it will free it as well.

Arguments

NLS (SUNNonlinearSolver)

Return value

None

F2003 Name

FSUNNonlinSolFreeEmpty

### 11.2 IDAS SUNNonlinearSolver interface

As discussed in Chapter 2 each integration step requires the (approximate) solution of the nonlinear system

\[ G(y_n) = F \left( t_n, y_n, h_n^{-1} \sum_{i=0}^{q} \alpha_{n,i} y_{n-i} \right) = 0. \]  

(11.1)

Rather than solving this system for the new state \( y_n \) IDA reformulates the system to solve for the correction \( y_{cor} \) to the predicted new state \( y_{pred} \) and its derivative \( \dot{y}_{pred} \) so that \( y_n = y_{pred} + y_{cor} \) and \( \dot{y}_n = \dot{y}_{pred} + h_n^{-1} \alpha_{n,0} y_{cor} \). The nonlinear system rewritten in terms of \( y_{cor} \) is

\[ G(y_{cor}) = F \left( t_n, y_{pred} + y_{cor}, \dot{y}_{pred} + \alpha y_{cor} \right) = 0. \]  

(11.2)

where \( \alpha = h_n^{-1} \alpha_{n,0} \). Similarly in the forward sensitivity analysis case the nonlinear system is also reformulated in terms of the correction to the predicted sensitivities.

The nonlinear system function provided by IDA to the nonlinear solver module internally updates the current value of the new state and its derivative based on the current correction passed to the function (as well as the sensitivities). These values are used when calling the DAE residual function and when setting up linear solves (e.g., for updating the Jacobian or preconditioner).

IDA provides several advanced functions that will not be needed by most users, but might be useful for users who choose to provide their own implementation of the SUNNonlinearSolver API. For example, such a user might need access to the current \( y \) and \( \dot{y} \) vectors to compute Jacobian data.

**IDAGetCurrentCj**

Call

```c
flag = IDAGetCurrentCj(ida_mem, &cj);
```

Description

The function IDAGetCurrentCj returns the scalar \( c_j \) which is proportional to the inverse of the step size (\( \alpha \) in Eq. (2.6)).

Arguments

ida_mem (void *) pointer to the IDA memory block.

cj (realtype) the value of \( c_j \).
Return value The return value flag (of type int) is one of
IDA_SUCCESS The optional output value has been successfully set.
IDA_MEM_NULL The ida_mem pointer is NULL.

[IDAGetCurrentY]
Call flag = IDAGetCurrentY(ida_mem, &y);
Description The function IDAGetCurrentY returns the current \( y \) vector.
Arguments ida_mem (void *) pointer to the IDA memory block.
y (N_Vector *) the current \( y \) vector
Return value The return value flag (of type int) is one of
IDA_SUCCESS The optional output value has been successfully set.
IDA_MEM_NULL The ida_mem pointer is NULL.

[IDAGetCurrentYp]
Call flag = IDAGetCurrentYp(ida_mem, &yp);
Description The function IDAGetCurrentYp returns the current \( \dot{y} \) vector.
Arguments ida_mem (void *) pointer to the IDA memory block.
yp (N_Vector *) the current \( \dot{y} \) vector
Return value The return value flag (of type int) is one of
IDA_SUCCESS The optional output value has been successfully set.
IDA_MEM_NULL The ida_mem pointer is NULL.

[IDAGetCurrentYSens]
Call flag = IDAGetCurrentYSens(ida_mem, &yyS);
Description The function IDAGetCurrentYSens returns the current sensitivity vector array.
Arguments ida_mem (void *) pointer to the IDA memory block.
yyS (N_Vector **) pointer to the vector array that is set to the array of sensitivity vectors
Return value The return value flag (of type int) is one of
IDA_SUCCESS The optional output value has been successfully set.
IDA_MEM_NULL The ida_mem pointer is NULL.

[IDAGetCurrentYpSens]
Call flag = IDAGetCurrentYpSens(ida_mem, &ypS);
Description The function IDAGetCurrentYpSens returns the derivative the current sensitivity vector array.
Arguments ida_mem (void *) pointer to the IDA memory block.
ypS (N_Vector **) pointer to the vector array that is set to the array of sensitivity vector derivatives
Return value The return value flag (of type int) is one of
IDA_SUCCESS The optional output value has been successfully set.
IDA_MEM_NULL The ida_mem pointer is NULL.
Description of the SUNNonlinearSolver module

**IDACalculateY**

Call  
\[ \text{flag} = \text{IDACalculateY}(\text{ida\_mem}, \text{ycor}, \text{y}); \]

Description  
The function computes the current \( y \) vector based on the given correction vector from the nonlinear solver.

Arguments  
\begin{align*}
\text{ida\_mem} & : \text{void *} \text{ pointer to the IDA memory block} \\
\text{ycor} & : \text{N\_Vector} \text{ the correction} \\
\text{y} & : \text{N\_Vector} \text{ the output vector}
\end{align*}

Return value  
The return value \text{flag} (of type \text{int}) is one of
\begin{align*}
\text{IDA\_SUCCESS} & : \text{The optional output value has been successfully set.} \\
\text{IDA\_MEM\_NULL} & : \text{The ida\_mem pointer is NULL.}
\end{align*}

**IDACalculateYp**

Call  
\[ \text{flag} = \text{IDACalculateYp}(\text{ida\_mem}, \text{ycor}, \text{yp}); \]

Description  
The function computes \( \dot{y} \) based on the given correction vector from the nonlinear solver.

Arguments  
\begin{align*}
\text{ida\_mem} & : \text{void *} \text{ pointer to the IDA memory block} \\
\text{ycor} & : \text{N\_Vector} \text{ the correction} \\
\text{yp} & : \text{N\_Vector} \text{ the output vector array}
\end{align*}

Return value  
The return value \text{flag} (of type \text{int}) is one of
\begin{align*}
\text{IDA\_SUCCESS} & : \text{The optional output value has been successfully set.} \\
\text{IDA\_MEM\_NULL} & : \text{The ida\_mem pointer is NULL.}
\end{align*}

**IDACalculateYSens**

Call  
\[ \text{flag} = \text{IDACalculateYSens}(\text{ida\_mem}, \text{ycorS}, \text{yyS}); \]

Description  
The function computes the sensitivities based on the given correction vector from the nonlinear solver.

Arguments  
\begin{align*}
\text{ida\_mem} & : \text{void *} \text{ pointer to the IDA memory block} \\
\text{ycorS} & : \text{N\_Vector *} \text{ the correction} \\
\text{yyS} & : \text{N\_Vector *} \text{ the output vector array}
\end{align*}

Return value  
The return value \text{flag} (of type \text{int}) is one of
\begin{align*}
\text{IDA\_SUCCESS} & : \text{The optional output value has been successfully set.} \\
\text{IDA\_MEM\_NULL} & : \text{The ida\_mem pointer is NULL.}
\end{align*}

**IDACalculateYpSens**

Call  
\[ \text{flag} = \text{IDACalculateYpSens}(\text{ida\_mem}, \text{ycorS}, \text{ypS}); \]

Description  
The function computes the sensitivity derivatives based on the given correction vector from the nonlinear solver.

Arguments  
\begin{align*}
\text{ida\_mem} & : \text{void *} \text{ pointer to the IDA memory block} \\
\text{ycorS} & : \text{N\_Vector *} \text{ the correction} \\
\text{ypS} & : \text{N\_Vector *} \text{ the output vector array}
\end{align*}

Return value  
The return value \text{flag} (of type \text{int}) is one of
\begin{align*}
\text{IDA\_SUCCESS} & : \text{The optional output value has been successfully set.} \\
\text{IDA\_MEM\_NULL} & : \text{The ida\_mem pointer is NULL.}
\end{align*}
11.3 The SUNNonlinearSolver_Newton implementation

This section describes the SUNNONLINSOL implementation of Newton’s method. To access the SUNNONLINSOL_NEWTON module, include the header file sunnonlinsol/sunnonlinsol_newton.h. We note that the SUNNONLINSOL_NEWTON module is accessible from SUNDIALS integrators without separately linking to the libsundials_sunnonlinsolnewton module library.

11.3.1 SUNNonlinearSolver_Newton description

To find the solution to

\[ F(y) = 0 \]  \hspace{1cm} (11.3)

given an initial guess \( y^{(0)} \), Newton’s method computes a series of approximate solutions

\[ y^{(m+1)} = y^{(m)} + \delta^{(m+1)} \]  \hspace{1cm} (11.4)

where \( m \) is the Newton iteration index, and the Newton update \( \delta^{(m+1)} \) is the solution of the linear system

\[ A(y^{(m)})\delta^{(m+1)} = -F(y^{(m)}), \]  \hspace{1cm} (11.5)

in which \( A \) is the Jacobian matrix

\[ A \equiv \frac{\partial F}{\partial y}. \]  \hspace{1cm} (11.6)

Depending on the linear solver used, the SUNNONLINSOL_NEWTON module will employ either a Modified Newton method, or an Inexact Newton method \([7, 11, 20, 22, 37]\). When used with a direct linear solver, the Jacobian matrix \( A \) is held constant during the Newton iteration, resulting in a Modified Newton method. With a matrix-free iterative linear solver, the iteration is an Inexact Newton method.

In both cases, calls to the integrator-supplied SUNNonlinSolLSetupFn function are made infrequently to amortize the increased cost of matrix operations (updating \( A \) and its factorization within direct linear solvers, or updating the preconditioner within iterative linear solvers). Specifically, SUNNONLINSOL_NEWTON will call the SUNNonlinSolLSetupFn function in two instances:

(a) when requested by the integrator (the input callLSetSetup is SUNTRUE) before attempting the Newton iteration, or

(b) when reattempting the nonlinear solve after a recoverable failure occurs in the Newton iteration with stale Jacobian information (jcur is SUNFALSE). In this case, SUNNONLINSOL_NEWTON will set jbad to SUNTRUE before calling the SUNNonlinSolLSetupFn function.

Whether the Jacobian matrix \( A \) is fully or partially updated depends on logic unique to each integrator-supplied SUNNonlinSolLSetupFn routine. We refer to the discussion of nonlinear solver strategies provided in Chapter 2 for details on this decision.

The default maximum number of iterations and the stopping criteria for the Newton iteration are supplied by the SUNDIALS integrator when SUNNONLINSOL_NEWTON is attached to it. Both the maximum number of iterations and the convergence test function may be modified by the user by calling the SUNNonlinSolSetMaxIters and/or SUNNonlinSolSetConvTestFn functions after attaching the SUNNONLINSOL_NEWTON object to the integrator.

11.3.2 SUNNonlinearSolver_Newton functions

The SUNNONLINSOL_NEWTON module provides the following constructors for creating a SUNNonlinearSolver object.
**SUNNonlinSol_Newton**

**Call**

\[
\text{NLS} = \text{SUNNonlinSol_Newton}(y);
\]

**Description**

The function **SUNNonlinSol_Newton** creates a **SUNNonlinearSolver** object for use with **SUNDIALS** integrators to solve nonlinear systems of the form \( F(y) = 0 \) using Newton's method.

**Arguments**

- \( y \) (\( N \text{Vector} \)) a template for cloning vectors needed within the solver.

**Return value**

The return value \( \text{NLS} \) (of type **SUNNonlinearSolver**) will be a **SUNNONLINSOL** object if the constructor exits successfully, otherwise \( \text{NLS} \) will be **NULL**.

F2003 Name **FSUNNonlinSol_Newton**

---

**SUNNonlinSol_NewtonSens**

**Call**

\[
\text{NLS} = \text{SUNNonlinSol_NewtonSens}(\text{count}, y);
\]

**Description**

The function **SUNNonlinSol_NewtonSens** creates a **SUNNonlinearSolver** object for use with **SUNDIALS** sensitivity enabled integrators (CVODES and IDAS) to solve nonlinear systems of the form \( F(y) = 0 \) using Newton's method.

**Arguments**

- \( \text{count} \) (\( \text{int} \)) the number of vectors in the nonlinear solve. When integrating a system containing \( Ns \) sensitivities the value of \( \text{count} \) is:
  - \( Ns+1 \) if using a simultaneous corrector approach.
  - \( Ns \) if using a staggered corrector approach.
- \( y \) (\( N \text{Vector} \)) a template for cloning vectors needed within the solver.

**Return value**

The return value \( \text{NLS} \) (of type **SUNNonlinearSolver**) will be a **SUNNONLINSOL** object if the constructor exits successfully, otherwise \( \text{NLS} \) will be **NULL**.

F2003 Name **FSUNNonlinSol_NewtonSens**

The **SUNNONLINSOL_NEWTON** module implements all of the functions defined in sections 11.1.1 – 11.1.3 except for the **SUNNonlinSolSetup** function. The **SUNNONLINSOL_NEWTON** functions have the same names as those defined by the generic **SUNNONLINSOL** API with **Newton** appended to the function name. Unless using the **SUNNONLINSOL_NEWTON** module as a standalone nonlinear solver the generic functions defined in sections 11.1.1 – 11.1.3 should be called in favor of the **SUNNONLINSOL_NEWTON**-specific implementations.

The **SUNNONLINSOL_NEWTON** module also defines the following additional user-callable function.

**SUNNonlinSolGetSysFn_Newton**

**Call**

\[
\text{retval} = \text{SUNNonlinSolGetSysFn_Newton}(\text{NLS}, \text{SysFn});
\]

**Description**

The function **SUNNonlinSolGetSysFn_Newton** returns the residual function that defines the nonlinear system.

**Arguments**

- \( \text{NLS} \) (\( \text{SUNNonlinearSolver} \)) a **SUNNONLINSOL** object
- \( \text{SysFn} \) (\( \text{SUNNonlinSolSysFn*} \)) the function defining the nonlinear system.

**Return value**

The return value \( \text{retval} \) (of type \( \text{int} \)) should be zero for a successful call, and a negative value for a failure.

**Notes**

This function is intended for users that wish to evaluate the nonlinear residual in a custom convergence test function for the **SUNNONLINSOL_NEWTON** module. We note that **SUNNONLINSOL_NEWTON** will not leverage the results from any user calls to **SysFn**.

F2003 Name **FSUNNonlinSolGetSysFn_Newton**

---

### 11.3.3 SUNNonlinearSolver_Newton Fortran interfaces

The **SUNNONLINSOL_NEWTON** module provides a **FORTRAN 2003** module as well as **FORTRAN 77** style interface functions for use from **FORTRAN** applications.
11.3 The SUNNonlinearSolver_Newton implementation

**FORTRAN 2003 interface module**

The fsunnonlinsol_newton_mod FORTRAN module defines interfaces to all SUNNONLINSOL_NEWTON C functions using the intrinsic iso_c_binding module which provides a standardized mechanism for interoperating with C. As noted in the C function descriptions above, the interface functions are named after the corresponding C function, but with a leading ‘F’. For example, the function SUNNonlinSol_Newton is interfaced as FSUNNonlinSol_Newton.

The FORTRAN 2003 SUNNONLINSOL_NEWTON interface module can be accessed with the use statement, i.e. use fsunnonlinsol_newton_mod, and linking to the library libsundials_fsunnonlinsolnewton_mod.lib in addition to the C library. For details on where the library and module file fsunnonlinsol_newton_mod.mod are installed see Appendix A. We note that the module is accessible from the FORTRAN 2003 SUNDIALS integrators without separately linking to the libsundials_fsunnonlinsolnewton_mod library.

**FORTRAN 77 interface functions**

For SUNDIALS integrators that include a FORTRAN 77 interface, the SUNNONLINSOL_NEWTON module also includes a Fortran-callable function for creating a SUNNonlinearSolver object.

```fortran
FSUNNEWTONINIT
Call FSUNNEWTONINIT(code, ier);
```

Description The function FSUNNEWTONINIT can be called for Fortran programs to create a SUNNonlinearSolver object for use with SUNDIALS integrators to solve nonlinear systems of the form \( F(y) = 0 \) with Newton’s method.

Arguments code (int*) is an integer input specifying the solver id (1 for cvode, 2 for ida, and 4 for arkode).

Return value ier is a return completion flag equal to 0 for a success return and -1 otherwise. See printed message for details in case of failure.

11.3.4 SUNNonlinearSolver_Newton content

The SUNNONLINSOL_NEWTON module defines the content field of a SUNNonlinearSolver as the following structure:

```c
struct _SUNNonlinearSolverContent_Newton {
    SUNNonlinSolSysFn Sys;
    SUNNonlinSolLSetupFn LSetup;
    SUNNonlinSolLSolveFn LSolve;
    SUNNonlinSolConvTestFn CTest;

    N_Vector delta;
    boolantype jcur;
    int curiter;
    int maxiters;
    long int niter;
    long int nconvfails;
    void* ctest_data;
};
```

These entries of the content field contain the following information:

Sys - the function for evaluating the nonlinear system,
LSetup - the package-supplied function for setting up the linear solver,
LSolve - the package-supplied function for performing a linear solve,
Description of the SUNNonlinearSolver module

- **CTest** - the function for checking convergence of the Newton iteration,
- **delta** - the Newton iteration update vector,
- **jcur** - the Jacobian status (SUNTRUE = current, SUNFALSE = stale),
- **curiter** - the current number of iterations in the solve attempt,
- **maxiters** - the maximum number of Newton iterations allowed in a solve,
- **nconvfails** - the total number of nonlinear convergence failures across all solves,
- **ctest.data** - the data pointer passed to the convergence test function.

### 11.4 The SUNNonlinearSolver_PetscSNES implementation

This section describes the SUNNONLINSOL interface to the PETSc SNES nonlinear solver(s). To enable the SUNNONLINSOL_PETSCSNES module, SUNDIALS must be configured to use PETSc. Instructions on how to do this are given in Chapter A.1.4. To access the module, users must include the header file sunnonlinsol/sunnonlinsol_petscsnes.h. The library to link to is libsundials_sunnonlinsol_petsc.lib where .lib is .so for shared libraries and .a for static libraries. Users of the SUNNONLINSOL_PETSCSNES should also see the section NVECTOR_PETSC 8.8 which discusses the NVECTOR interface to the PETSc Vec API.

#### 11.4.1 SUNNonlinearSolver_PetscSNES description

The SUNNONLINSOL_PETSCSNES implementation allows users to utilize a PETSc SNES nonlinear solver to solve the nonlinear systems that arise in the SUNDIALS integrators. Since SNES uses the KSP linear solver interface underneath it, the SUNNONLINSOL_PETSCSNES implementation does not interface with SUNDIALS linear solvers. Instead, users should set nonlinear solver options, linear solver options, and preconditioner options through the PETSc SNES, KSP, and PC APIs [5].

Important usage notes for the SUNNONLINSOL_PETSCSNES implementation are provided below:

- The SUNNONLINSOL_PETSCSNES implementation handles calling SNESSetFunction at construction. The actual residual function $F(y)$ is set by the SUNDIALS integrator when the SUNNONLINSOL_PETSCSNES object is attached to it. Therefore, a user should not call SNESSetFunction on a SNES object that is being used with SUNNONLINSOL_PETSCSNES. For these reasons, it is recommended, although not always necessary, that the user calls SUNNonlinSol_PetscSNES with the new SNES object immediately after calling.

- The number of nonlinear iterations is tracked by SUNDIALS separately from the count kept by SNES. As such, the function SUNNonlinSolGetNumIters reports the cumulative number of iterations across the lifetime of the SUNNONLINSOL object.

- Some “converged” and “diverged” convergence reasons returned by SNES are treated as recoverable convergence failures by SUNDIALS. Therefore, the count of convergence failures returned by SUNNonlinSolGetNumConvFails will reflect the number of recoverable convergence failures as determined by SUNDIALS, and may differ from the count returned by SNESGetNonlinearStepFailures.

- The SUNNONLINSOL_PETSCSNES module is not currently compatible with the CVODES or IDAS staggered or simultaneous sensitivity strategies.

#### 11.4.2 SUNNonlinearSolver_PetscSNES functions

The SUNNONLINSOL_PETSCSNES module provides the following constructor for creating a SUNNonlinearSolver object.
11.4 The SUNNonlinearSolver_PetscSNES implementation

**SUNNonlinSol_PetscSNES**

Call

```
NLS = SUNNonlinSol_PetscSNES(y, snes);
```

Description The function SUNNonlinSol_PetscSNES creates a SUNNonlinearSolver object that wraps a PETSc SNES object for use with SUNDIALS. This will call SNESSetFunction on the provided SNES object.

Arguments

- `snes` (SNES) a PETSc SNES object
- `y` (N_Vector) a N_Vector object of type NVECTOR_PETSC that used as a template for the residual vector

Return value A SUNNONLINSOL object if the constructor exits successfully, otherwise NLS will be NULL.

Notes This function calls SNESSetFunction and will overwrite whatever function was previously set. Users should not call SNESSetFunction on the SNES object provided to the constructor.

The SUNNONLINSOL_PETSCSNES module implements all of the functions defined in sections 11.1.1 – 11.1.3 except for SUNNonlinSolSetup, SUNNonlinSolSetLSetupFn, SUNNonlinSolSetLSolveFn, SUNNonlinSolSetConvTestFn, and SUNNonlinSolSetMaxIters.

The SUNNONLINSOL_PETSCSNES functions have the same names as those defined by the generic SUNNONLINSOL API with _PetscSNES appended to the function name. Unless using the SUNNONLINSOL_PETCSNES module as a standalone nonlinear solver the generic functions defined in sections 11.1.1 – 11.1.3 should be called in favor of the SUNNONLINSOL_PETCSNES-specific implementations.

The SUNNONLINSOL_PETCSNES module also defines the following additional user-callable functions.

**SUNNonlinSolGetSNES_PetscSNES**

Call

```
retval = SUNNonlinSolGetSNES_PetscSNES(NLS, SNES* snes);
```

Description The function SUNNonlinSolGetSNES_PetscSNES gets the SNES context that was wrapped.

Arguments

- `NLS` (SUNNonlinearSolver) a SUNNONLINSOL object
- `snes` (SNES*) a pointer to a PETSc SNES object that will be set upon return

Return value The return value `retval` (of type int) should be zero for a successful call, and a negative value for a failure.

**SUNNonlinSolGetPetscErrorCode_PetscSNES**

Call

```
retval = SUNNonlinSolGetPetscErrorCode_PetscSNES(NLS, PestcErrorCode* error);
```

Description The function SUNNonlinSolGetPetscErrorCode_PetscSNES gets the last error code returned by the last internal call to a PETSc API function.

Arguments

- `NLS` (SUNNonlinearSolver) a SUNNONLINSOL object
- `error` (PestcErrorCode*) a pointer to a PETSc error integer that will be set upon return

Return value The return value `retval` (of type int) should be zero for a successful call, and a negative value for a failure.

**SUNNonlinSolGetSysFn_PetscSNES**

Call

```
retval = SUNNonlinSolGetSysFn_PetscSNES(NLS, SysFn);
```

Description The function SUNNonlinSolGetSysFn_PetscSNES returns the residual function that defines the nonlinear system.

Arguments

- `NLS` (SUNNonlinearSolver) a SUNNONLINSOL object
- `SysFn` (SUNNonlinSolSysFn*) the function defining the nonlinear system

Return value The return value `retval` (of type int) should be zero for a successful call, and a negative value for a failure.
11.4.3 SUNNonlinearSolver_PetscSNES content

The SUNNONLINSOL_PETSCSNES module defines the *content* field of a SUNNonlinearSolver as the following structure:

```c
struct _SUNNonlinearSolverContent_PetscSNES {
    int sysfn_last_err;
    PetscErrorCode petsc_last_err;
    long int nconvfails;
    long int nni;
    void *imem;
    SNES snes;
    Vec r;
    N_Vector y, f;
    SUNNonlinSolSysFn Sys;
};
```

These entries of the *content* field contain the following information:
- `sysfn_last_err` - last error returned by the system defining function,
- `petsc_last_err` - last error returned by PETSc
- `nconvfails` - number of nonlinear converge failures (recoverable or not),
- `nni` - number of nonlinear iterations,
- `imem` - Sundials integrator memory,
- `snes` - PETSc SNES context,
- `r` - the nonlinear residual,
- `y` - wrapper for PETSc vectors used in the system function,
- `f` - wrapper for PETSc vectors used in the system function,
- `Sys` - nonlinear system defining function.
Appendix A

SUNDIALS Package Installation Procedure

The installation of any SUNDIALS package is accomplished by installing the SUNDIALS suite as a whole, according to the instructions that follow. The same procedure applies whether or not the downloaded file contains one or all solvers in SUNDIALS.

The SUNDIALS suite (or individual solvers) are distributed as compressed archives (.tar.gz). The name of the distribution archive is of the form solver-x.y.z.tar.gz, where solver is one of: sundials, cvode, cvodes, arkode, ida, idas, or kinsol, and x.y.z represents the version number (of the SUNDIALS suite or of the individual solver). To begin the installation, first uncompress and expand the sources, by issuing

```
% tar xzf solver-x.y.z.tar.gz
```

This will extract source files under a directory solver-x.y.z.

Starting with version 2.6.0 of SUNDIALS, CMake is the only supported method of installation. The explanations of the installation procedure begins with a few common observations:

- The remainder of this chapter will follow these conventions:
  - solverdir is the directory solver-x.y.z created above; i.e., the directory containing the SUNDIALS sources.
  - builddir is the (temporary) directory under which SUNDIALS is built.
  - instdir is the directory under which the SUNDIALS exported header files and libraries will be installed. Typically, header files are exported under a directory instdir/include while libraries are installed under instdir/CMAKE_INSTALL_LIBDIR, with instdir and CMAKE_INSTALL_LIBDIR specified at configuration time.
  - For SUNDIALS CMake-based installation, in-source builds are prohibited; in other words, the build directory builddir can not be the same as solverdir and such an attempt will lead to an error. This prevents “polluting” the source tree and allows efficient builds for different configurations and/or options.
  - The installation directory instdir can not be the same as the source directory solverdir.
  - By default, only the libraries and header files are exported to the installation directory instdir. If enabled by the user (with the appropriate toggle for CMake), the examples distributed with SUNDIALS will be built together with the solver libraries but the installation step will result in exporting (by default in a subdirectory of the installation directory) the example sources and sample outputs together with automatically generated configuration files that reference the installed SUNDIALS headers and libraries. As such, these configuration files for the SUNDIALS examples can be used as “templates” for your own problems. CMake installs CMakeLists.txt files
and also (as an option available only under Unix/Linux) Makefile files. Note this installation approach also allows the option of building the SUNDIALS examples without having to install them. (This can be used as a sanity check for the freshly built libraries.)

- Even if generation of shared libraries is enabled, only static libraries are created for the FCMIX modules. (Because of the use of fixed names for the Fortran user-provided subroutines, FCMIX shared libraries would result in “undefined symbol” errors at link time.)

### A.1 CMake-based installation

CMake-based installation provides a platform-independent build system. CMake can generate Unix and Linux Makefiles, as well as KDevelop, Visual Studio, and (Apple) XCode project files from the same configuration file. In addition, CMake also provides a GUI front end and which allows an interactive build and installation process.

The SUNDIALS build process requires CMake version 3.1.3 or higher and a working C compiler. On Unix-like operating systems, it also requires Make (and curses, including its development libraries, for the GUI front end to CMake, \texttt{ccmake}), while on Windows it requires Visual Studio. CMake is continually adding new features, and the latest version can be downloaded from \url{http://www.cmake.org}. Build instructions for CMake (only necessary for Unix-like systems) can be found on the CMake website. Once CMake is installed, Linux/Unix users will be able to use \texttt{ccmake}, while Windows users will be able to use \texttt{CMakeSetup}.

As previously noted, when using CMake to configure, build and install SUNDIALS, it is always required to use a separate build directory. While in-source builds are possible, they are explicitly prohibited by the SUNDIALS CMake scripts (one of the reasons being that, unlike autotools, CMake does not provide a \texttt{make distclean} procedure and it is therefore difficult to clean-up the source tree after an in-source build). By ensuring a separate build directory, it is an easy task for the user to clean-up all traces of the build by simply removing the build directory. CMake does generate a \texttt{make clean} which will remove files generated by the compiler and linker.

#### A.1.1 Configuring, building, and installing on Unix-like systems

The default CMake configuration will build all included solvers and associated examples and will build static and shared libraries. The \texttt{instdir} defaults to /usr/local and can be changed by setting the \texttt{CMAKE_INSTALL_PREFIX} variable. Support for FORTRAN and all other options are disabled.

CMake can be used from the command line with the \texttt{cmake} command, or from a curses-based GUI by using the \texttt{ccmake} command. Examples for using both methods will be presented. For the examples shown it is assumed that there is a top level SUNDIALS directory with appropriate source, build and install directories:

```
% mkdir (...)sundials/instdir
% mkdir (...)sundials/builddir
% cd (...)sundials/builddir
```

**Building with the GUI**

Using CMake with the GUI follows this general process:

- Select and modify values, run configure (c key)
- New values are denoted with an asterisk
- To set a variable, move the cursor to the variable and press enter
  - If it is a boolean (ON/OFF) it will toggle the value
  - If it is string or file, it will allow editing of the string
– For file and directories, the <tab> key can be used to complete

• Repeat until all values are set as desired and the generate option is available (g key)

• Some variables (advanced variables) are not visible right away

• To see advanced variables, toggle to advanced mode (t key)

• To search for a variable press / key, and to repeat the search, press the n key

To build the default configuration using the GUI, from the builddir enter the ccmake command and point to the solverdir:

% ccmake ../solverdir

The default configuration screen is shown in Figure A.1.

The default instdir for both sundials and corresponding examples can be changed by setting the 
CMAKE_INSTALL_PREFIX and the EXAMPLES_INSTALL_PATH as shown in figure A.2.

Pressing the (g key) will generate makefiles including all dependencies and all rules to build sundials on this system. Back at the command prompt, you can now run:
Figure A.2: Changing the *instdir* for SUNDIALS and corresponding *examples*.

% make

To install SUNDIALS in the installation directory specified in the configuration, simply run:

% make install

### Building from the command line

Using CMake from the command line is simply a matter of specifying CMake variable settings with the *cmake* command. The following will build the default configuration:

```
% cmake -DCMAKE_INSTALL_PREFIX=/home/myname/sundials/instdir \
> -DEXAMPLES_INSTALL_PATH=/home/myname/sundials/instdir/examples \
> ../solverdir
% make
% make install
```

### A.1.2 Configuration options (Unix/Linux)

A complete list of all available options for a CMake-based SUNDIALS configuration is provided below. Note that the default values shown are for a typical configuration on a Linux system and are provided as illustration only.
A.1 CMake-based installation

BUILD_ARCODE - Build the ARKODE library
Default: ON

BUILD_CVODE - Build the CVODE library
Default: ON

BUILD_CVODES - Build the CVODES library
Default: ON

BUILD_IDA - Build the IDA library
Default: ON

BUILD_IDAS - Build the IDAS library
Default: ON

BUILD_KINSOL - Build the KINSOL library
Default: ON

BUILD_SHARED_LIBS - Build shared libraries
Default: ON

BUILD_STATIC_LIBS - Build static libraries
Default: ON

CMAKE_BUILD_TYPE - Choose the type of build, options are: None (CMAKE_C_FLAGS used), Debug, Release, RelWithDebInfo, and MinSizeRel
Default:
Note: Specifying a build type will trigger the corresponding build type specific compiler flag options below which will be appended to the flags set by CMAKE_<language>_FLAGS.

CMAKE_C_COMPILER - C compiler
Default: /usr/bin/cc

CMAKE_C_FLAGS - Flags for C compiler
Default:

CMAKE_C_FLAGS_DEBUG - Flags used by the C compiler during debug builds
Default: -g

CMAKE_C_FLAGS_MINSIZEREL - Flags used by the C compiler during release minsize builds
Default: -Os -DNDEBUG

CMAKE_C_FLAGS_RELEASE - Flags used by the C compiler during release builds
Default: -O3 -DNDEBUG

CMAKE_CXX_COMPILER - C++ compiler
Default: /usr/bin/c++
Note: A C++ compiler (and all related options) are only triggered if C++ examples are enabled (EXAMPLES_ENABLE_CXX is ON). All sundials solvers can be used from C++ applications by default without setting any additional configuration options.

CMAKE_CXX_FLAGS - Flags for C++ compiler
Default:

CMAKE_CXX_FLAGS_DEBUG - Flags used by the C++ compiler during debug builds
Default: -g

CMAKE_CXX_FLAGS_MINSIZEREL - Flags used by the C++ compiler during release minsize builds
Default: -Os -DNDEBUG
CMAKE_CXX_FLAGS_RELEASE - Flags used by the C++ compiler during release builds
   Default: -O3 -DNDEBUG

CMAKE_Fortran_COMPILER - Fortran compiler
   Default: /usr/bin/gfortran
   Note: Fortran support (and all related options) are triggered only if either Fortran-C support is enabled (FCMIX_ENABLE is ON) or LAPACK support is enabled (LAPACK_ENABLE is ON).

CMAKE_Fortran_FLAGS - Flags for Fortran compiler
   Default:

CMAKE_Fortran_FLAGS_DEBUG - Flags used by the Fortran compiler during debug builds
   Default: -g

CMAKE_Fortran_FLAGS_MINSIZEREL - Flags used by the Fortran compiler during release minsize builds
   Default: -Os

CMAKE_Fortran_FLAGS_RELEASE - Flags used by the Fortran compiler during release builds
   Default: -O3

CMAKE_INSTALL_PREFIX - Install path prefix, prepended onto install directories
   Default: /usr/local
   Note: The user must have write access to the location specified through this option. Exported SUNDIALS header files and libraries will be installed under subdirectories include and CMAKE_INSTALL_LIBDIR of CMAKE_INSTALL_PREFIX, respectively.

CMAKE_INSTALL_LIBDIR - Library installation directory
   Default:
   Note: This is the directory within CMAKE_INSTALL_PREFIX that the SUNDIALS libraries will be installed under. The default is automatically set based on the operating system using the GNUInstallDirs CMake module.

Fortran_INSTALL_MODDIR - Fortran module installation directory
   Default: fortran

CUDA_ENABLE - Build the SUNDIALS CUDA modules.
   Default: OFF

CUDA_ARCH - Specifies the CUDA architecture to compile for.
   Default: sm_30

EXAMPLES_ENABLE_C - Build the SUNDIALS C examples
   Default: ON

EXAMPLES_ENABLE_CUDA - Build the SUNDIALS CUDA examples
   Default: OFF
   Note: You need to enable CUDA support to build these examples.

EXAMPLES_ENABLE_CXX - Build the SUNDIALS C++ examples
   Default: OFF unless Trilinos_ENABLE is ON.

EXAMPLES_ENABLE_F77 - Build the SUNDIALS Fortran77 examples
   Default: ON (if F77_INTERFACE_ENABLE is ON)

EXAMPLES_ENABLE_F90 - Build the SUNDIALS Fortran90 examples
   Default: ON (if F77_INTERFACE_ENABLE is ON)

EXAMPLES_ENABLE_F2003 - Build the SUNDIALS Fortran2003 examples
   Default: ON (if F2003_INTERFACE_ENABLE is ON)
EXAMPLES_INSTALL - Install example files
   Default: ON
   Note: This option is triggered when any of the SUNDIALS example programs are enabled
   (EXAMPLES_ENABLE_<language> is ON). If the user requires installation of example programs
   then the sources and sample output files for all SUNDIALS modules that are currently enabled
   will be exported to the directory specified by EXAMPLES_INSTALL_PATH. A CMake configuration
   script will also be automatically generated and exported to the same directory. Additionally, if
   the configuration is done under a Unix-like system, makefiles for the compilation of the example
   programs (using the installed SUNDIALS libraries) will be automatically generated and exported
   to the directory specified by EXAMPLES_INSTALL_PATH.

EXAMPLES_INSTALL_PATH - Output directory for installing example files
   Default: /usr/local/examples
   Note: The actual default value for this option will be an examples subdirectory created under
   CMAKE_INSTALL_PREFIX.

F77_INTERFACE_ENABLE - Enable Fortran-C support via the Fortran 77 interfaces
   Default: OFF

F2003_INTERFACE_ENABLE - Enable Fortran-C support via the Fortran 2003 interfaces
   Default: OFF

HYPRE_ENABLE - Enable hypre support
   Default: OFF
   Note: See additional information on building with hypre enabled in A.1.4.

HYPRE_INCLUDE_DIR - Path to hypre header files

HYPRE_LIBRARY_DIR - Path to hypre installed library files

KLU_ENABLE - Enable KLU support
   Default: OFF
   Note: See additional information on building with KLU enabled in A.1.4.

KLU_INCLUDE_DIR - Path to SuiteSparse header files

KLU_LIBRARY_DIR - Path to SuiteSparse installed library files

LAPACK_ENABLE - Enable LAPACK support
   Default: OFF
   Note: Setting this option to ON will trigger additional CMake options. See additional informa-
   tion on building with LAPACK enabled in A.1.4.

LAPACK_LIBRARIES - LAPACK (and BLAS) libraries
   Default: /usr/lib/liblapack.so;/usr/lib/libblas.so
   Note: CMake will search for libraries in your LD_LIBRARY_PATH prior to searching default system
   paths.

MPI_ENABLE - Enable MPI support. This will build the parallel NVVECTOR and the MPI-aware version
   of the ManyVector library.
   Default: OFF
   Note: Setting this option to ON will trigger several additional options related to MPI.

MPI_C_COMPILER - mpicc program
   Default:

MPI_CXX_COMPILER - mpicxx program
   Default:
Note: This option is triggered only if MPI is enabled (MPI_ENABLE is ON) and C++ examples are enabled (EXAMPLES_ENABLE_CXX is ON). All SUNDIALS solvers can be used from C++ MPI applications by default without setting any additional configuration options other than MPI_ENABLE.

MPI_Fortran_COMPILER - mpif77 or mpif90 program
Default: 
Note: This option is triggered only if MPI is enabled (MPI_ENABLE is ON) and Fortran-C support is enabled (F77_INTERFACE_ENABLE or F2003_INTERFACE_ENABLE is ON).

MPIEXEC_EXECUTABLE - Specify the executable for running MPI programs
Default: mpirun
Note: This option is triggered only if MPI is enabled (MPI_ENABLE is ON).

OPENMP_ENABLE - Enable OpenMP support (build the OpenMP nvector).
Default: OFF

OPENMP_DEVICE_ENABLE - Enable OpenMP device offloading (build the OpenMPDEV nvector) if supported by the provided compiler.
Default: OFF

SKIP_OPENMP_DEVICE_CHECK - advanced option - Skip the check done to see if the OpenMP provided by the compiler supports OpenMP device offloading.
Default: OFF

PETSC_ENABLE - Enable PETSc support
Default: OFF
Note: See additional information on building with PETSc enabled in ??.

PETSC_DIR - Path to PETSc installation
Default: 

PETSC_LIBRARIES - advanced option - Semi-colon separated list of PETSc link libraries. Unless provided by the user, this is autopopulated based on the PETSc installation found in PETSC_DIR.
Default: 

PETSC_INCLUDES - advanced option - Semi-colon separated list of PETSc include directories. Unless provided by the user, this is autopopulated based on the PETSc installation found in PETSC_DIR.
Default: 

PTHREAD_ENABLE - Enable Pthreads support (build the Pthreads nvector).
Default: OFF

RAJA_ENABLE - Enable RAJA support (build the RAJA nvector).
Default: OFF
Note: You need to enable CUDA in order to build the RAJA vector module.

SUNDIALS_F77_FUNC_CASE - advanced option - Specify the case to use in the Fortran name-mangling scheme, options are: lower or upper
Default: 
Note: The build system will attempt to infer the Fortran name-mangling scheme using the Fortran compiler. This option should only be used if a Fortran compiler is not available or to override the inferred or default (lower) scheme if one can not be determined. If used, SUNDIALS_F77_FUNC_UNDERSCORES must also be set.

SUNDIALS_F77_FUNC_UNDERSCORES - advanced option - Specify the number of underscores to append in the Fortran name-mangling scheme, options are: none, one, or two
Default: 
Note: The build system will attempt to infer the Fortran name-mangling scheme using the Fortran compiler. This option should only be used if a Fortran compiler is not available.
or to override the inferred or default (one) scheme if one can not be determined. If used, \texttt{SUNDIALS\_F77\_FUNC\_CASE} must also be set.

\texttt{SUNDIALS\_INDEX\_TYPE} - \textbf{advanced option} - Integer type used for \texttt{SUNDIALS} indices. The size must match the size provided for the \texttt{SUNDIALS\_INDEX\_SIZE} option.

\textbf{Default:}

Note: In past \texttt{SUNDIALS} versions, a user could set this option to \texttt{INT64\_T} to use 64-bit integers, or \texttt{INT32\_T} to use 32-bit integers. Starting in \texttt{SUNDIALS} 3.2.0, these special values are deprecated. For \texttt{SUNDIALS} 3.2.0 and up, a user will only need to use the \texttt{SUNDIALS\_INDEX\_SIZE} option in most cases.

\texttt{SUNDIALS\_INDEX\_SIZE} - Integer size (in bits) used for indices in \texttt{SUNDIALS}, options are: 32 or 64

\textbf{Default:} 64

Note: The build system tries to find an integer type of appropriate size. Candidate 64-bit integer types are (in order of preference): \texttt{int64\_t}, \texttt{\_int64}, \texttt{long long}, and \texttt{long}. Candidate 32-bit integers are (in order of preference): \texttt{int32\_t}, \texttt{int}, and \texttt{long}. The advanced option, \texttt{SUNDIALS\_INDEX\_TYPE} can be used to provide a type not listed here.

\texttt{SUNDIALS\_PRECISION} - Precision used in \texttt{SUNDIALS}, options are: \texttt{double}, \texttt{single}, or \texttt{extended}

\textbf{Default:} \texttt{double}

\texttt{SUPERLUDIST\_ENABLE} - Enable SuperLU\_DIST support

\textbf{Default:} OFF

Note: See additional information on building with SuperLU\_DIST enabled in \texttt{A.1.4}.

\texttt{SUPERLUDIST\_INCLUDE\_DIR} - Path to SuperLU\_DIST header files (typically SRC directory)

\texttt{SUPERLUDIST\_LIBRARY\_DIR} - Path to SuperLU\_DIST installed library files

\texttt{SUPERLUDIST\_LIBRARIES} - Semi-colon separated list of libraries needed for SuperLU\_DIST

\texttt{SUPERLUDIST\_OpenMP} - Enable \texttt{SUNDIALS} support for SuperLU\_DIST built with OpenMP

\textbf{Default:} OFF

Note: SuperLU\_DIST must be built with OpenMP support for this option to function properly. Additionally the environment variable \texttt{OMP\_NUM\_THREADS} must be set to the desired number of threads.

\texttt{SUPERLUMT\_ENABLE} - Enable SuperLU\_MT support

\textbf{Default:} OFF

Note: See additional information on building with SuperLU\_MT enabled in \texttt{A.1.4}.

\texttt{SUPERLUMT\_INCLUDE\_DIR} - Path to SuperLU\_MT header files (typically SRC directory)

\texttt{SUPERLUMT\_LIBRARY\_DIR} - Path to SuperLU\_MT installed library files

\texttt{SUPERLUMT\_LIBRARIES} - Semi-colon separated list of libraries needed for SuperLU\_MT

\texttt{SUPERLUMT\_THREAD\_TYPE} - Must be set to Pthread or OpenMP

\textbf{Default:} Pthread

\texttt{Trilinos\_ENABLE} - Enable Trilinos support (build the Tpetra NVVECTOR).

\textbf{Default:} OFF

\texttt{Trilinos\_DIR} - Path to the Trilinos install directory.

\textbf{Default:}
TRILINOS_INTERFACE_C_COMPILER - advanced option - Set the C compiler for building the Trilinos interface (i.e., nvector_trilinos and the examples that use it).
Default: The C compiler exported from the found Trilinos installation if USE_XSDK_DEFAULTS=OFF.
CMAKE_C_COMPILER or MPI_C_COMPILER if USE_XSDK_DEFAULTS=ON.
Note: It is recommended to use the same compiler that was used to build the Trilinos library.

TRILINOS_INTERFACE_C_COMPILER_FLAGS - advanced option - Set the C compiler flags for Trilinos interface (i.e., nvector_trilinos and the examples that use it).
Default: The C compiler flags exported from the found Trilinos installation if USE_XSDK_DEFAULTS=OFF.
CMAKE_C_FLAGS if USE_XSDK_DEFAULTS=ON.
Note: It is recommended to use the same flags that were used to build the Trilinos library.

TRILINOS_INTERFACE_CXX_COMPILER - advanced option - Set the C++ compiler for building Trilinos interface (i.e., nvector_trilinos and the examples that use it).
Default: The C++ compiler exported from the found Trilinos installation if USE_XSDK_DEFAULTS=OFF.
CMAKE_CXX_COMPILER or MPI_CXX_COMPILER if USE_XSDK_DEFAULTS=ON.
Note: It is recommended to use the same compiler that was used to build the Trilinos library.

TRILINOS_INTERFACE_CXX_COMPILER_FLAGS - advanced option - Set the C++ compiler flags for Trilinos interface (i.e., nvector_trilinos and the examples that use it).
Default: The C++ compiler flags exported from the found Trilinos installation if USE_XSDK_DEFAULTS=OFF.
CMAKE_CXX_FLAGS if USE_XSDK_DEFAULTS=ON.
Note: It is recommended to use the same flags that were used to build the Trilinos library.

USE_GENERIC_MATH - Use generic (stdc) math libraries
Default: ON

xSDK Configuration Options

SUNDIALS supports CMake configuration options defined by the Extreme-scale Scientific Software Development Kit (xSDK) community policies (see https://xsdk.info for more information). xSDK CMake options are unused by default but may be activated by setting USE_XSDK_DEFAULTS to ON.

When xSDK options are active, they will overwrite the corresponding SUNDIALS option and may have different default values (see details below). As such the equivalent SUNDIALS options should not be used when configuring with xSDK options. In the GUI front end to CMake (ccmake), setting USE_XSDK_DEFAULTS to ON will hide the corresponding SUNDIALS options as advanced CMake variables. During configuration, messages are output detailing which xSDK flags are active and the equivalent SUNDIALS options that are replaced. Below is a complete list xSDK options and the corresponding SUNDIALS options if applicable.

TPL_ENABLE_HYPRE - Enable hypre support
Default: OFF
SUNDIALS equivalent: HYPRE_ENABLE

TPL_ENABLE_KLU - Enable KLU support
Default: OFF
SUNDIALS equivalent: KLU_ENABLE

TPL_ENABLE_PETSC - Enable PETSc support
Default: OFF
SUNDIALS equivalent: PETSC_ENABLE

TPL_ENABLE_LAPACK - Enable LAPACK support
Default: OFF
SUNDIALS equivalent: LAPACK_ENABLE
A.1 CMake-based installation

TPL_ENABLE_SUPERLUDIST - Enable SuperLU_DIST support
  Default: OFF
  SUNDIALS equivalent: SUPERLUDIST_ENABLE

TPL_ENABLE_SUPERLUMT - Enable SuperLU_MT support
  Default: OFF
  SUNDIALS equivalent: SUPERLUMT_ENABLE

TPL_HYPRE_INCLUDE_DIRS - Path to hypre header files
  SUNDIALS equivalent: HYPRE_INCLUDE_DIR

TPL_HYPRE_LIBRARIES - hypre library
  SUNDIALS equivalent: N/A

TPL_KLU_INCLUDE_DIRS - Path to KLU header files
  SUNDIALS equivalent: KLU_INCLUDE_DIR

TPL_KLU_LIBRARIES - KLU library
  SUNDIALS equivalent: N/A

TPL_LAPACK_LIBRARIES - LAPACK (and BLAS) libraries
  Default: /usr/lib/liblapack.so:/usr/lib/libblas.so
  SUNDIALS equivalent: LAPACK_LIBRARIES
  Note: CMake will search for libraries in your LD_LIBRARY_PATH prior to searching default system paths.

TPL_PETSC_DIR - Path to PETSc installation
  SUNDIALS equivalent: PETSC_DIR

TPL_SUPERLUDIST_INCLUDE_DIRS - Path to SuperLU_DIST header files
  SUNDIALS equivalent: SUPERLUDIST_INCLUDE_DIR

TPL_SUPERLUDIST_LIBRARIES - Semi-colon separated list of libraries needed for SuperLU_DIST including the SuperLU_DIST library itself
  SUNDIALS equivalent: SUPERLUDIST_LIBRARIES

TPL_SUPERLUDIST_OPENMP - Enable SUNDIALS support for SuperLU_DIST built with OpenMP
  SUNDIALS equivalent: SUPERLUDIST_OPENMP

TPL_SUPERLUMT_LIBRARIES - SuperLU_MT library
  SUNDIALS equivalent: N/A

TPL_SUPERLUMT_THREAD_TYPE - SuperLU_MT library thread type
  SUNDIALS equivalent: SUPERLUMT_THREAD_TYPE

USE_XSDK_DEFAULTS - Enable xSDK default configuration settings
  Default: OFF
  SUNDIALS equivalent: N/A
  Note: Enabling xSDK defaults also sets CMAKE_BUILD_TYPE to Debug

XSDK_ENABLE_FORTRAN - Enable SUNDIALS Fortran interfaces
  Default: OFF
  SUNDIALS equivalent: F77_INTERFACE_ENABLE/F2003_INTERFACE_ENABLE

XSDK_INDEX_SIZE - Integer size (bits) used for indices in SUNDIALS, options are: 32 or 64
  Default: 32
  SUNDIALS equivalent: SUNDIALS_INDEX_SIZE

XSDK_PRECISION - Precision used in SUNDIALS, options are: double, single, or quad
  Default: double
  SUNDIALS equivalent: SUNDIALS_PRECISION
A.1.3 Configuration examples

The following examples will help demonstrate usage of the CMake configure options.

To configure sundials using the default C and Fortran compilers, and default mpicc and mpif77 parallel compilers, enable compilation of examples, and install libraries, headers, and example sources under subdirectories of /home/myname/sundials/, use:

```bash
% cmake
> -DCMAKE_INSTALL_PREFIX=/home/myname/sundials/instdir
> -DEXAMPLES_INSTALL_PATH=/home/myname/sundials/instdir/examples
> -DMPI_ENABLE=ON
> -DFCMIX_ENABLE=ON
> /home/myname/sundials/solverdir
%
% make install
%
```

To disable installation of the examples, use:

```bash
% cmake
> -DCMAKE_INSTALL_PREFIX=/home/myname/sundials/instdir
> -DEXAMPLES_INSTALL_PATH=/home/myname/sundials/instdir/examples
> -DMPI_ENABLE=ON
> -DFCMIX_ENABLE=ON
> -DEXAMPLES_INSTALL=OFF
> /home/myname/sundials/solverdir
%
% make install
%
```

A.1.4 Working with external Libraries

The sundials suite contains many options to enable implementation flexibility when developing solutions. The following are some notes addressing specific configurations when using the supported third party libraries. When building sundials as a shared library any external libraries used with sundials must also be build as a shared library or as a static library compiled with the -fPIC flag.

Building with LAPACK

To enable LAPACK, set the LAPACK_ENABLE option to ON. If the directory containing the LAPACK library is in the LD_LIBRARY_PATH environment variable, CMake will set the LAPACK_LIBRARIES variable accordingly, otherwise CMake will attempt to find the LAPACK library in standard system locations. To explicitly tell CMake what library to use, the LAPACK_LIBRARIES variable can be set to the desired libraries required for LAPACK.

```bash
% cmake
> -DCMAKE_INSTALL_PREFIX=/home/myname/sundials/instdir
> -DEXAMPLES_INSTALL_PATH=/home/myname/sundials/instdir/examples
> -DLAPACK_ENABLE=ON
> -DLAPACK_LIBRARIES=/mypackpath/lib/libblas.so;/mypackpath/lib/liblapack.so
> /home/myname/sundials/solverdir
%
% make install
%
```
A.1 CMake-based installation

If a working Fortran compiler is not available to infer the Fortran name-mangling scheme, the options SUNDIALS_F77_FUNC_CASE and SUNDIALS_F77_FUNC_UNDERSCORES must be set in order to bypass the check for a Fortran compiler and define the name-mangling scheme. The defaults for these options in earlier versions of SUNDIALS were lower and one respectively.

Building with KLU

The KLU libraries are part of SuiteSparse, a suite of sparse matrix software, available from the Texas A&M University website: http://faculty.cse.tamu.edu/davis/suitesparse.html. SUNDIALS has been tested with SuiteSparse version 5.3.0. To enable KLU, set KLU_ENABLE to ON, set KLU_INCLUDE_DIR to the include path of the KLU installation and set KLU_LIBRARY_DIR to the lib path of the KLU installation. The CMake configure will result in populating the following variables: AMD_LIBRARY, AMD_LIBRARY_DIR, BTF_LIBRARY, BTF_LIBRARY_DIR, COLAMD_LIBRARY, COLAMD_LIBRARY_DIR, and KLU_LIBRARY.

Building with SuperLU_MT

The SuperLU_MT libraries are available for download from the Lawrence Berkeley National Laboratory website: http://crd-legacy.lbl.gov/~xiaoye/SuperLU/#superlu_mt. SUNDIALS has been tested with SuperLU_MT version 3.1. To enable SuperLU_MT, set SUPERLUMT_ENABLE to ON, set SUPERLUMT_INCLUDE_DIR to the SRC path of the SuperLU_MT installation, and set the variable SUPERLUMT_LIBRARY_DIR to the lib path of the SuperLU_MT installation. At the same time, the variable SUPERLUMT_LIBRARIES must be set to a semi-colon separated list of other libraries SuperLU_MT depends on. For example, if SuperLU_MT was build with an external blas library, then include the full path to the blas library in this list. Additionally, the variable SUPERLUMT_THREAD_TYPE must be set to either Pthread or OpenMP. Do not mix thread types when building SUNDIALS solvers. If threading is enabled for SUNDIALS by having either OPENMP_ENABLE or PTHREAD_ENABLE set to ON then SuperLU_MT should be set to use the same threading type.

Building with SuperLU_DIST

The SuperLU_DIST libraries are available for download from the Lawrence Berkeley National Laboratory website: http://crd-legacy.lbl.gov/~xiaoye/SuperLU/#superlu_dist. SUNDIALS has been tested with SuperLU_DIST 6.1.1. To enable SuperLU_DIST, set SUPERLUDIST_ENABLE to ON, set SUPERLUDIST_INCLUDE_DIR to the include directory of the SuperLU_DIST installation (typically SRC), and set the variable SUPERLUDIST_LIBRARY_DIR to the path to library directory of the SuperLU_DIST installation (typically 1ib). At the same time, the variable SUPERLUDIST_LIBRARIES must be set to a semi-colon separated list of other libraries SuperLU_DIST depends on. For example, if SuperLU_DIST was built with LAPACK, then include the LAPACK library in this list. If SuperLU_DIST was built with OpenMP support, then you may set SUPERLUDIST_OPENMP to ON to utilize the OpenMP functionality of SuperLU_DIST. Do not mix thread types when building SUNDIALS solvers. If threading is enabled for SUNDIALS by having PTHREAD_ENABLE set to ON then SuperLU_DIST should not be set to use OpenMP.

Building with PETSc

The PETSc libraries are available for download from the Argonne National Laboratory website: http://www.mcs.anl.gov/petsc. SUNDIALS has been tested with PETSc version 3.10.0–3.12.1. To enable PETSc, set PETSC_ENABLE to ON and then set PETSC_DIR to the path of the PETSc installation. Alternatively, a user can provide a list of include paths in PETSC_INCLUDES, and a list of complete paths to the libraries needed in PETSC_LIBRARIES.
Building with hypre

The hypre libraries are available for download from the Lawrence Livermore National Laboratory website: [http://computing.llnl.gov/projects/hypre](http://computing.llnl.gov/projects/hypre). SUNDIALS has been tested with hypre version 2.14.0–2.18.0. To enable hypre, set HYPRE_ENABLE to ON, set HYPRE_INCLUDE_DIR to the include path of the hypre installation, and set the variable HYPRE_LIBRARY_DIR to the lib path of the hypre installation.

Note: SUNDIALS must be configured so that SUNDIALS_INDEX_SIZE (or equivalently, XSDK_INDEX_SIZE) equals the precision of HYPRE_BigInt in the corresponding hypre installation.

Building with CUDA

SUNDIALS CUDA modules and examples have been tested with versions 9 through 10.1 of the CUDA toolkit. To build them, you need to install the Toolkit and compatible NVIDIA drivers. Both are available for download from the NVIDIA website: [https://developer.nvidia.com/cuda-downloads](https://developer.nvidia.com/cuda-downloads). To enable CUDA, set CUDA_ENABLE to ON. If CUDA is installed in a nonstandard location, you may be prompted to set the variable CUDA_TOOLKIT_ROOT_DIR with your CUDA Toolkit installation path. To enable CUDA examples, set EXAMPLES_ENABLE_CUDA to ON.

Building with RAJA

RAJA is a performance portability layer developed by Lawrence Livermore National Laboratory and can be obtained from [https://github.com/LLNL/RAJA](https://github.com/LLNL/RAJA). SUNDIALS RAJA modules and examples have been tested with RAJA up to version 0.9. Building SUNDIALS RAJA modules requires a CUDA-enabled RAJA installation. To enable RAJA, set CUDA_ENABLE and RAJA_ENABLE to ON. If RAJA is installed in a nonstandard location you will be prompted to set the variable RAJA_DIR with the path to the RAJA CMake configuration file. To enable building the RAJA examples set EXAMPLES_ENABLE_CUDA to ON.

Building with Trilinos

Trilinos is a suite of numerical libraries developed by Sandia National Laboratories. It can be obtained at [https://github.com/trilinos/Trilinos](https://github.com/trilinos/Trilinos). SUNDIALS Trilinos modules and examples have been tested with Trilinos version 12.14.1. To enable Trilinos, set Trilinos_ENABLE to ON. If Trilinos is installed in a nonstandard location you will be prompted to set the variable Trilinos_DIR with the path to the Trilinos CMake configuration file. It is desireable to build the Trilinos vector interface with same compiler and options that were used to build Trilinos. CMake will try to find the correct compiler settings automatically from the Trilinos configuration file. If that is not successful, the compilers and options can be manually set with the following CMake variables:

- TrilinosINTERFACE_C_COMPILER
- TrilinosINTERFACE_C_COMPILER_FLAGS
- TrilinosINTERFACE_CXX_COMPILER
- TrilinosINTERFACE_CXX_COMPILER_FLAGS

A.1.5 Testing the build and installation

If SUNDIALS was configured with EXAMPLES_ENABLE_<language> options to ON, then a set of regression tests can be run after building with the make command by running:

```
% make test
```

Additionally, if EXAMPLES_INSTALL was also set to ON, then a set of smoke tests can be run after installing with the make install command by running:

```
% make test_install
```
A.2 Building and Running Examples

Each of the SUNDIALS solvers is distributed with a set of examples demonstrating basic usage. To build and install the examples, set at least one of the EXAMPLES_ENABLE_<language> options to ON, and set EXAMPLES_INSTALL to ON. Specify the installation path for the examples with the variable EXAMPLES_INSTALL_PATH. CMake will generate CMakeLists.txt configuration files (and Makefile files if on Linux/Unix) that reference the installed SUNDIALS headers and libraries.

Either the CMakeLists.txt file or the traditional Makefile may be used to build the examples as well as serve as a template for creating user developed solutions. To use the supplied Makefile simply run make to compile and generate the executables. To use CMake from within the installed example directory, run cmake (or ccmake to use the GUI) followed by make to compile the example code. Note that if CMake is used, it will overwrite the traditional Makefile with a new CMake-generated Makefile. The resulting output from running the examples can be compared with example output bundled in the SUNDIALS distribution.

NOTE: There will potentially be differences in the output due to machine architecture, compiler versions, use of third party libraries etc.

A.3 Configuring, building, and installing on Windows

CMake can also be used to build SUNDIALS on Windows. To build SUNDIALS for use with Visual Studio the following steps should be performed:

1. Unzip the downloaded tar file(s) into a directory. This will be the solverdir
2. Create a separate builddir
3. Open a Visual Studio Command Prompt and cd to builddir
4. Run cmake-gui ../solverdir
   (a) Hit Configure
   (b) Check/Uncheck solvers to be built
   (c) Change CMAKE_INSTALL_PREFIX to instdir
   (d) Set other options as desired
   (e) Hit Generate
5. Back in the VS Command Window:
   (a) Run msbuild ALL_BUILD.vcxproj
   (b) Run msbuild INSTALL.vcxproj

The resulting libraries will be in the instdir. The SUNDIALS project can also now be opened in Visual Studio. Double click on the ALL_BUILD.vcxproj file to open the project. Build the whole solution to create the SUNDIALS libraries. To use the SUNDIALS libraries in your own projects, you must set the include directories for your project, add the SUNDIALS libraries to your project solution, and set the SUNDIALS libraries as dependencies for your project.

A.4 Installed libraries and exported header files

Using the CMake SUNDIALS build system, the command

```
% make install
```
will install the libraries under $\texttt{libdir}$ and the public header files under $\texttt{includedir}$. The values for these directories are $\texttt{instdir/CMAKE\_INSTALL\_LIBDIR}$ and $\texttt{instdir/include}$, respectively. The location can be changed by setting the CMake variable $\texttt{CMAKE\_INSTALL\_PREFIX}$. Although all installed libraries reside under $\texttt{libdir/CMAKE\_INSTALL\_LIBDIR}$, the public header files are further organized into subdirectories under $\texttt{includedir/include}$.

The installed libraries and exported header files are listed for reference in Table A.1. The file extension $\texttt{.lib}$ is typically $\texttt{.so}$ for shared libraries and $\texttt{.a}$ for static libraries. Note that, in the Tables, names are relative to $\texttt{libdir}$ for libraries and to $\texttt{includedir}$ for header files.

A typical user program need not explicitly include any of the shared SUNDIALS header files from under the $\texttt{includedir/include/sundials}$ directory since they are explicitly included by the appropriate solver header files (e.g., $\texttt{cvode_dense.h}$ includes $\texttt{sundials_dense.h}$). However, it is both legal and safe to do so, and would be useful, for example, if the functions declared in $\texttt{sundials_dense.h}$ are to be used in building a preconditioner.
<table>
<thead>
<tr>
<th>Libraries</th>
<th>Header files</th>
<th>Module files</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>SHARED</strong></td>
<td>n/a</td>
<td>n/a</td>
</tr>
<tr>
<td>sundials/sundials_config.h</td>
<td>sundials/sundials_fconfig.h</td>
<td>sundials/sundials_types.h</td>
</tr>
<tr>
<td>sundials/sundials_math.h</td>
<td>sundials/sundials_nvector.h</td>
<td>sundials/sundials_fvector.h</td>
</tr>
<tr>
<td>sundials/sundials_matrix.h</td>
<td>sundials/sundials_linear_solver.h</td>
<td>sundials/sundials_iterative.h</td>
</tr>
<tr>
<td>sundials/sundials_direct.h</td>
<td>sundials/sundials_dense.h</td>
<td>sundials/sundials_band.h</td>
</tr>
<tr>
<td>sundials/sundials_nonlinear_solver.h</td>
<td>sundials/sundials_version.h</td>
<td>sundials/sundials_mpi_types.h</td>
</tr>
<tr>
<td><strong>NVECTOR_SERIAL</strong></td>
<td>lib sundials_nvector_serial.lib</td>
<td>nvector/nvector_serial.h</td>
</tr>
<tr>
<td>lib sundials_fnvector_serial.mod.lib</td>
<td>lib sundials_fnvector_serial.a</td>
<td>fnvector_serial_mod.mod</td>
</tr>
<tr>
<td><strong>NVECTOR_PARALLEL</strong></td>
<td>lib sundials_nvector_parallel.lib</td>
<td>nvector/nvector_parallel.h</td>
</tr>
<tr>
<td>lib sundials_fnvector_parallel.a</td>
<td>lib sundials_fnvector_parallel_mod.lib</td>
<td>fnvector_parallel_mod.mod</td>
</tr>
<tr>
<td><strong>NVECTOR_MANYVECTOR</strong></td>
<td>lib sundials_nvector_manyvector.lib</td>
<td>nvector/nvector_manyvector.h</td>
</tr>
<tr>
<td>lib sundials_nvector_manyvector_mod.lib</td>
<td>nvector_manyvector_mod.mod</td>
<td>fnvector_manyvector_mod.mod</td>
</tr>
<tr>
<td><strong>NVECTOR_MPI MANYVECTOR</strong></td>
<td>lib sundials_nvector_mpi_manyvector.lib</td>
<td>nvector/nvector_mpi_manyvector.h</td>
</tr>
<tr>
<td>lib sundials_nvector_mpi_manyvector_mod.lib</td>
<td>nvector_mpi_manyvector_mod.mod</td>
<td>fnvector_mpi_manyvector_mod.mod</td>
</tr>
<tr>
<td><strong>NVECTOR_MPI PLUS X</strong></td>
<td>lib sundials_nvector_mpi_plus_x.lib</td>
<td>nvector/nvector_mpi_plus_x.h</td>
</tr>
<tr>
<td>lib sundials_nvector_mpi_plus_x_mod.lib</td>
<td>nvector_mpi_plus_x_mod.mod</td>
<td><strong>continued on next page</strong></td>
</tr>
</tbody>
</table>
### Header files
- `nvector/nvector_mpiplusx.h`  
- `mpiplusx.h`  

### Module files
- `fnvector_mpiplusx.mod`  

### Libraries
- `libsundials_nvecopenmp.lib`  
- `libsundials_nvecopenmp_mod.lib`  
- `libsundials_nvecopenmp.a`  

### Header files
- `nvector/nvector_openmp.h`  

### Module files
- `fnvector_openmp.mod`  

### Libraries
- `libsundials_nvecopenmpdev.lib`  

### Header files
- `nvector/nvector_openmpdev.h`  

### Module files
- `fnvector_openmpdev.mod`  

### Libraries
- `libsundials_nvec pthreads.lib`  
- `libsundials_nvec pthreads_mod.lib`  
- `libsundials_nvec pthreads.a`  

### Header files
- `nvector/nvector pthreads.h`  

### Module files
- `fnvector pthreads.mod`  

### Libraries
- `libsundials_nvec parhyp.lib`  

### Header files
- `nvector/nvector parhyp.h`  

### Libraries
- `libsundials_nvec petsc.lib`  

### Header files
- `nvector/nvector petsc.h`  

### Libraries
- `libsundials_nvec cuda.lib`  

### Header files
- `nvector/nvector cuda.h`  
- `nvector/cuda/ThreadPartitioning.hpp`  
- `nvector/cuda/Vector.hpp`  
- `nvector/cuda/VectorKernels.cuh`  

### Libraries
- `libsundials_nvec raja.lib`  

### Header files
- `nvector/nvector raja.h`  
- `nvector/raja/Vector.hpp`  

### Libraries
- `libsundials_nvec trilinos.lib`  

### Header files
- `nvector/nvector trilinos.h`  
- `nvector/trilinos/SundialsTpetraVectorInterface.hpp`  
- `nvector/trilinos/SundialsTpetraVectorKernels.hpp`  

### Libraries
- `libsundials_sunmatrix band.lib`  
- `libsundials_fsunmatrix band_mod.lib`  
- `libsundials_fsunmatrix band.a`  

### Header files
- `sunmatrix/sunmatrix_band.h`  

### Module files
- `fsunmatrix_band.mod`  

### Libraries
- `libsundials_sunmatrix dense.lib`  
- `libsundials_fsunmatrix dense_mod.lib`  
- `libsundials_fsunmatrix dense.a`  

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*continued on next page*
A.4 Installed libraries and exported header files

<table>
<thead>
<tr>
<th>Libraries</th>
<th>File Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>libsundials, sunlinsol_sparse.lib</td>
<td>libsunlinsol_sparse.a</td>
</tr>
<tr>
<td>libsundials, sunlinsoljson.lib</td>
<td>libsunlinsoljson.a</td>
</tr>
<tr>
<td>libsundials, sunlinsoljson_mod.lib</td>
<td>libsunlinsoljson_mod.a</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Header files</th>
<th>File Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>sunlinsol/sunlinsol_sparse.h</td>
<td>sunlinsol/sunlinsoljson.h</td>
</tr>
<tr>
<td>sunlinsol/sunlinsolSparse_mod.mod</td>
<td>sunlinsol/sunlinsoljson_mod.mod</td>
</tr>
</tbody>
</table>

continued on next page
### SUNLINSOLSPBCGS
- **Libraries**: `libsundials_fsunlinsolspbcs_mod.lib`
- `libsundials_fsunlinsolspbcs.a`
- **Header files**: `sunlinsol/sunlinsol_spbcs.h`
- **Module files**: `fsunlinsol_spbcs_mod.mod`

### SUNLINSOLSPFGMR
- **Libraries**: `libsundials_sunlinsolspgmr.lib`
- `libsundials_fsunlinsolspgmr_mod.lib`
- `libsundials_fsunlinsolspgmr.a`
- **Header files**: `sunlinsol/sunlinsol_spgmr.h`
- **Module files**: `fsunlinsol_spgmr_mod.mod`

### SUNLINSOLSPTFQMR
- **Libraries**: `libsundials_sunlinsolsptfqmr.lib`
- `libsundials_fsunlinsolsptfqmr_mod.lib`
- `libsundials_fsunlinsolsptfqmr.a`
- **Header files**: `sunlinsol/sunlinsol_sptfqmr.h`
- **Module files**: `fsunlinsol_sptfqmr_mod.mod`

### SUNLINSOLSUPERLUMT
- **Libraries**: `libsundials_sunlinsolsuperlumt.lib`
- `libsundials_fsunlinsolsuperlumt.a`
- **Header files**: `sunlinsol/sunlinsol_superlumt.h`

### SUNLINSOLSUPERLUDIST
- **Libraries**: `libsundials_sunlinsolsuperludist.lib`
- **Header files**: `sunlinsol/sunlinsol_superludist.h`

### SUNLINSOLCUSOLVERSP_BATCHQR
- **Libraries**: `libsundials_sunlinsolcusolverp_batchqr.lib`
- **Header files**: `sunlinsol/sunlinsolcusolverp_batchqr.h`

### SUNNONLINSONLNEWTON
- **Libraries**: `libsundials_sunnonsolinlholnewton.lib`
- `libsundials_fsunnonlimsolinlholnewton_mod.lib`
- `libsundials_fsunnonlimsolinlholnewton.a`
- **Header files**: `sunnonlinsonl/sunnonlinsonl_newton.h`
- **Module files**: `fsunnonlinsonl_newton_mod.mod`

### SUNNONLINSONLFIXEDPOINT
- **Libraries**: `libsundials_sunnonsolinlfixedpoint.lib`
- `libsundials_fsunnonlimsolinlfixedpoint_mod.lib`
- `libsundials_fsunnonlimsolinlfixedpoint.a`
- **Header files**: `sunnonlinsonl/sunnonlinsonl_fixedpoint.h`
- **Module files**: `fsunnonlinsonl_fixedpoint_mod.mod`

*continued on next page*
### Installed libraries and exported header files

<table>
<thead>
<tr>
<th>SUNNONLINSOL_PETSCSNES</th>
<th>Libraries</th>
<th>Header files</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>libsundials_sunnonsol_petscsnes.lib</td>
<td>sunnonlinsol/sunnonsol_petscsnes.h</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>CVODE</th>
<th>Libraries</th>
<th>Header files</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>libsundials_cvode.lib</td>
<td>cvode/cvode.h</td>
</tr>
<tr>
<td></td>
<td>libsundials_fcvode.a</td>
<td>cvode/cvode_direct.h</td>
</tr>
<tr>
<td></td>
<td>libsundials_fcvode_mod.lib</td>
<td>cvode/cvode_splis.h</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>CVODES</th>
<th>Libraries</th>
<th>Header files</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>libsundials_cvodes.lib</td>
<td>cvodes/cvodes.h</td>
</tr>
<tr>
<td></td>
<td>libsundials_fcvodes_mod.lib</td>
<td>cvodes/cvodes_direct.h</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ARKODE</th>
<th>Libraries</th>
<th>Header files</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>libsundials_arkode.lib</td>
<td>arkode/arkode.h</td>
</tr>
<tr>
<td></td>
<td>libsundials_farkode.a</td>
<td>arkode/arkode_direct.h</td>
</tr>
<tr>
<td></td>
<td>libsundials_farkode_mod.lib</td>
<td>arkode/arkode_splis.h</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>IDA</th>
<th>Libraries</th>
<th>Header files</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>libsundials_ida.lib</td>
<td>ida/ida.h</td>
</tr>
<tr>
<td></td>
<td>libsundials_fida.a</td>
<td>ida/ida_direct.h</td>
</tr>
<tr>
<td></td>
<td>libsundials_fida_mod.lib</td>
<td>ida/ida_splis.h</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>IDAS</th>
<th>Libraries</th>
<th>Header files</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>libsundials_idas.lib</td>
<td>idas/idas.h</td>
</tr>
<tr>
<td></td>
<td>libsundials_fidas_mod.lib</td>
<td>idas/idas_direct.h</td>
</tr>
</tbody>
</table>

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*continued from last page*

### Module files

<table>
<thead>
<tr>
<th>SUNNONLINSOL_PETSCSNES</th>
<th>Module files</th>
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<tbody>
<tr>
<td></td>
<td>fcvode_mod.mod</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>CVODE</th>
<th>Module files</th>
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<tr>
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<td>fcvode_mod.mod</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>CVODES</th>
<th>Module files</th>
</tr>
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<tbody>
<tr>
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<td>fcvodes_mod.mod</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>ARKODE</th>
<th>Module files</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>farkode_mod.mod</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>IDA</th>
<th>Module files</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>fida_mod.mod</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>IDAS</th>
<th>Module files</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>fidas_mod.mod</td>
</tr>
</tbody>
</table>

*continued on next page*
### KINSOL

<table>
<thead>
<tr>
<th>Libraries</th>
<th><code>libsundials_kinsol.lib</code></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><code>libsundials_fkinsol.a</code></td>
</tr>
<tr>
<td></td>
<td><code>libsundials_fkinsol_mod.lib</code></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Header files</th>
<th><code>kinsol/kinsol.h</code></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><code>kinsol/kinsol_impl.h</code></td>
</tr>
<tr>
<td></td>
<td><code>kinsol/kinsol_direct.h</code></td>
</tr>
<tr>
<td></td>
<td><code>kinsol/kinsol_ls.h</code></td>
</tr>
<tr>
<td></td>
<td><code>kinsol/kinsol_spils.h</code></td>
</tr>
<tr>
<td></td>
<td><code>kinsol/kinsol_bbdpre.h</code></td>
</tr>
</tbody>
</table>

| Module files    | `fkinsol_mod.mod`        |
Appendix B

IDAS Constants

Below we list all input and output constants used by the main solver and linear solver modules, together with their numerical values and a short description of their meaning.

### B.1 IDAS input constants

<table>
<thead>
<tr>
<th>IDAS main solver module</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>IDA_NORMAL</td>
<td>1 Solver returns at specified output time.</td>
</tr>
<tr>
<td>IDA_ONE_STEP</td>
<td>2 Solver returns after each successful step.</td>
</tr>
<tr>
<td>IDA_SIMULTANEOUS</td>
<td>1 Simultaneous corrector forward sensitivity method.</td>
</tr>
<tr>
<td>IDA_STAGGERED</td>
<td>2 Staggered corrector forward sensitivity method.</td>
</tr>
<tr>
<td>IDA_CENTERED</td>
<td>1 Central difference quotient approximation (2\textsuperscript{nd} order) of the sensitivity RHS.</td>
</tr>
<tr>
<td>IDA_FORWARD</td>
<td>2 Forward difference quotient approximation (1\textsuperscript{st} order) of the sensitivity RHS.</td>
</tr>
<tr>
<td>IDA_YA_YDP_INIT</td>
<td>1 Compute $y_a$ and $\dot{y}_d$, given $y_d$.</td>
</tr>
<tr>
<td>IDA_Y_INIT</td>
<td>2 Compute $y$, given $\dot{y}$.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>IDAS adjoint solver module</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>IDA_HERMITE</td>
<td>1 Use Hermite interpolation.</td>
</tr>
<tr>
<td>IDA_POLYNOMIAL</td>
<td>2 Use variable-degree polynomial interpolation.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Iterative linear solver module</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>PREC_NONE</td>
<td>0 No preconditioning</td>
</tr>
<tr>
<td>PREC_LEFT</td>
<td>1 Preconditioning on the left.</td>
</tr>
<tr>
<td>MODIFIED_GS</td>
<td>1 Use modified Gram-Schmidt procedure.</td>
</tr>
<tr>
<td>CLASSICAL_GS</td>
<td>2 Use classical Gram-Schmidt procedure.</td>
</tr>
</tbody>
</table>

### B.2 IDAS output constants

<table>
<thead>
<tr>
<th>IDAS main solver module</th>
<th>Description</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>IDA Constant</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>IDA_SUCCESS</td>
<td>0</td>
<td>Successful function return.</td>
</tr>
<tr>
<td>IDA_TSTOP_RETURN</td>
<td>1</td>
<td>IDASolve succeeded by reaching the specified stopping point.</td>
</tr>
<tr>
<td>IDA_ROOT_RETURN</td>
<td>2</td>
<td>IDASolve succeeded and found one or more roots.</td>
</tr>
<tr>
<td>IDA_WARNING</td>
<td>99</td>
<td>IDASolve succeeded but an unusual situation occurred.</td>
</tr>
<tr>
<td>IDA_TOO_MUCH_WORK</td>
<td>-1</td>
<td>The solver took \texttt{mxstep} internal steps but could not reach \texttt{tout}.</td>
</tr>
<tr>
<td>IDA_TOO_MUCH_ACC</td>
<td>-2</td>
<td>The solver could not satisfy the accuracy demanded by the user for some internal step.</td>
</tr>
<tr>
<td>IDA_ERR_FAIL</td>
<td>-3</td>
<td>Error test failures occurred too many times during one internal time step or minimum step size was reached.</td>
</tr>
<tr>
<td>IDA_CONV_FAIL</td>
<td>-4</td>
<td>Convergence test failures occurred too many times during one internal time step or minimum step size was reached.</td>
</tr>
<tr>
<td>IDA_LINIT_FAIL</td>
<td>-5</td>
<td>The linear solver’s initialization function failed.</td>
</tr>
<tr>
<td>IDA_LSETUP_FAIL</td>
<td>-6</td>
<td>The linear solver’s setup function failed in an unrecoverable manner.</td>
</tr>
<tr>
<td>IDA_LSOLVE_FAIL</td>
<td>-7</td>
<td>The linear solver’s solve function failed in an unrecoverable manner.</td>
</tr>
<tr>
<td>IDA_RES_FAIL</td>
<td>-8</td>
<td>The user-provided residual function failed in an unrecoverable manner.</td>
</tr>
<tr>
<td>IDA_REP_RES_FAIL</td>
<td>-9</td>
<td>The user-provided residual function repeatedly returned a recoverable error flag, but the solver was unable to recover.</td>
</tr>
<tr>
<td>IDA_RTFUNC_FAIL</td>
<td>-10</td>
<td>The rootfinding function failed in an unrecoverable manner.</td>
</tr>
<tr>
<td>IDA_CONSTR_FAIL</td>
<td>-11</td>
<td>The inequality constraints were violated and the solver was unable to recover.</td>
</tr>
<tr>
<td>IDA_FIRST_RES_FAIL</td>
<td>-12</td>
<td>The user-provided residual function failed recoverably on the first call.</td>
</tr>
<tr>
<td>IDA_LINESearch_FAIL</td>
<td>-13</td>
<td>The line search failed.</td>
</tr>
<tr>
<td>IDA_NO_RECOVERY</td>
<td>-14</td>
<td>The residual function, linear solver setup function, or linear solver solve function had a recoverable failure, but IDACalcIC could not recover.</td>
</tr>
<tr>
<td>IDA_NLS_INIT_FAIL</td>
<td>-15</td>
<td>The nonlinear solver’s init routine failed.</td>
</tr>
<tr>
<td>IDA_NLS_SETUP_FAIL</td>
<td>-16</td>
<td>The nonlinear solver’s setup routine failed.</td>
</tr>
<tr>
<td>IDA_MEM_NULL</td>
<td>-20</td>
<td>The \texttt{ida_mem} argument was \texttt{NULL}.</td>
</tr>
<tr>
<td>IDA_MEM_FAIL</td>
<td>-21</td>
<td>A memory allocation failed.</td>
</tr>
<tr>
<td>IDA_IILL_INPUT</td>
<td>-22</td>
<td>One of the function inputs is illegal.</td>
</tr>
<tr>
<td>IDA_NO_MALLOC</td>
<td>-23</td>
<td>The IDAS memory was not allocated by a call to IDAInit.</td>
</tr>
<tr>
<td>IDA_BAD_EWT</td>
<td>-24</td>
<td>Zero value of some error weight component.</td>
</tr>
<tr>
<td>IDA_BAD_K</td>
<td>-25</td>
<td>The ( k )-th derivative is not available.</td>
</tr>
<tr>
<td>IDA_BAD_T</td>
<td>-26</td>
<td>The time ( t ) is outside the last step taken.</td>
</tr>
<tr>
<td>IDA_BAD_DKY</td>
<td>-27</td>
<td>The vector argument where derivative should be stored is \texttt{NULL}.</td>
</tr>
<tr>
<td>IDA_NO_QUAD</td>
<td>-30</td>
<td>Quadratures were not initialized.</td>
</tr>
<tr>
<td>IDA_QRHS_FAIL</td>
<td>-31</td>
<td>The user-provided right-hand side function for quadratures failed in an unrecoverable manner.</td>
</tr>
<tr>
<td>IDA_FIRST_QRHS_ERR</td>
<td>-32</td>
<td>The user-provided right-hand side function for quadratures failed in an unrecoverable manner on the first call.</td>
</tr>
</tbody>
</table>
### B.2 IDAS output constants

<table>
<thead>
<tr>
<th>Constant</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>IDA_REP_QRHS_ERR</td>
<td>-33</td>
<td>The user-provided right-hand side repeatedly returned a recoverable error flag, but the solver was unable to recover.</td>
</tr>
<tr>
<td>IDA_NO_SENS</td>
<td>-40</td>
<td>Sensitivities were not initialized.</td>
</tr>
<tr>
<td>IDA_SRES_FAIL</td>
<td>-41</td>
<td>The user-provided sensitivity residual function failed in an unrecoverable manner.</td>
</tr>
<tr>
<td>IDA_REP_SRES_ERR</td>
<td>-42</td>
<td>The user-provided sensitivity residual function repeatedly returned a recoverable error flag, but the solver was unable to recover.</td>
</tr>
<tr>
<td>IDA_BAD_IS</td>
<td>-43</td>
<td>The sensitivity identifier is not valid.</td>
</tr>
<tr>
<td>IDA_NO_QUADSENS</td>
<td>-50</td>
<td>Sensitivity-dependent quadratures were not initialized.</td>
</tr>
<tr>
<td>IDA_QSRHS_FAIL</td>
<td>-51</td>
<td>The user-provided sensitivity-dependent quadrature right-hand side function failed in an unrecoverable manner.</td>
</tr>
<tr>
<td>IDA_FIRST_QSRHS_ERR</td>
<td>-52</td>
<td>The user-provided sensitivity-dependent quadrature right-hand side function failed in an unrecoverable manner on the first call.</td>
</tr>
<tr>
<td>IDA_REP_QSRHS_ERR</td>
<td>-53</td>
<td>The user-provided sensitivity-dependent quadrature right-hand side repeatedly returned a recoverable error flag, but the solver was unable to recover.</td>
</tr>
</tbody>
</table>

**IDAS adjoint solver module**

<table>
<thead>
<tr>
<th>Constant</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>IDA_NO_ADJ</td>
<td>-101</td>
<td>The combined forward-backward problem has not been initialized.</td>
</tr>
<tr>
<td>IDA_NO_FWD</td>
<td>-102</td>
<td>IDASolveF has not been previously called.</td>
</tr>
<tr>
<td>IDA_NO_BCK</td>
<td>-103</td>
<td>No backward problem was specified.</td>
</tr>
<tr>
<td>IDA_BAD_TBO</td>
<td>-104</td>
<td>The desired output for backward problem is outside the interval over which the forward problem was solved.</td>
</tr>
<tr>
<td>IDA_REIFWD_FAIL</td>
<td>-105</td>
<td>No checkpoint is available for this hot start.</td>
</tr>
<tr>
<td>IDA_FWD_FAIL</td>
<td>-106</td>
<td>IDASolveB failed because IDASolve was unable to store data between two consecutive checkpoints.</td>
</tr>
<tr>
<td>IDA_GETY_BADT</td>
<td>-107</td>
<td>Wrong time in interpolation function.</td>
</tr>
</tbody>
</table>

**IDALS linear solver interface**

<table>
<thead>
<tr>
<th>Constant</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>IDALS_SUCCESS</td>
<td>0</td>
<td>Successful function return.</td>
</tr>
<tr>
<td>IDALS_MEM_NULL</td>
<td>-1</td>
<td>The ida_mem argument was NULL.</td>
</tr>
<tr>
<td>IDALS_LMEM_NULL</td>
<td>-2</td>
<td>The IDALS linear solver has not been initialized.</td>
</tr>
<tr>
<td>IDALS_IILL_INPUT</td>
<td>-3</td>
<td>The IDALS solver is not compatible with the current NVECTOR module, or an input value was illegal.</td>
</tr>
<tr>
<td>IDALS_MEM_FAIL</td>
<td>-4</td>
<td>A memory allocation request failed.</td>
</tr>
<tr>
<td>IDALS_PMEM_NULL</td>
<td>-5</td>
<td>The preconditioner module has not been initialized.</td>
</tr>
<tr>
<td>IDALS_JACFUNC_UNRECVR</td>
<td>-6</td>
<td>The Jacobian function failed in an unrecoverable manner.</td>
</tr>
<tr>
<td>IDALS_JACFUNC_RECVR</td>
<td>-7</td>
<td>The Jacobian function had a recoverable error.</td>
</tr>
<tr>
<td>IDALS_SUNMAT_FAIL</td>
<td>-8</td>
<td>An error occurred with the current SUNMATRIX module.</td>
</tr>
<tr>
<td>IDALS_SUNLS_FAIL</td>
<td>-9</td>
<td>An error occurred with the current SUNLINSOL module.</td>
</tr>
<tr>
<td>IDALS_NO_ADJ</td>
<td>-101</td>
<td>The combined forward-backward problem has not been initialized.</td>
</tr>
<tr>
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## Appendix C

### SUNDIALS Release History

Table C.1: Release History

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¹CVODE written, ²PVODE written, ³CVODE and PVODE combined, ⁴IDA written, ⁵KINSOL written
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